Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
Trigonometric Exam Skills [3.3]
Workbook

Outline:

Learning Objectives:

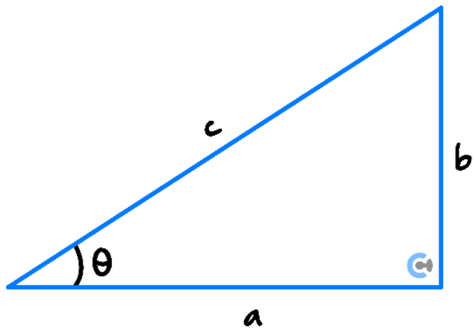
- ☐ **SM12 [3.3.1]** - Apply Trigonometry to Solve Problems in 3D
- ☐ **SM12 [3.3.2]** - Apply Trigonometry to Find the Angle between Planes

Section A: Recap

If you were here last week, skip to Section B – Warmup Test.

Definition

Trigonometric Ratios



$$\sin(\theta) = \frac{b}{c} = \frac{opposite}{hypotenuse}$$

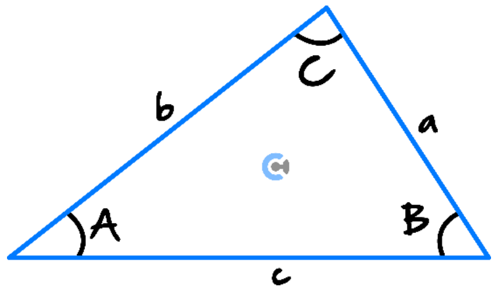
$$\cos(\theta) = \frac{a}{c} = \frac{adjacent}{hypotenuse}$$

$$\tan(\theta) = \frac{b}{a} = \frac{opposite}{adjacent}$$

Space for Personal Notes

<u>The Sine Rule</u>

- The sine rule states that for a triangle ABC:



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

<u>Application of Sine Rule</u>

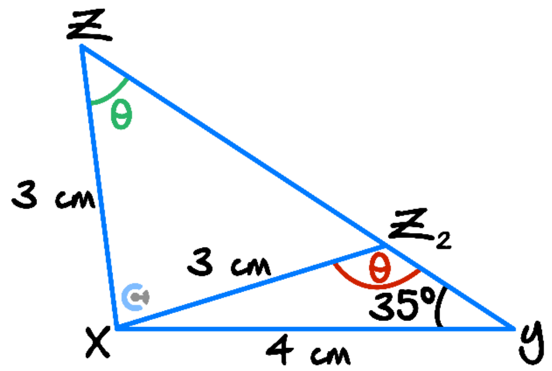
- We can use it to solve for lengths or angles within the triangle.
- CASE 1: One side and two angles are given.
  - In CASE 1, the triangle is uniquely defined up to congruence (AAS congruence test).
- CASE 2: Two sides and a non-included angle are given (the angle is not 'between' the two sides).
  - In CASE 2, there may be two possible triangles.

Space for Personal Notes

Question 1

Case 2: Two sides and a non-included angle are given.

Consider a triangle XYZ. Find the magnitude of angle Z in the triangle, given that $Y = 35°$, $XZ = 3\ cm$, and $XY = 4\ cm$. Give your answer in degrees, correct to two decimal places.

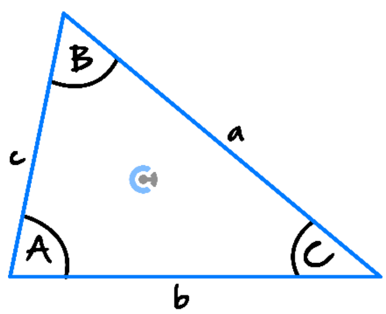


$$\boxed{\frac{4}{\sin(\theta)} = \frac{3}{\sin(35°)}}$$

$\theta = 49.89°$, $\theta = 130.11°$

The Cosine Rule

- The cosine rule states that for a triangle ABC:



$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Definition

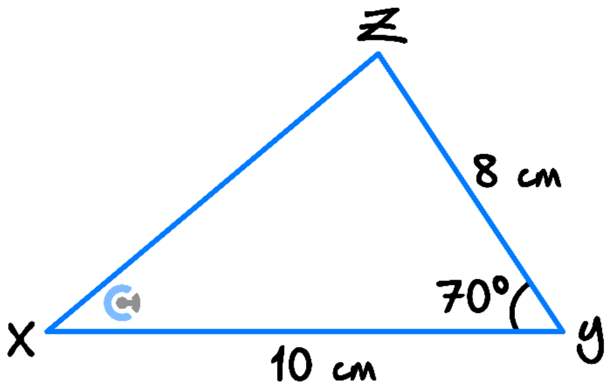
Application of Cosine Rule

- We can use it to solve for lengths or angles within the triangle.
  - Same as the sine rule in terms of the aim.
    - CASE 1: Three sides are given.
    - CASE 2: Two sides and the included angle are given (the angle **IS** between the two sides).
- In each case, the triangles are uniquely defined up to congruence.

Question 2

Case 2: Two sides and the included angle are given.

Find the length of XZ using the sine rule, in cm correct to two decimal places.

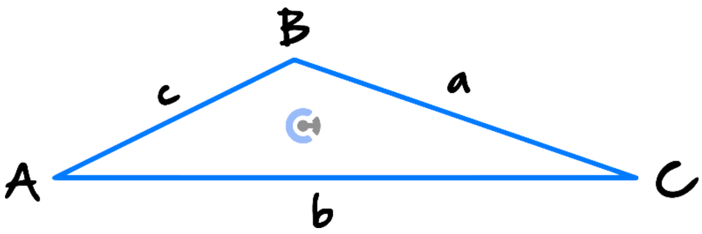



```
64 + 100 - 2 * 8 * 10 * Cos[70 Degree]
                        [코사인    [도

164 - 160 Sin[20 °]

√(164 - 160 Sin[20 °])  // N
                          [수치

10.4536
```

Definition

Area of a Triangle

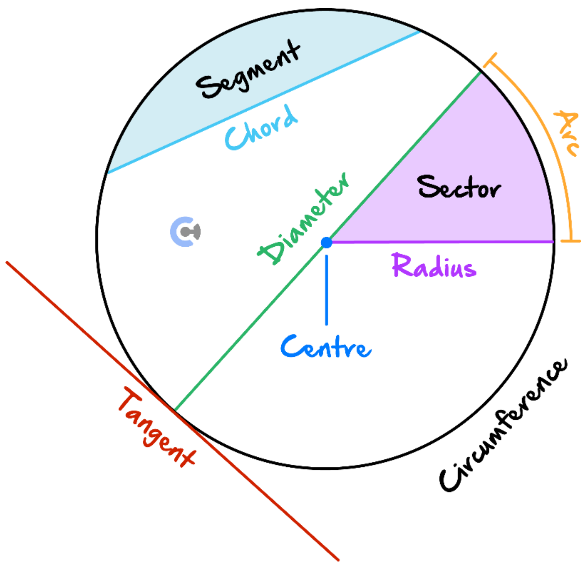
- In terms of two given sides, and the included angle:



$$\text{Area} = \frac{1}{2}bc\sin(A)$$

Definition

Mensuration



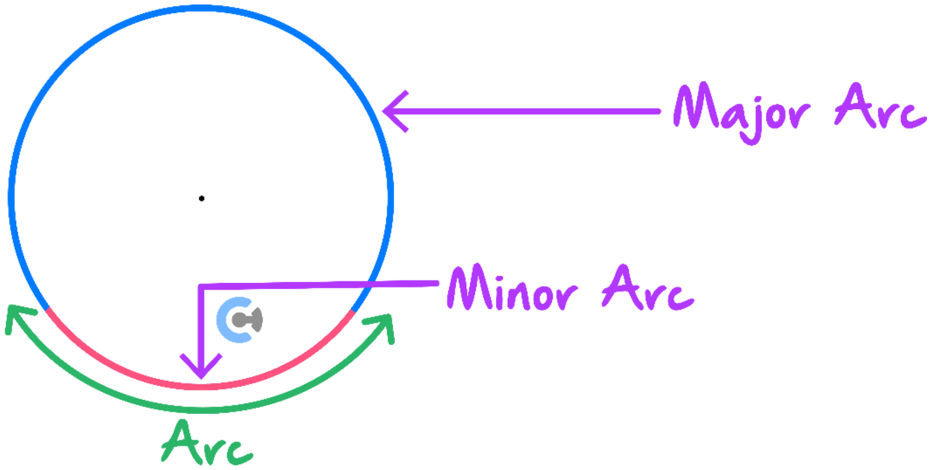
- Part of geometry concerned with finding lengths, areas, and volumes of shapes and objects.
- Circle mensuration is about finding the lengths and areas of different features on circles.

Space for Personal Notes

Definition

Key Terminology

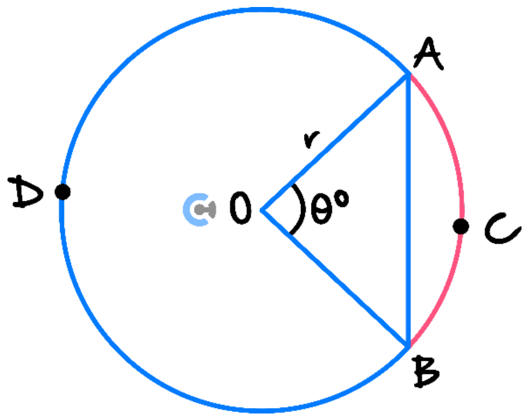
- In the diagram below, the circle has a centre.



- **Chord** = Line segment with endpoints on the circle.
    - Chord passing through the centre is called the diameter.
- **Arc** = Any curved part of the circle.
    - The shorter arc is called the **minor arc** and the longer is the **major arc.**
- **Segment** = Every chord divides the interior of a circle into two segments.
    - The smaller segment is called the **minor segment** and the larger is the **major segment**.
- **Sector** = Pizza slice. Two radii and an arc define a sector.
- **Tangent** = Line outside a circle that touches the circle exactly once (and does not pass through it).

Space for Personal Notes

Arc Length



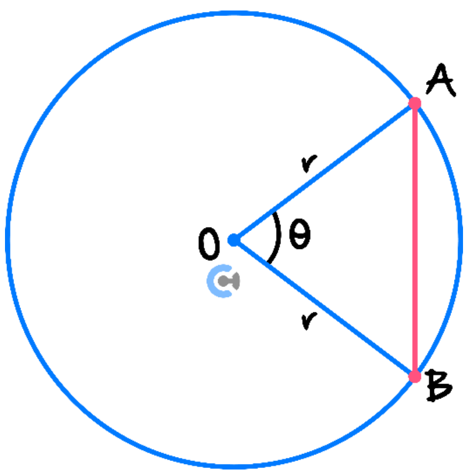
- The arc ACB and the corresponding chord AB are said to subtend the angle $\angle AOB$ at the centre of the circle.

$$l = 2\pi r \times \%$$

$$\text{Where, } \% = \frac{\theta^c}{2\pi} = \frac{\theta^\circ}{360}$$

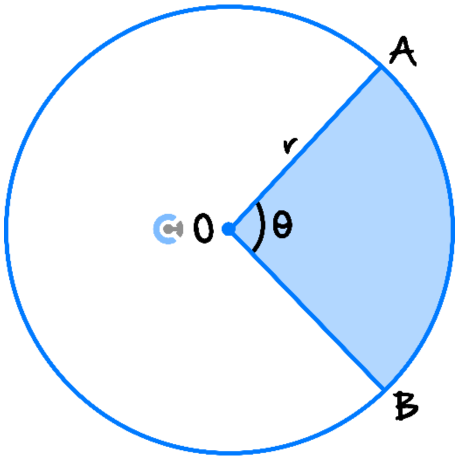
- We simply find the % of circumference.
  - % is defined by the angle θ divided by the entire rotation.

Chord Length



$$AB = 2r\sin\left(\frac{\theta}{2}\right)$$

Area of Sector



$$l = \pi r^2 \times \%$$

$$\text{Where, } \% = \frac{\theta^c}{2\pi} = \frac{\theta^\circ}{360}$$

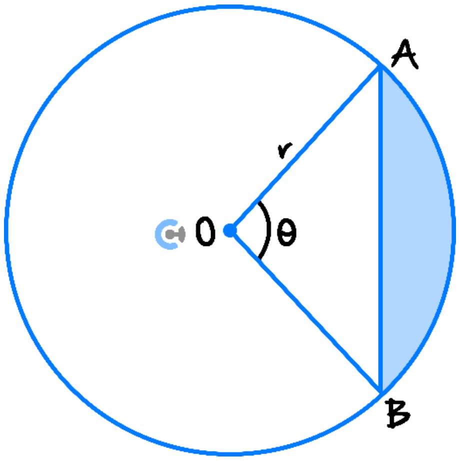
- We simply find the % of the circle area.
  - % is defined by the angle θ divided by the entire rotation.

$$\text{Using degrees: Area of sector} = \frac{\pi r^2 \theta^\circ}{360}$$

$$\text{Using radians: Area of sector} = \frac{1}{2} r^2 \theta^c$$

Space for Personal Notes

Area of Segment



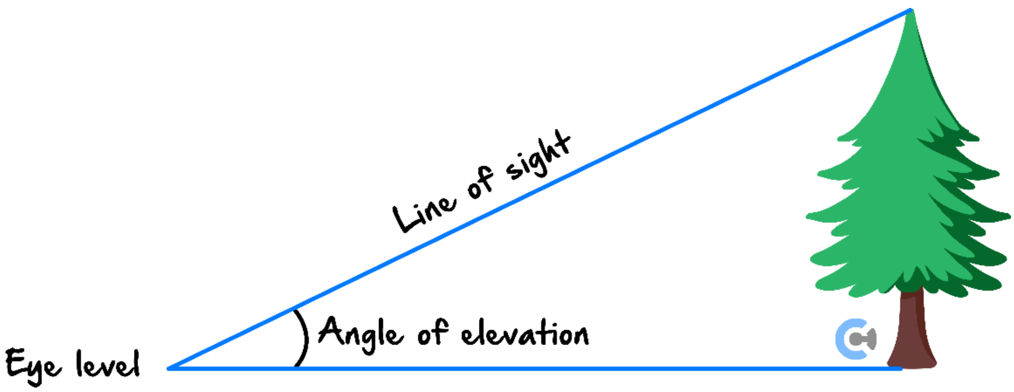
- The **area of the segment** is the area of the sector OAB minus the area of the triangle OAB.
- Using the area of a triangle formula, the area of triangle OAB is $\frac{1}{2}r^2\sin(\theta)$.

$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta) = \frac{1}{2}r^2(\theta - \sin(\theta))\ (\text{radians})$$

$$A = \left(\frac{\theta}{360}\right) \times (\pi r^2) - \frac{1}{2}r^2\sin(\theta)\ \ (\text{degrees})$$

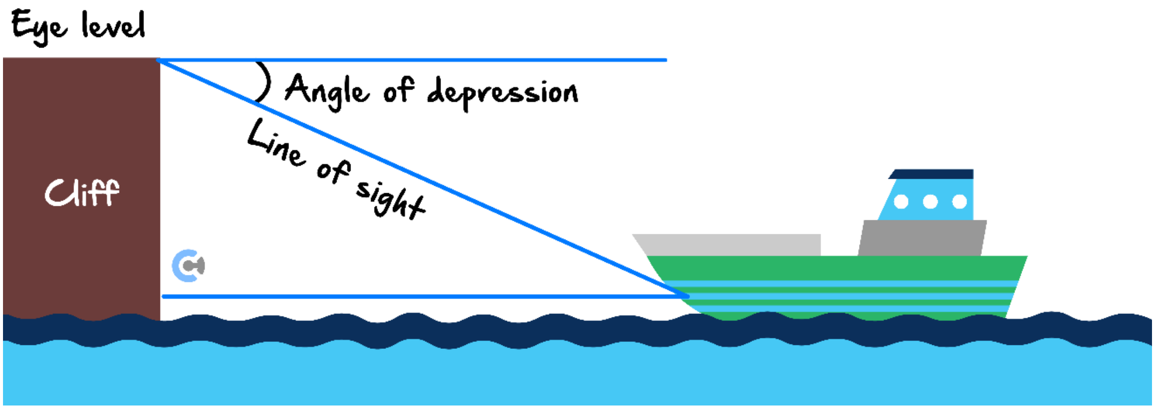
Angle of Elevation, Angle of Depression

- **Angle of Elevation**
  - The angle of elevation is the angle between the **horizontal** and a **direction** above the horizontal.



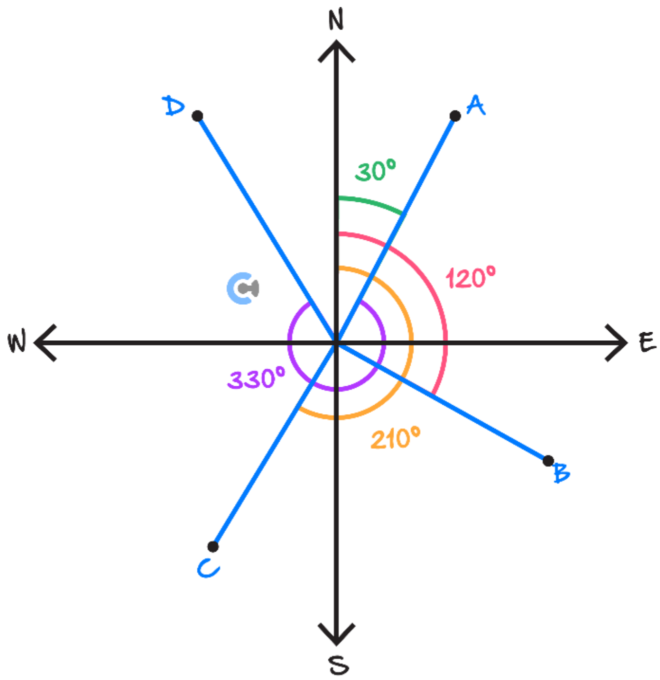
- **Angle of Depression**

  - The angle of depression is the angle between the **horizontal** and a **direction** below the horizontal.



Bearing

- The **bearing** is the angle measured from north **in the clockwise direction.**



Space for Personal Notes

Definition

Radians and Degrees

$$1^c = \left(\frac{180}{\pi}\right)^\circ$$

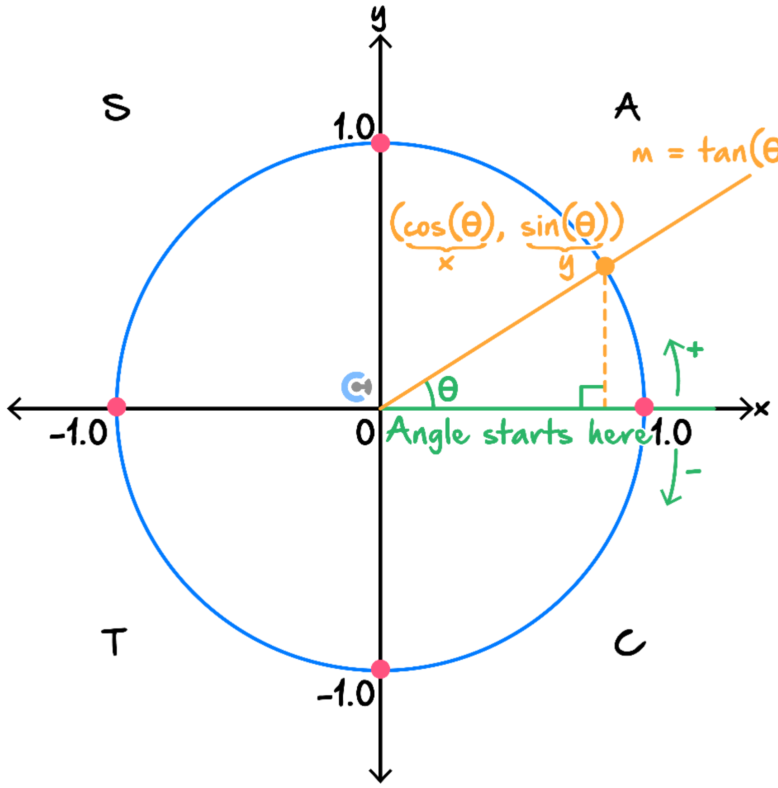
$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$180^\circ = \pi^c$$

Definition

Unit Circle

- The unit circle is simply a circle of radius 1.



$$\sin(\theta) = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \text{gradient}$$

Definition

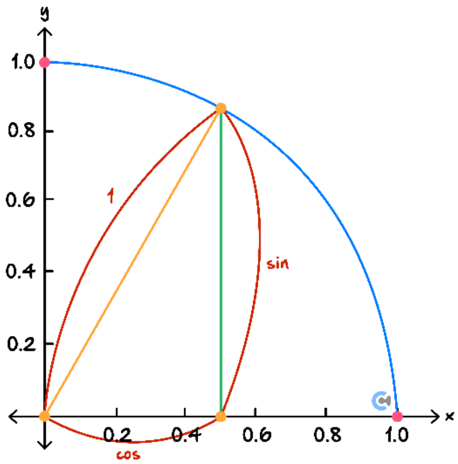
Period of a Trigonometric Function

$$\text{Period of } \sin(nx) \text{ and } \cos(nx) \text{ functions} = \frac{2\pi}{|n|}$$

$$\text{Period of } \tan(nx) \text{ functions} = \frac{\pi}{|n|}$$

where, n = coefficient of x.

Pythagorean Identities



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

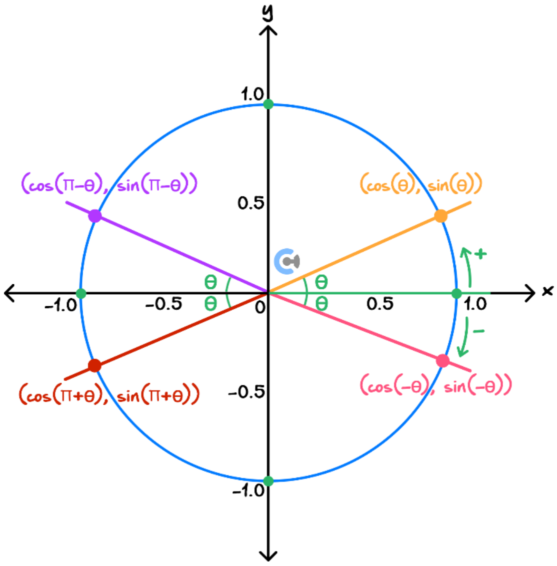
- Can be used for finding one trigonometry function by using the other.

The Exact Values Table

x	$0\ (0^\circ)$	$\frac{\pi}{6}\ (30^\circ)$	$\frac{\pi}{4}\ (45^\circ)$	$\frac{\pi}{3}\ (60^\circ)$	$\frac{\pi}{2}\ (90^\circ)$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Definition

Supplementary Relationships



- Simply look at the quadrant to find the correct sign.
  - Second Quadrant $(\pi - \theta)$:

$$\cos(\pi - \theta) = -\cos(\theta)$$

$$\sin(\pi - \theta) = +\sin(\theta)$$

$$\tan(\pi - \theta) = -\tan(\theta)$$

  - Third Quadrant $(\pi + \theta)$:

$$\cos(\pi + \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta)$$

$$\tan(\pi + \theta) = +\tan(\theta)$$

  - Fourth Quadrant $(-\theta)$:

$$\cos(-\theta) = +\cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

<u>Particular Solutions</u>

- Solving trigonometric equations **for finite solutions.**
- **Steps:**
  - Make the trigonometric function the subject.
  - Find the necessary angle for one period.
  - Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  - Add and subtract the period to find all other solutions in the domain.

Question 3

Solve the following equations for x over the domains specified.

a. $\sin\left(x - \frac{\pi}{2}\right) = -\frac{1}{2}$ for $x \in [-\pi, \pi]$.

```
Solve[Sin[x - π / 2] + 1 / 2 == 0 && -π ≤ x ≤ π, x]
|풀이 함수 |사인
{{x → -π/3}, {x → π/3}}
```

b. $2\cos\left(2x + \frac{\pi}{6}\right) + 1 = 0$ for $x \in [0, 2\pi]$.

```
Solve[2 Cos[2 x + π / 6] + 1 == 0 && 0 ≤ x ≤ 2 π, x]
|코사인
{{x → π/4}, {x → 7π/12}, {x → 5π/4}, {x → 19π/12}}
```

Definition

General Solutions

- Finding infinite solutions to a trigonometric equation.
- **Steps:**
  - Make the trigonometric function the subject.
  - Find the necessary angle for one period.
  - Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  - Add $\text{Period} \cdot n$ where $n \in Z$.

Question 4

Find the general solutions to the following equation:

$$2\sin\left(-2x+\frac{\pi}{4}\right)=\sqrt{2}$$

```
Solve[2 Sin[-2 x + π / 4] == √2 , x] // Expand
풀이 함수  사인  확장
```

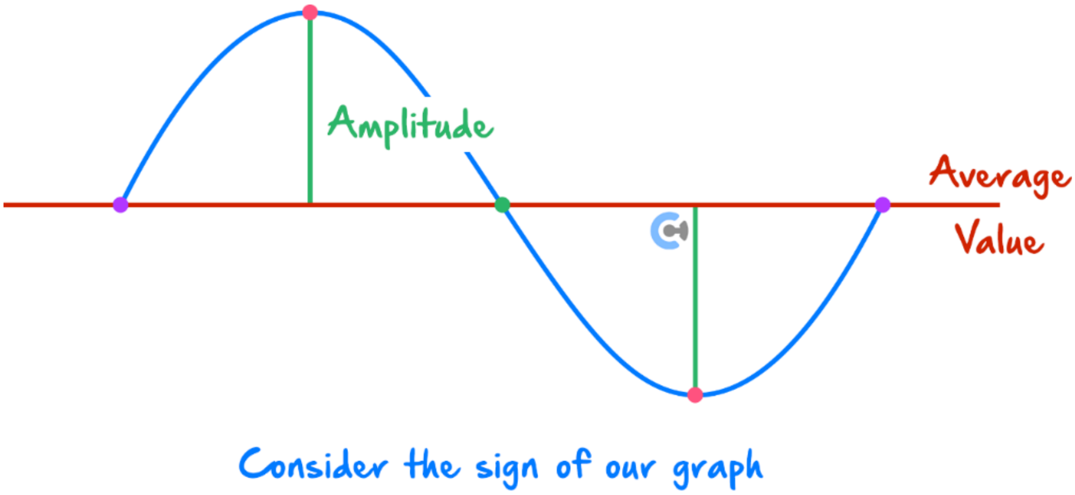
$$\left\{\left\{x \to -\pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\}, \left\{x \to -\frac{\pi}{4} - \pi c_1 \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$$

Space for Personal Notes

Definition

Amplitude, Period, and Average Value

- For $y = A\sin/\cos(nx + b) + k$,

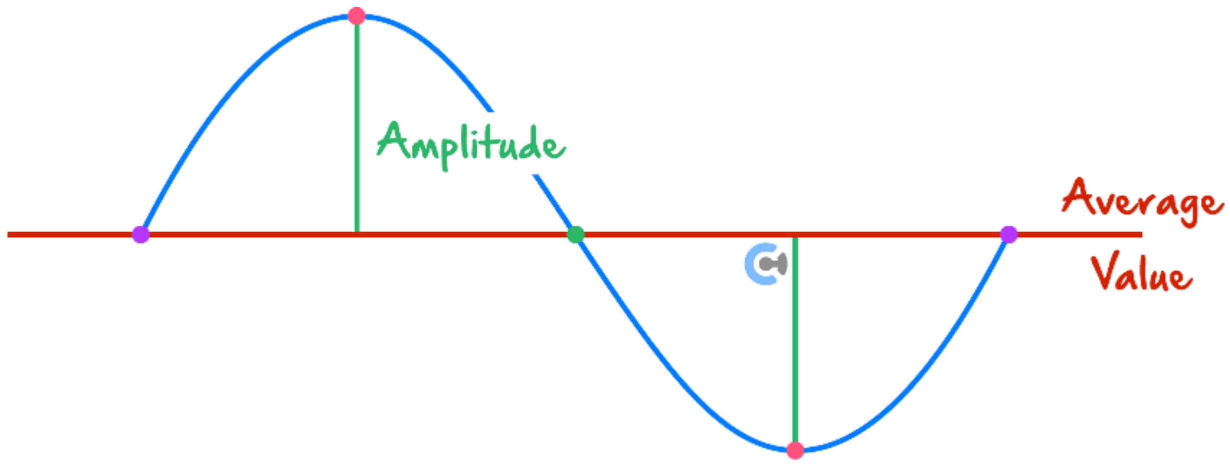


- Amplitude $= |A|$
- Period $= \frac{2\pi}{n}$
- Average Value $= k$

Steps for Sketching Transformations of sin and cos Functions

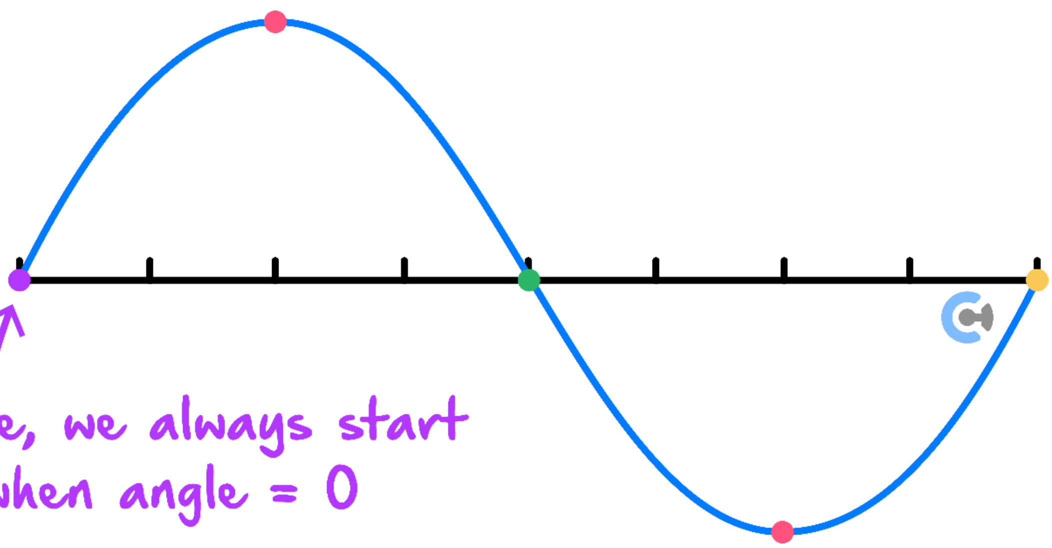
- Identify:
  - Amplitude.
  - Period.
  - Mean Value.
  - Positive/Negative Shape.

- And create a "mini version" of the graph you are about to draw.



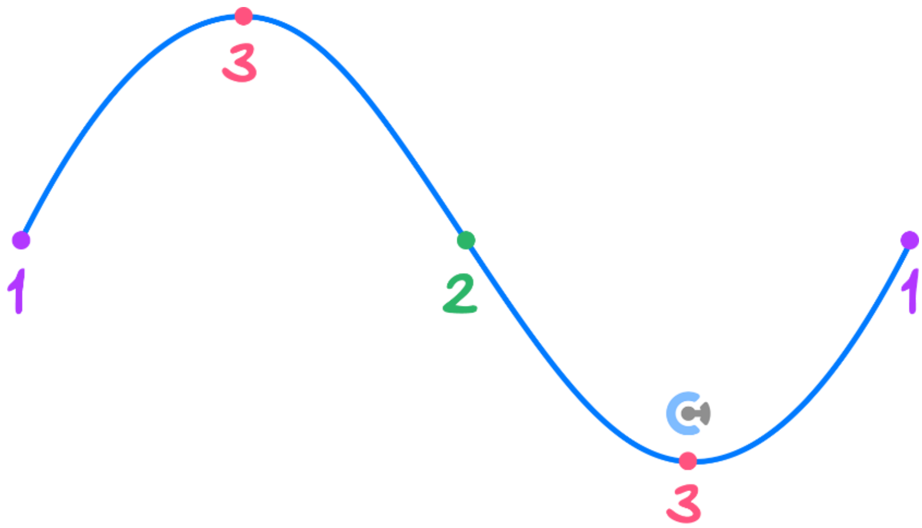
Consider the sign of our graph

- Start plotting the function from when the angle $= 0$.
  - For instance, for $\sin\left(2x - \frac{\pi}{3}\right)$, start from $x = \frac{\pi}{6}$.
  - Why?



eg: for sine, we always start here when angle = 0

- Draw the start and end of the periods, and plot the halves (turning points).



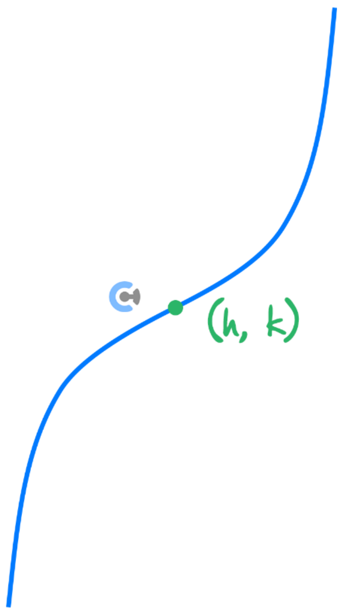
- Find any x-intercepts.
- Join all the points!

Space for Personal Notes

Steps for Sketching tan Functions

- Identify:
  - The period $= \frac{\pi}{n}$.
- Find the vertical asymptotes by solving for the angle $= \frac{\pi}{2}$.
- Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
  - For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x.
- Plot the inflection point (h, k) (Midpoint of the two vertical asymptotes).
  - x-value of inflection point $=$ x-value, which makes an angle $= 0$.
  - y-value of inflection point $=$ Vertical translation of the function.

eg: tan(x−h)+k

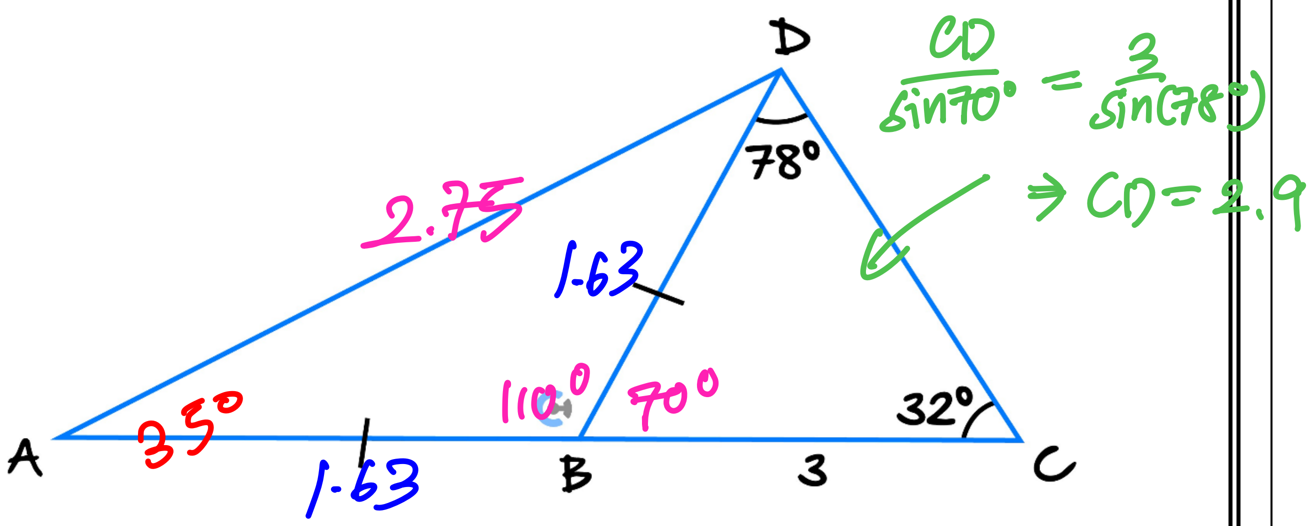


- Find any x-intercepts.
- Sketch a "cubic-like" shape.

Space for Personal Notes

Section B: Warm Up Test (19 Marks)

Question 5 (6 marks) **Tech-Active.**



ACD is a triangle and B is a point on AC. The triangle ABD is an isosceles triangle, and the length $BC = 3\ cm$, angle $BCD = 32°$, and the angle $BDC = 78°$.

a. Find the length BD. Round your answer to 2 decimal places. (2 marks)

$\frac{BD}{\sin 32°} = \frac{3}{\sin 78°} \Rightarrow BD = 1.63\text{cm}$

b. Find the length AD. Round your answer to 2 decimal places. (2 marks)

$\angle CBD = 70° \Rightarrow \angle ABD = 110°$

$AD^2 = 1.63^2 + 1.63^2 - 2(1.63)(1.63)\cos 110°$

$AD^2 = 7.09$

$\Rightarrow AD = 2.66\ \text{cm}$

c. Find the area of triangle ACD. Round your answer to 2 decimal places. (2 marks)

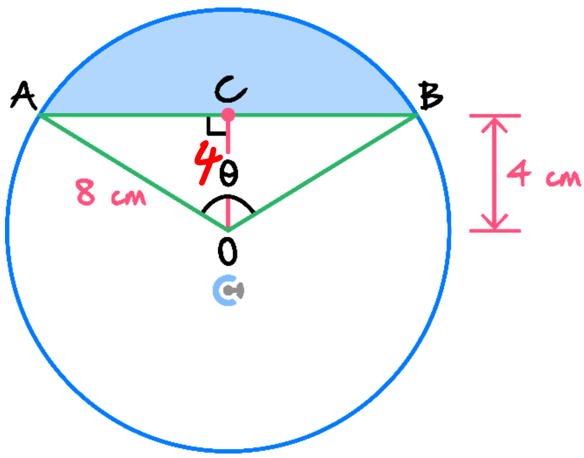
$\text{Area} = \frac{1}{2} \cdot AC \cdot CD \cdot \sin C)$

$= \frac{1}{2} \cdot (3 + 1.63)(2.9)\sin(32°)$

$= 3.53\ \text{cm}^2$

Question 6 (8 marks)

Find the following:

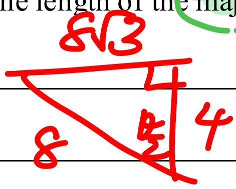


a. The length of chord AB. (2 marks)

chord $= 2\times\sqrt{8^2-4^2}$

$= 8\sqrt{3}$ cm

b. The length of the major arc AB. (2 marks)



$\cos\frac{\theta}{2} = \frac{4}{8}$

$\cos\frac{\theta}{2} = \frac{1}{2}$

$\frac{\theta}{2} = \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

Arc $= 2\pi r \times \frac{\theta}{2\pi}$

$= 2\pi \times 8 \times \frac{2\pi}{3} \times \frac{1}{2\pi}$

$= \frac{16\pi}{3}$

major arc $= 2\pi r - \frac{16\pi}{3}$

$= 16\pi - \frac{16\pi}{3}$

$= \frac{32\pi}{3}$

c. The area of the major sector AOB. (2 marks)

$\theta = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

Area $= \pi r^2 \times \frac{\frac{4\pi}{3}}{2\pi} = 64\pi \times \frac{2}{3} = \frac{128\pi}{3}$

d. The area of the minor segment formed by chord AB. (2 marks)

$$\text{Area} = \frac{1}{2}(8)^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$$

$$= 32\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{64\pi}{3} - 16\sqrt{3}$$

Space for Personal Notes

Question 7 (5 marks)

Consider the equation below:

$$-2\sin\left(\frac{\pi}{3}-\frac{x}{2}\right)+\sqrt{3}=0$$

a. Solve for the value(s) of x. (4 marks)

$\sin\left(\frac{\pi}{3}-\frac{x}{2}\right)=\frac{\sqrt{3}}{2}$ period $=\frac{2\pi}{1/2}=4\pi$

$\frac{\pi}{3}-\frac{x}{2}=\frac{\pi}{3}, \frac{2\pi}{3}$ one period

$-\frac{x}{2}=0, \frac{\pi}{3}$

$x=0, -\frac{2\pi}{3}$

$x=4\pi n$

$-\frac{2\pi}{3}+4\pi n, \quad n\in\mathbb{Z}$

b. Solve for the value(s) of x where $x \in [-4\pi, \pi]$. (1 mark)

$x=-4\pi, -\frac{2\pi}{3}, 0$

Space for Personal Notes

Sub-Section: Problems in 3D

Problems in 3D

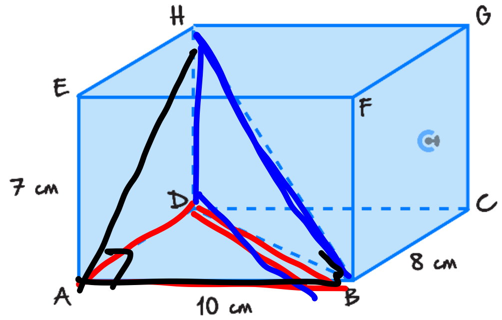
- 3D trigonometry problems can be broken down into multiple 2D problems.
- Main Tools:

1. Pythagoras' theorem.
2. SOH CAH TOA.
3. The sine and cosine rule.

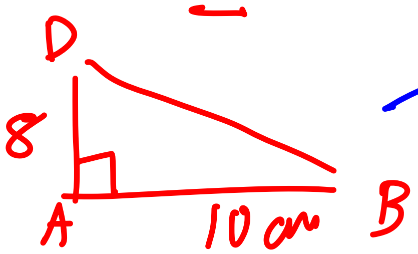
Space for Personal Notes

Question 8 Walkthrough. Tech Active.

ABCDEFGH is a cuboid. Find:

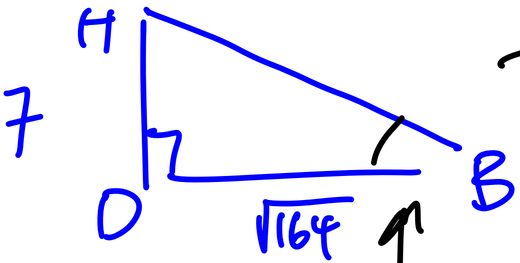


a. The distance *DB*, correct to two decimal places.



$$BD = \sqrt{10^2 + 8^2}$$
$$= \sqrt{164}$$
$$= 12.81 \text{ cm}$$

b. The distance *HB*, correct to two decimal places.

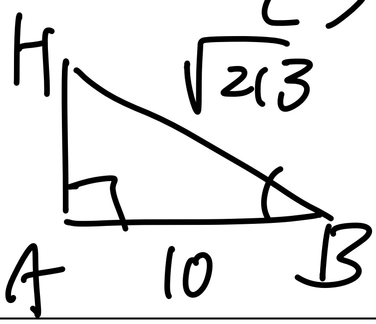


$$HB = \sqrt{164 + 49}$$
$$= \sqrt{213}$$
$$= 14.59 \text{ cm}$$

c. The magnitude of angle *HBD*, in degrees correct to two decimal places.

$$\tan(\angle HBD) = \frac{7}{\sqrt{164}} \Rightarrow \angle HBD = \tan^{-1}\left(\frac{7}{\sqrt{164}}\right)$$
$$= 28.66^\circ$$

d. The magnitude of angle *HBA*, in degrees correct to two decimal places.

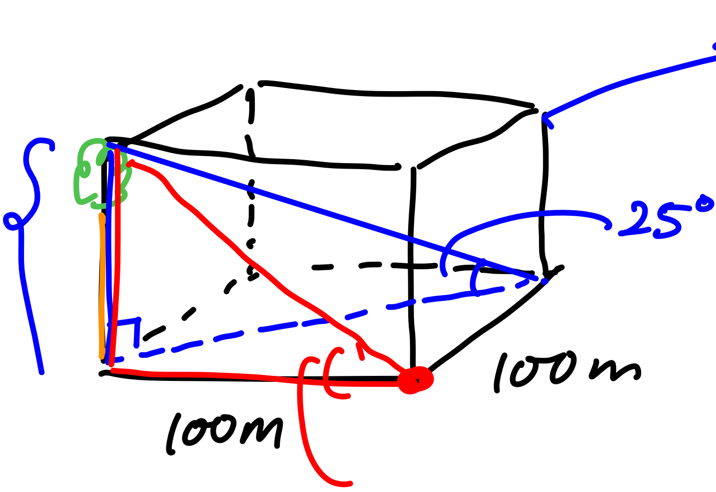


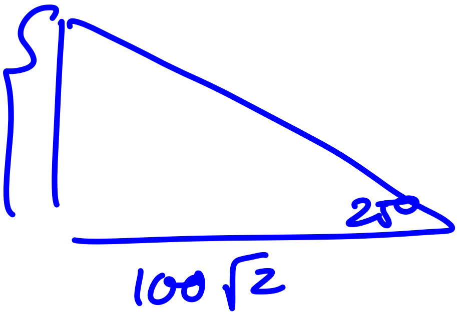
$$\cos(\angle HBA) = \frac{10}{\sqrt{213}}$$

$$\angle HBA = 46.75^\circ$$

Question 9 Tech-Active.

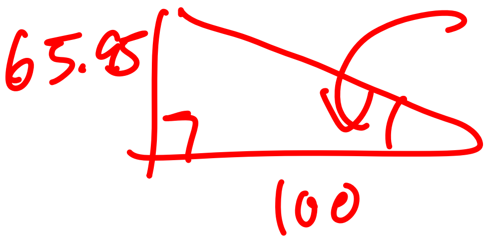
There is a tree (vertical) that stands at the corner of a square paddock of side 100 m. Given that the angle of elevation of the top of the tree from the diagonally opposite corner is 25°, what is the angle of elevation of the top of the tree from the other corners? Give your answer correct to two decimal places.





$\tan 25° = \dfrac{\text{tree}}{100\sqrt{2}}$

$\text{tree} = 65.95$



$\tan\theta = \dfrac{65.95}{100}$

$\theta = 33.40°$

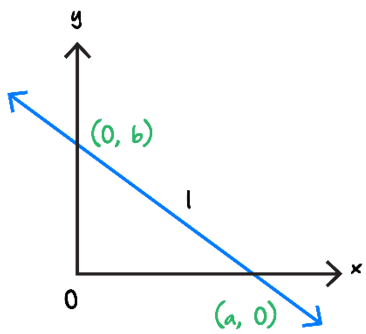
Space for Personal Notes

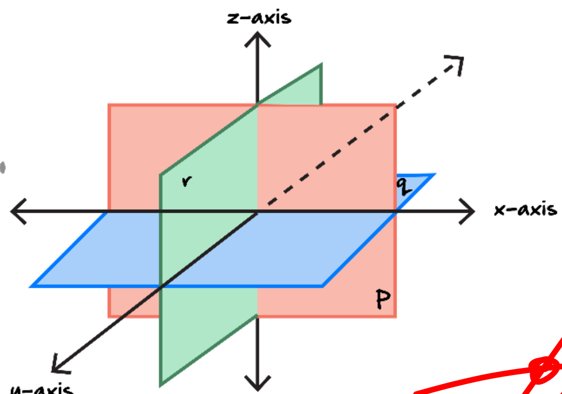
Sub-Section: Angle Between Planes

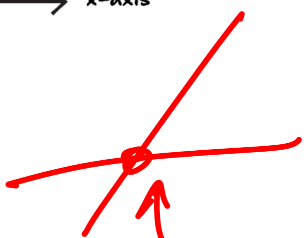
How can we visualise planes?

Exploration: Visualising Planes and Lines

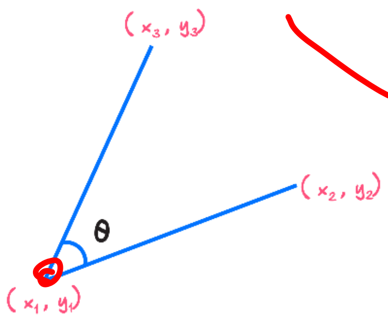
- In maths, a plane is a flat, two-dimensional surface that extends indefinitely.
- Think of it as an infinitely large rectangle.
- Just like how **lines** are defined as **one-dimensional** on a **two-dimensional surface**, planes are usually **two-dimensional surfaces** defined on a **three-dimensional space**.

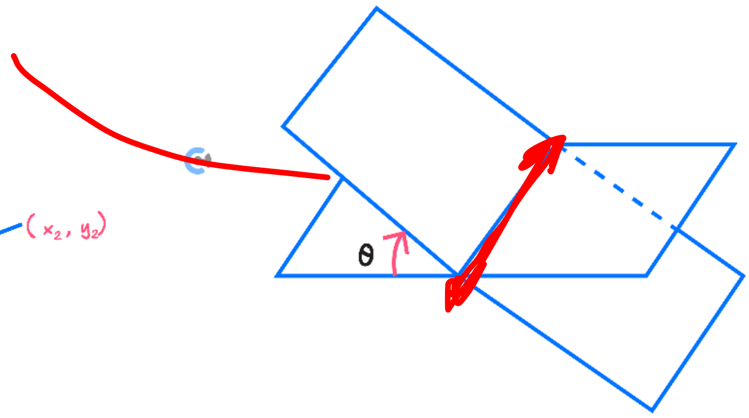






- If you want to construct an angle between two lines, they have to intersect at a common point.
- Similarly, if we want to construct an angle between two planes they have to intersect at a common line.

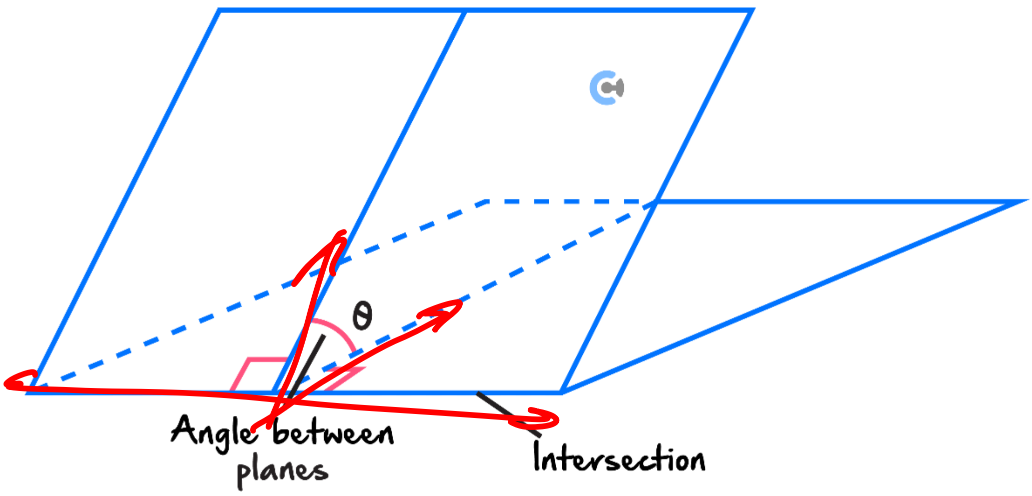




How can we find angles between two planes?

Angles Between Planes

- When finding the angle between two planes, it is important to consider where the planes intersect and the line that this intersection forms.

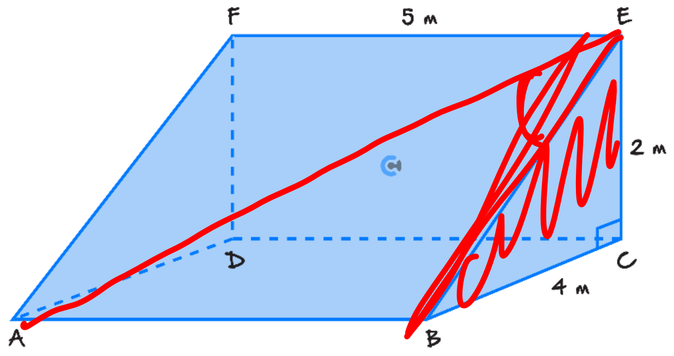


- The angle between the two planes is equal to the angle between the lines in each plane that are perpendicular to the line intersection between the planes.

Space for Personal Notes

Question 10 Walkthrough. Tech-Active.

Consider the wedge shown. Find, correct to one decimal place, the angles between:

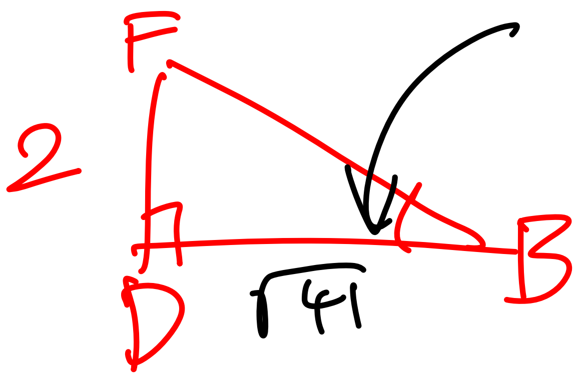


a. The line BE at the plane $ABCD$.

$$\tan\theta = \frac{2}{4}$$

$$\theta = 26.6^\circ$$

b. The line BF and the plane $ABCD$.



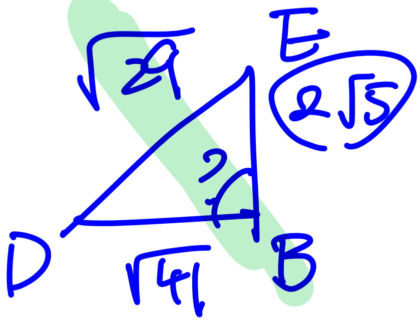
$$\tan\theta = \frac{2}{\sqrt{41}}$$

$$\theta = 17.3^\circ$$

c. Planes $ABCD$ and $ABEF$.

$26.6°$ (a)

d. The lines BD and BE.

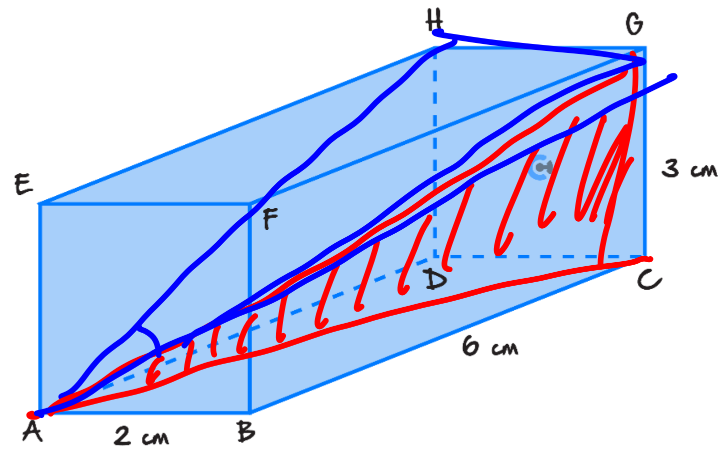


$$\sqrt{29}^2 = (2\sqrt{5})^2 + (\sqrt{41})^2 - 2(2\sqrt{5})(\sqrt{41})\cos(\theta)$$

$$\hookrightarrow \theta = 56.0°$$

Question 11

The diagram shows a cuboid.



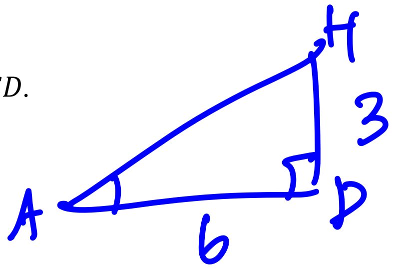
Find, correct to two decimal places, the angle between:

a. The line AG and the plane $ABCD$.

$\tan^{-1}\left(\frac{3}{\sqrt{40}}\right) = 25.38°$

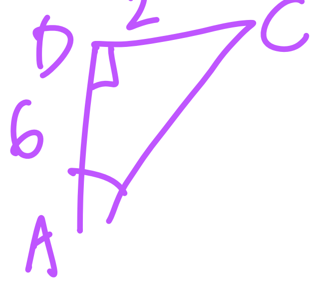
b. The plane $ABGH$ and the plane $ABCD$.

$\overrightarrow{AH}$ $\quad$ $\overrightarrow{AD}$



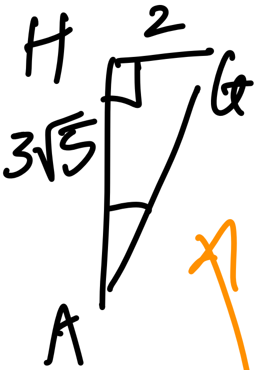
$\tan^{-1}\left(\frac{3}{6}\right) = 26.57°$

c. The line AC and the plane $ADEH$.



$\tan^{-1}\left(\frac{2}{6}\right) = 18.43°$

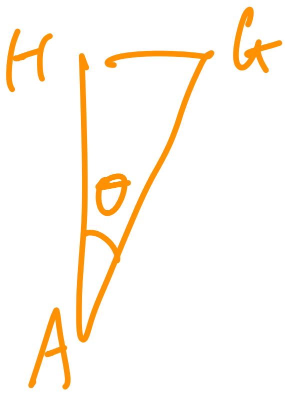
d. The line AG and the plane $ADEH$.



$$\tan\theta = \frac{2}{3\sqrt{5}}$$

$$\theta = 16.60°$$

e. The plane $ACGE$ and the line AH.

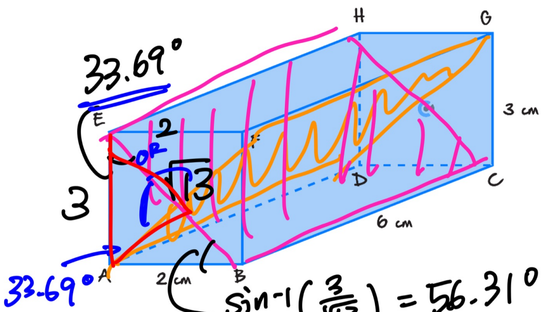


$16.60°$

f. The plane $ADGF$ and the plane $BCHE$.

$112.62°$, $67.38°$

diagram shows a cuboid.



$\sin^{-1}\left(\frac{3}{\sqrt{13}}\right) = 56.31°$

, correct to two decimal places, the angle between:

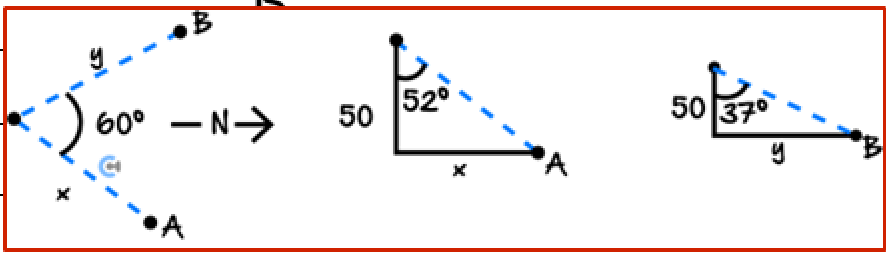
The line AG and the plane $ABCD$.

Space for Personal Notes

Section C: Exam 1 Questions (16 Marks)

Question 12 (3 marks) **Tech-Active.**

An observer facing north is on a cliff 50 m above sea level, and sights two ships. One ship is located on a bearing of $N035W$, and another is located on a bearing of $N025E$. The angles of depression of the ships are 37° and 52° respectively. Find in metres correct to one decimal place, the distance between the two ships.



$$x = 50\tan(52°)$$

$$y = 50\tan(37°)$$

$$AB = \sqrt{x^2+y^2-2xy\cos(60°)}$$

$$\approx 55.7\ m$$

Space for Personal Notes

Question 13 (3 marks)

It is known that $\cos(a) = -\frac{3}{5}$ where a is a second quadrant angle.

Evaluate the following:

a. $\cos(\pi + a)$. (1 mark)

$\boxed{\frac{3}{5}}$

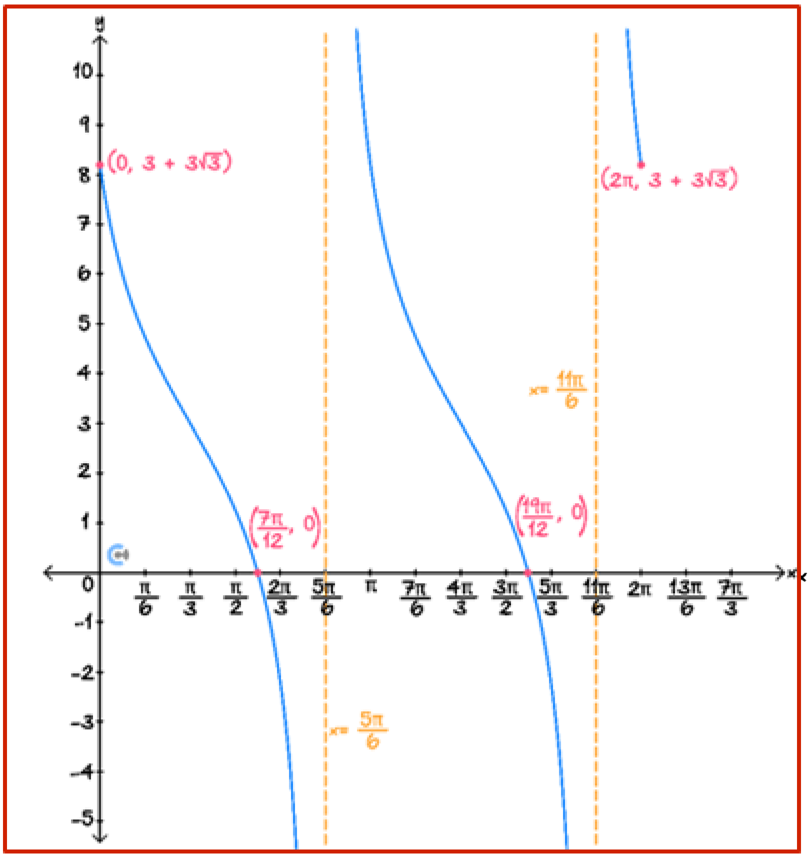
b. $\sin\left(\frac{\pi}{2} - a\right)$. (2 marks)

$\boxed{-\frac{3}{5}}$

Space for Personal

Question 14 (3 marks)

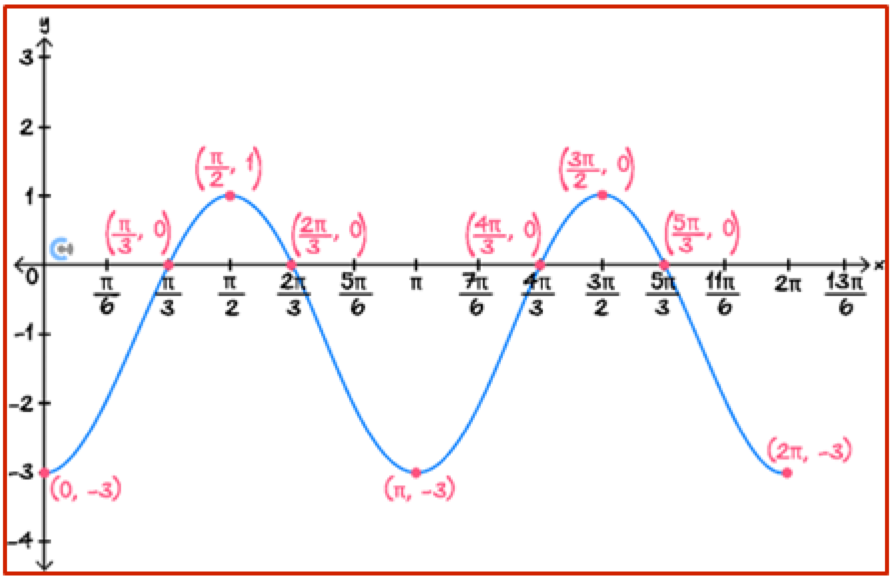
Sketch the graph of $f(x) = 3\tan\left(\frac{\pi}{3} - x\right) + 3$ for $x \in [0, 2\pi]$ on the axis below, labelling all asymptotes, intercepts and endpoints with their coordinates.



Space for Personal

Question 15 (7 marks)

a. Sketch the graph of $f(x) = -2\sin\left(\frac{\pi}{2} - 2x\right) - 1$ for $x \in [0, 2\pi]$ on the axis below, labelling all intercepts and endpoints with their coordinates. (3 marks)



b. Solve $f(x) = -2$ for $x \in [0, 2\pi]$. (3 marks)

$$x = \frac{(6n \pm 1)\pi}{6}, n \in Z$$

c. Hence, solve $f(x) \leq -2$ for $x \in [0, 2\pi]$. (1 mark)

$$x \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$$

Section D: Exam 2 Questions (26 Marks)

Question 16 (1 mark)

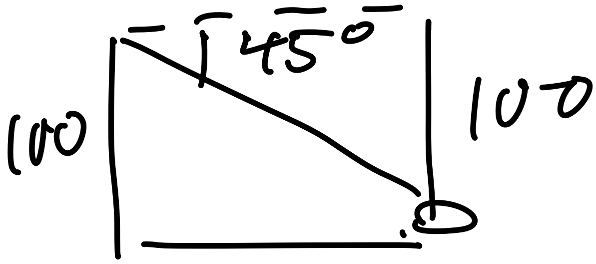
A cliff is 100 metres high. The angle of depression of a boat in the water at the base of the cliff is 45°. What is the distance between the boat and the top of the cliff?

A. 70.7 metres

B. 100 metres

C. 141.4 metres

D. 200 metres



$\sin 45° = \frac{100}{H}$

$H = 141$

Question 17 (1 mark)

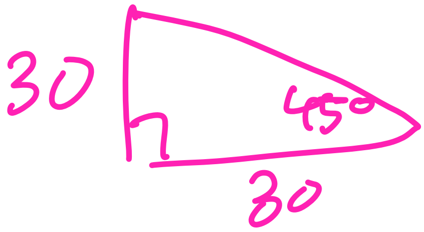
The angles of elevation of top and bottom of a flag at a distance of 30 m is 45° and 30° respectively. What is the height of the flag AB?

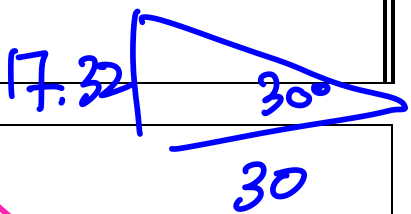
A. 10.54 metres

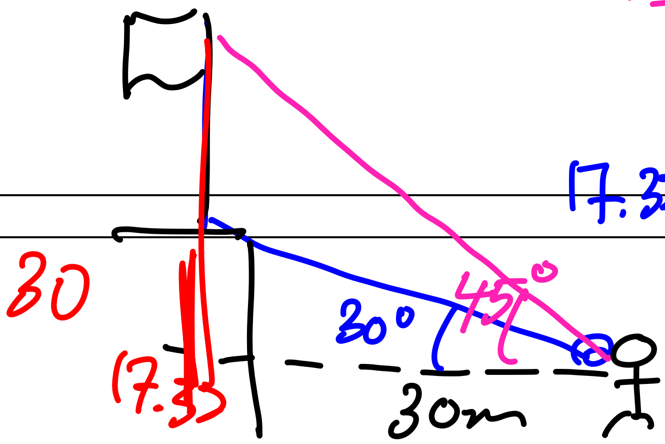
B. 10 metres

C. 20 metres

D. 12.68 metres



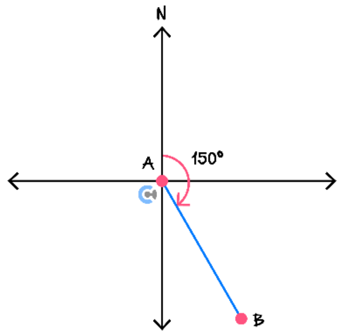


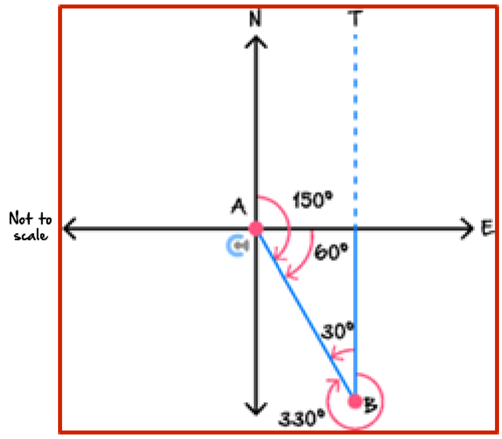


Space for Personal

Question 18 (1 mark)

A helicopter flies on a bearing of 150° from A to B.





What is the bearing of A from B?

A. 30°

B. 150°

C. 210°

D. 330°

$\angle TBA = 30^\circ \quad (180^\circ \text{ in } \Delta)$

$\therefore$ Bearing of A from B

$= 360 - 30$

$= 330^\circ$

$\Rightarrow D$

Question 19 (1 mark)

A poster is on top of a building. Rajesh is standing on the ground at a distance of 50 m from the building. The angles of elevation to the top of the poster and bottom of the poster are 45° and 30° respectively. What is the height of the poster?

A. $\frac{50(\sqrt{3}-1)}{\sqrt{3}}\ m$

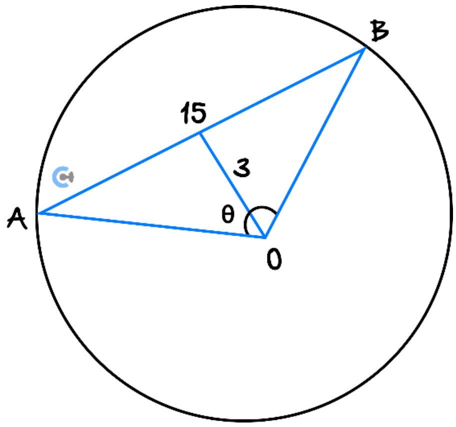
B. $50\sqrt{3}\ m$

C. $\frac{25}{\sqrt{3}}\ m$

D. None of these.

Space for Personal

Question 20 (8 marks)



Find:

a. The radius of the circle. (2 marks)

$$\sqrt{7.5^2 + 3^2} = \frac{3\sqrt{29}}{2}$$

b. The length of the minor arc AB. Round your answer correct to 2 decimal places. (2 marks)

$\theta = 2\tan^{-1}\left(\frac{15}{6}\right) = 136°,$

Thus, major arc $AB = \frac{3\sqrt{29}}{2}\left(\frac{136}{180}\pi\right) = 19.23$

c. The area of the major sector AOB. Provide your answer correct to 2 decimal places. (2 marks)

$\theta_{sector} = 223°,$

Thus, area of major sector $AOB = \frac{3\sqrt{29}}{2}^2\left(\frac{223}{360}\pi\right) = 127.32$

d. The area of the minor segment formed by chord AB. (2 marks)

Area of minor segment $= 0.5 \times \left(\frac{3\sqrt{29}}{2}\right)^2 \left(\frac{136...}{180}\pi - \sin(136\ ...)\right) = 55.17$

Question 21 (4 marks)

A man standing on a cliff observes a ship at an angle of depression $30°$, approaching the shore just beneath him. Three minutes later, the angle of depression of the ship is $60°$. How soon will it reach the shore?

Let distances from shore at the two observations be x and x' respectively.

$$\tan(30) = \frac{h}{x} \Rightarrow x = h\sqrt{3}$$

$$\tan(60) = \frac{h}{x'} \Rightarrow x' = \frac{h}{\sqrt{3}}$$

In three minutes, ship has moved $d = x - x' = \frac{2h}{\sqrt{3}}$

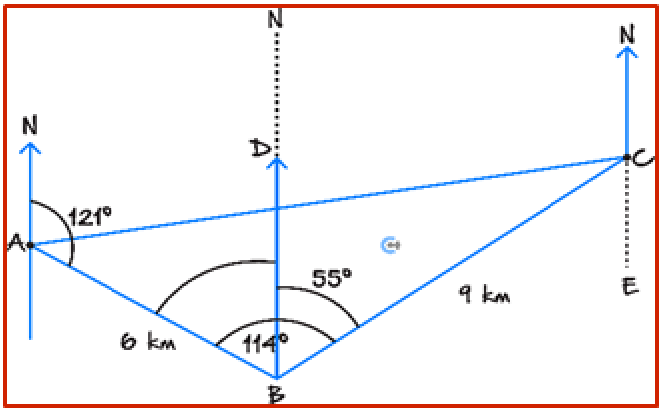
So, ship velocity $v = \frac{2h}{3\sqrt{3}}$

Then time to move x' is $t = \frac{x'}{v} = \frac{3}{2}$

So, it reaches shore 4.5 minutes after first observation.

Question 22 (6 marks)

A ship sails 6 km from A to B on a bearing of 121°. It then sails 9 km to C. The size of angle ABC is 114°.



Not to scale

a. What is the bearing of C from B? (1 mark)

Let point D be due North of point B

$\angle ABD = 180 - 121$ (cointerior with $\angle A$)

$= 59^\circ$

$\angle DBC = 114 - 59$

$= 55^\circ$

$\therefore$ Bearing of C from B is 055°

b. Find the distance AC. Give your answer correct to the nearest kilometre. (2 marks)

Using cosine rule

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC$$
$$= 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 114^\circ$$
$$= 160.9275...$$
$$\therefore AC = 12.685... \quad (\text{Noting } AC > 0)$$
$$= 13 \text{ km (nearest km)}$$

c. What is the bearing of A from C? Give your answer correct to the nearest degree. (3 marks)

(iii) Need to find $\angle ACB$ (see diagram)

$$\cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}$$
$$= \frac{(12.685...)^2 + 9^2 - 6^2}{2 \times (12.685..) \times 9}$$
$$= 0.9018...$$
$$\angle ACB = 25.6^\circ \text{ (to 1 d.p.)}$$

From diagram

$\angle BCE = 55^\circ$ (alternate to $\angle DBC$)

$\therefore$ Bearing of A from C

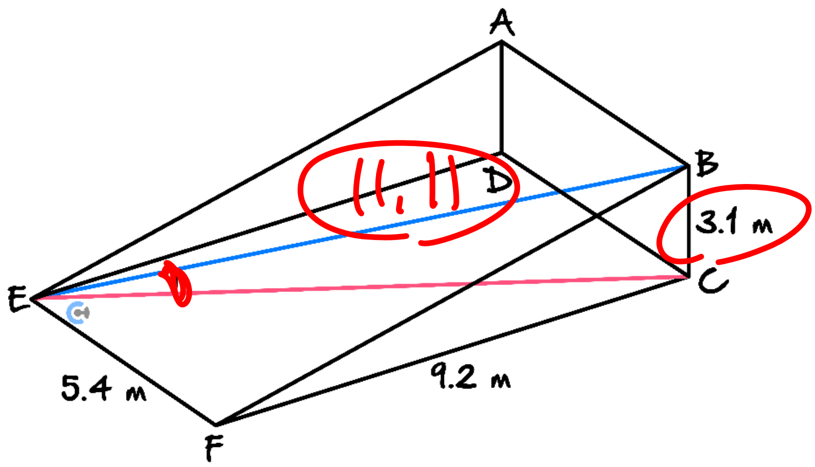
$= 180 + 55 + 25.6$

$= 260.6$

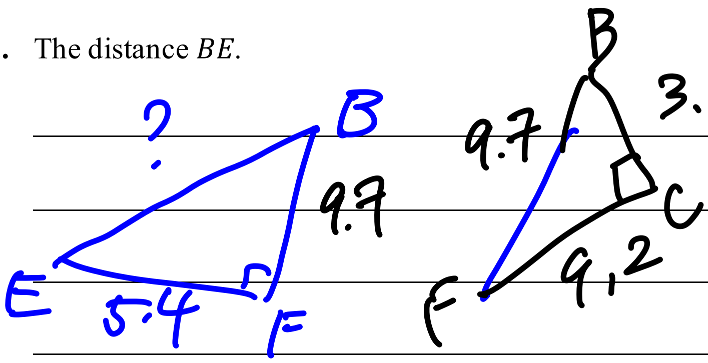
$= 261^\circ$ (nearest degree)

Question 23 (4 marks)

The diagram below shows a wedge in which rectangle $ABCD$ is perpendicular to rectangle $CDEF$. The distances are as indicated on the diagram. From the diagram find, correct to two decimal places:

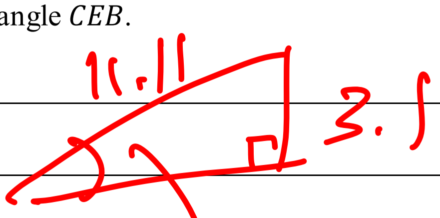


a. The distance BE.



$BE = 11.11$

b. The angle CEB.



$\sin\theta = \frac{3.1}{11.11} \Rightarrow 16.20°$

Contour Check

Learning Objective: [3.3.1] – Apply trigonometry to solve problems in $3D$

Key Takeaways

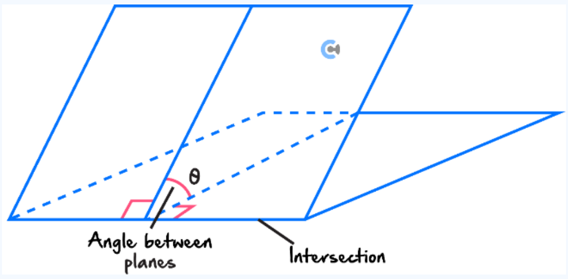
- $3D$ trigonometry problems can be broken down into ___Multiple $2D$___ problems.
- Main Tools:
  1. Pythagoras' theorem.
  2. SOH CAH TOA.
  3. The sine and cosine rule.

Learning Objective: [3.3.2] – Apply trigonometry to find the angle between planes

Key Takeaways

Angles Between Planes

- When finding the angle between two planes, it is important to consider where the planes ___intersect___ and the ___line___ that this intersection forms.



- The angle between the two planes is equal to the ___angle___ between the ___lines___ in ___each plane___ that are ___perpendicular___ to the ___line of intersection___ between the planes.

CONTOUREDUCATION
Website: contoureducation.com.au | Phone: 1800 888 300 | Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45 + raw scores and 99 + ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after school weekdays, and all-day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-specialist-consult-2025