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VCE Specialist Mathematics ½
Trigonometric Exam Skills [3.3]
Homework Solutions

Admin Info & Homework Outline:

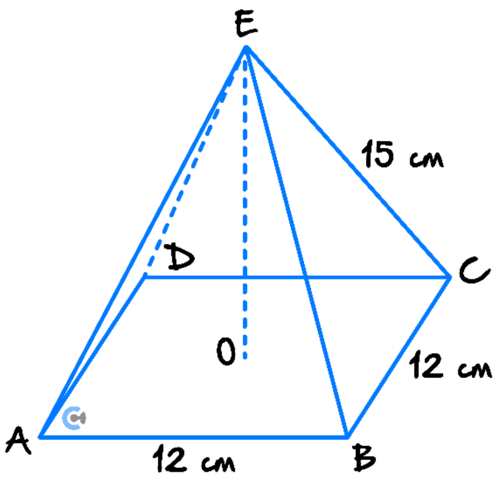
Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 21

Section A: Compulsory Questions

Sub-Section [3.3.1] and [3.3.2]: Apply Trigonometry to Solve Problems in 3D and Find the Angle between Planes

Question 1

A square pyramid $ABCDE$ stands on level horizontal ground. The vertex of the pyramid is at E. The points A, B, C, D are the corners of a square of side 12 cm, whose diagonals intersect at the point O. Each of the sloping edges of the pyramid has a length of 15 cm.



a. Calculate the length OC.

By Pythagoras, $OC^2 = 6^2 + 6^2 \implies OC = 6\sqrt{2}$.

b. Calculate the volume of the pyramid. (Recall $V = \frac{1}{3} \times \text{base} \times \text{height}$)

$OE^2 = EC^2 - OC^2$
$OE^2 = 225 - 72 = 153 \implies OE = \sqrt{153} = 3\sqrt{17}.$
Base area $= 12 \times 12 = 144.$
Therefore $V = \frac{1}{3} \times 144 \times 3\sqrt{17} = 144\sqrt{17}$

c. Calculate the total surface area of the pyramid.

Use Pythagoras to find the height of the triangular face.
$h^2 = 15^2 - 6^2 = 189 \implies h = 3\sqrt{21}.$
Then face area $= \frac{1}{2} \times 12 \times 3\sqrt{21} = 18\sqrt{21}.$
Surface area $= 144 + 4 \times 18\sqrt{21} = 144 + 72\sqrt{21}.$

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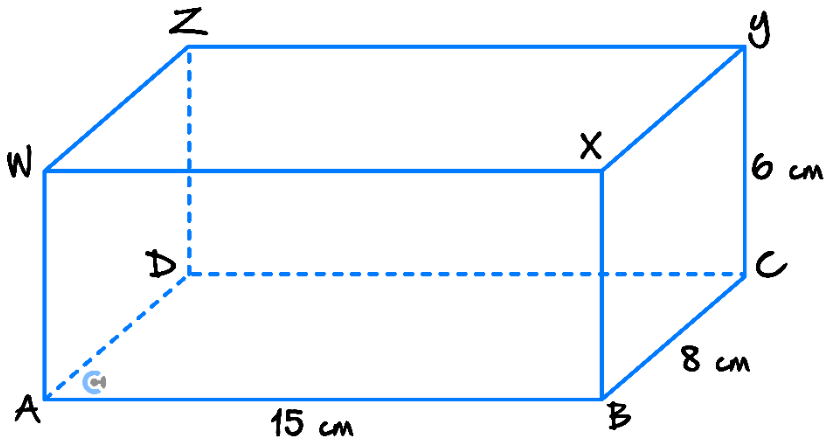
Question 2

The figure shows a cuboid $ABCDWXYZ$ standing on level horizontal ground. The lengths of AB, BC and CY are $15\ cm, 8\ cm$ and $6\ cm$, respectively.



a. Find the length of AY.

$AC = \sqrt{15^2 + 8^2} = 17.$
Then $AY = \sqrt{6^2 + 17^2} = 5\sqrt{13}$

b. Calculate the angle AY makes with the ground, correct to two decimal places.

$\tan(\theta) = \frac{6}{17} \implies \theta \approx 19.44°$

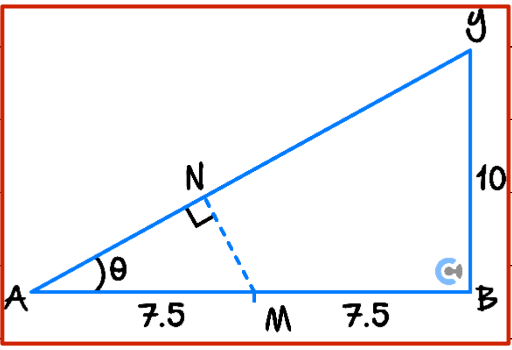
c. Determine the area of the triangle ABY.

$AY^2 = AB^2 + BY^2$
$BY = \sqrt{25 \times 13 - 15^2} = 10$
$\text{Area } ABY = \frac{1}{2} \times 10 \times 15 = 75$

The point M is the midpoint of AB and the point N lies on AY.

d. The point M is the midpoint of AB and the point N lies on AY. Calculate the length of MN, given that MN is perpendicular to AY . Give your answer correct to two decimal places.

$\tan(\theta) = \frac{10}{15} = \frac{2}{3} \implies \theta = 33.69°.$
$\text{Then } \sin(\theta) = \frac{NM}{7.5} \implies NM = 4.16 \text{ cm}$



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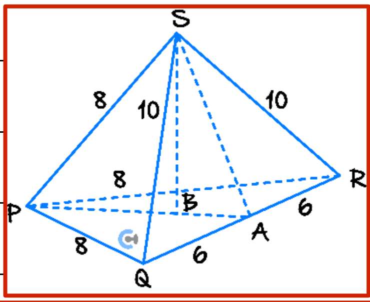
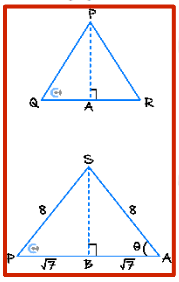
Question 3

A pyramid $PQRS$ has a triangular horizontal base PQR, where $PQ = PR = 8\ m$ and $RQ = 12\ m$. The vertex of the pyramid S lies directly above the level of PQR so that $SQ = SR = 10\ m$ and $SP = 8\ m$.

a. Show that the shortest distance of S from the base PQR is $\sqrt{57}\ m$.



The shortest distance is perpendicular to the base. We must find SB.



$|AP| = \sqrt{8^2 - 6^2} = 2\sqrt{7}$

$|AS| = \sqrt{10^2 - 6^2} = 8$

PQR is isosceles so B is the midpoint of AP

$|SB| = \sqrt{64 - 7} = \sqrt{57}$

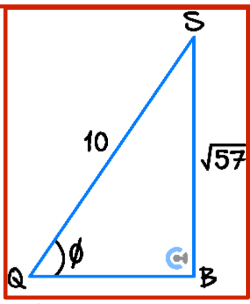
b. Calculate, in degrees correct to two decimal places, the acute angle between:

i. The plane SQR and the plane PQR.

The angle θ in the diagram above.

$\cos(\theta) = \dfrac{\sqrt{7}}{8} \implies \theta = 70.69°$

ii. The edge SQ and the plane PQR.



$\sin\phi = \dfrac{\sqrt{57}}{10}$

$\phi = 49.02°$

$\sin(\phi) = \dfrac{\sqrt{57}}{10} \implies \phi = 49.02°$

c. Determine, as an exact surd, the shortest distance of P from the plane SQR.

HINT: Compute the volume of the pyramid in two different ways.

$$\text{Area of } \triangle PQR = \frac{1}{2}|QR||AP| = \frac{1}{2} \times 12 \times 2\sqrt{7} = 12\sqrt{7}$$

$$\begin{aligned}\text{Volume of Pyramid} &= \frac{1}{3}(\text{Base Area}) \times \text{Height} \\ &= \frac{1}{3} \times 12\sqrt{7} \times \sqrt{57} \\ &= 4\sqrt{2}\sqrt{57}\end{aligned}$$

$$\text{Area of } \triangle SQR = \frac{1}{2}|QR||AS| = \frac{1}{2} \times 12 \times 8 = 48$$

Volume of Pyramid is also equal to:

$$\frac{1}{3}(\text{Area of base } SQR) \times (\text{Height from } P \text{ to } SQR)$$

$$\begin{aligned}\implies \frac{1}{3} \times 48 \times h &= 4\sqrt{2}\sqrt{57} \\ 16h &= 4\sqrt{2}\sqrt{57} \\ h &= \frac{1}{4}\sqrt{399}\end{aligned}$$

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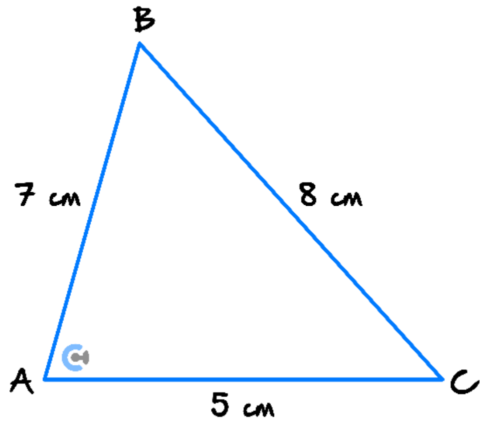
Sub-Section: Exam 1 Questions

Question 4

The figure below shows a triangle ABC where the following information is given.

$$|AB| = 7\ cm, |BC| = 8\ cm, |AC| = 5\ cm.$$



a. Find the size of the angle $\angle ACB$ in degrees.

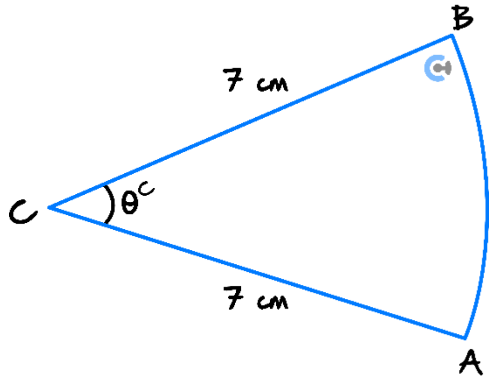
Let $\angle ACB = \theta$.
$\cos(\theta) = \frac{8^2 + 5^2 - 7^2}{80} = \frac{40}{80} = \frac{1}{2}$.
Therefore $\theta = 60^\circ$

b. Hence, determine as an exact surd the area of the triangle ABC.

$\text{Area} = \frac{1}{2} \times 5 \times 8 \sin(60^\circ) = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

Question 5

The figure below shows a circular sector ABC of radius $7\ cm$ subtending an angle θ radius at C. Given that the perimeter of the sector is equal to the area of the sector, find the value of θ in radians.



Perimiter $= 14 + 7\theta$
Area $= \frac{1}{2} \times 7^2\theta = \frac{49}{2}\theta$.
Solve $14 + 7\theta = \frac{49}{2}\theta \implies \frac{35}{2}\theta = 14 \implies 35\theta = 28$.
$\theta = \frac{28}{35} = \frac{4}{5}$.

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Question 6

Prove the identity $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$.

We proceed as follows:

$$\begin{aligned}\frac{\cos\theta}{1-\sin\theta} &= \frac{\cos\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta} \\ &= \frac{\cos\theta + \sin\theta\cos\theta}{1-\sin^2\theta} \\ &= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} \\ &= \frac{1+\sin\theta}{\cos\theta}\end{aligned}$$

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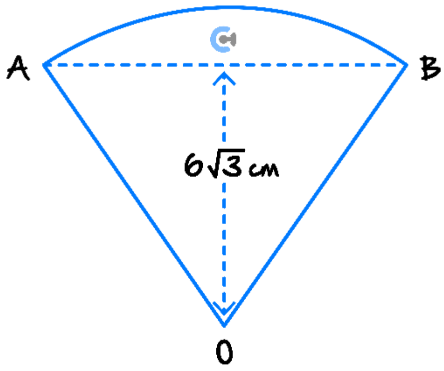
Question 7

The figure above shows a badge in the shape of a circular sector OAB, centred at O. The triangle OAB is equilateral and its perpendicular height is $6\sqrt{3}$ cm.



a. Find the length of OA.

$$\sin\left(\frac{\pi}{3}\right) = \frac{6\sqrt{3}}{OA} \implies OA = \frac{2 \times 6\sqrt{3}}{\sqrt{3}} = 12$$

b. Determine in terms of π:

i. The area of the badge.

$$\text{Area} = \frac{1}{2}r^2\theta = 12 \times \frac{1}{2} \times \frac{\pi}{3} = 2\pi$$

ii. The perimeter of the badge.

Length of arc $AB = r\theta = 12 \times \frac{\pi}{3} = 4\pi$.
Therefore perimeter $= 24 + 4\pi$.

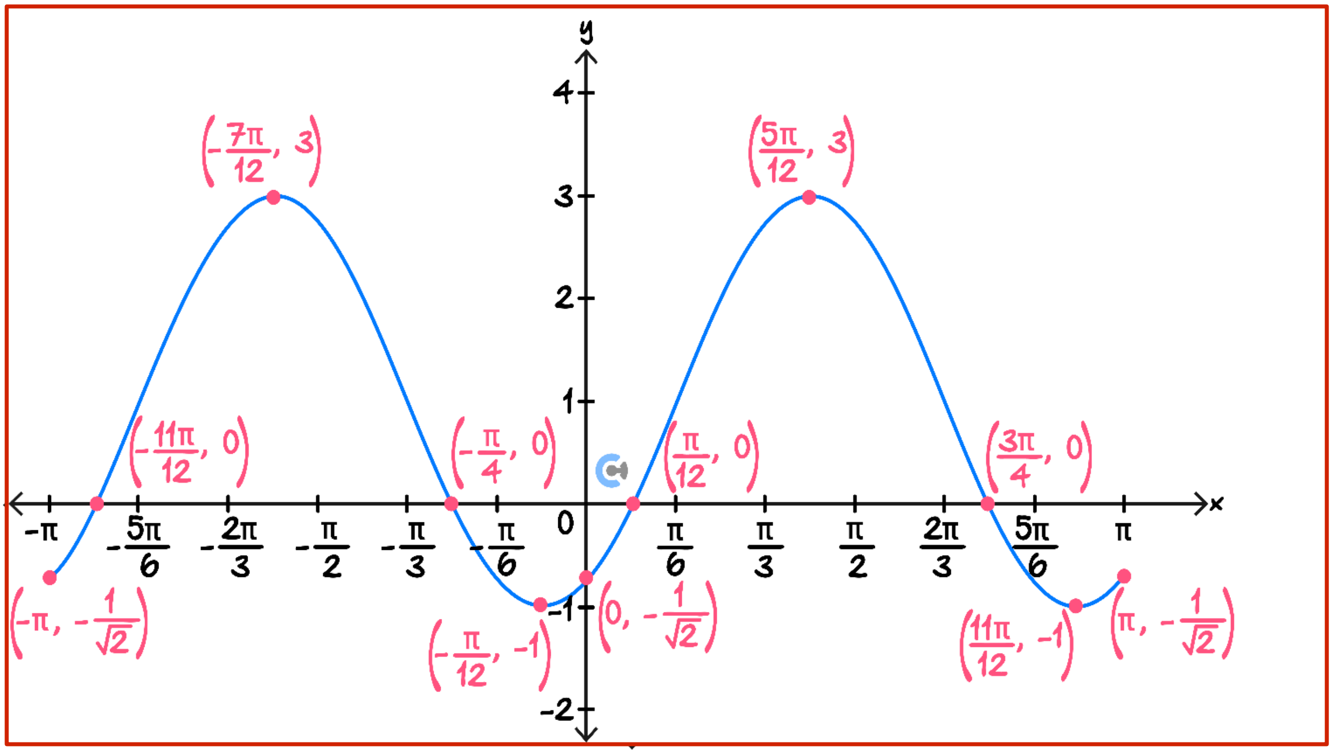
Question 8

Consider the function $f(x) = 2\sin\left(2x - \frac{\pi}{3}\right) + 1$.

a. Find the general solution to $f(x) = 0$.

$$\sin\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$$
$$2x - \frac{\pi}{3} = -\frac{5\pi}{6} + 2n\pi, -\frac{\pi}{6} + 2n\pi$$
$$2x = -\frac{\pi}{2} + 2n\pi \quad \text{or} \quad 2x = \frac{\pi}{6} + 2n\pi$$
$$x = -\frac{\pi}{4} + n\pi \quad \text{or} \quad x = \frac{\pi}{12} + n\pi, n \in \mathbb{Z}.$$

b. Sketch the graph of $y = f(x)$ for $x \in [-\pi, \pi]$ on the axes below. Label all axes intercepts, turning points and endpoints with coordinates.



c. Find the values of x for which $f(x) > 2$.

We solve $f(x) = 2 \implies 2\sin\left(2x - \frac{\pi}{3}\right) = 1$.

$$\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}$$
$$2x - \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$$
$$2x = \frac{\pi}{2}, \frac{7\pi}{6}$$
$$x = \frac{\pi}{4}, \frac{7\pi}{12}$$

The period is π, so other solutions are $x = -\frac{3\pi}{4}$ and $x = -\frac{5\pi}{12}$.

Then by the shape of the graph we see that $f(x) > 2$ for $x \in \left(-\frac{3\pi}{4}, -\frac{5\pi}{12}\right) \cup \left(\frac{\pi}{4}, \frac{7\pi}{12}\right)$

d. The function $f(x)$ has an equivalent expression $f(x) = 2\cos\left(2x + \frac{a\pi}{6}\right) + 1$, where $0 < a < 12$. State the value of a.

Note that $\sin\left(2x - \frac{\pi}{3}\right) = \cos\left(2x - \frac{\pi}{2} - \frac{\pi}{3}\right) = \cos\left(2x - \frac{5\pi}{6}\right)$

Then $-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$

$a = 7$

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Sub-Section: Exam 2 Questions

Question 9

A building is 60 metres tall. From a certain point, the angle of elevation to the top of the building is 30°. How far is the point from the building?

A. $60\sqrt{3}$ metres.

B. $45\sqrt{3}$ metres.

C. $30\sqrt{3}$ metres.

D. $40\sqrt{3}$ metres.

Question 10

If $\tan(\theta) = -\frac{3}{4}$ and $\theta \in [0,2\pi]$, then $\cos(\theta)$ is equal to:

A. $\frac{3}{5}$ or $-\frac{3}{5}$.

B. $\frac{4}{3}$ or $-\frac{4}{3}$.

C. $-\frac{3}{5}$ or $-\frac{4}{5}$.

D. $\frac{4}{5}$ or $-\frac{4}{5}$.

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Question 11

The solutions of the equation

$$2\cos\left(2x - \frac{\pi}{3}\right) + 1 = 0$$

are:

A. $x = \frac{\pi(6k-2)}{6}$ or $x = \frac{\pi(6k-3)}{6}$, for $k \in \mathbb{Z}$.

B. $x = \frac{\pi(6k-2)}{6}$ or $x = \frac{\pi(6k+5)}{6}$, for $k \in \mathbb{Z}$.

C. $x = \frac{\pi(6k-1)}{6}$ or $x = \frac{\pi(6k+2)}{6}$, for $k \in \mathbb{Z}$.

D. $x = \frac{\pi(6k-1)}{6}$ or $x = \frac{\pi(6k+3)}{6}$, for $k \in \mathbb{Z}$.

Question 12

Let $\cos(x) = -\frac{3}{5}$ and $\sin^2(y) = \frac{25}{169}$, where $x \in \left[\frac{\pi}{2}, \pi\right]$ and $y \in \left[\frac{3\pi}{2}, 2\pi\right]$.

The value of $\sin(x) + \cos(y)$ is:

A. $\frac{8}{65}$

B. $-\frac{112}{65}$

C. $\frac{112}{65}$

D. $-\frac{8}{65}$

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Question 13

Ryan's line of sight, while looking at a bird on a tree top, makes a $45°$ angle of elevation. He walks 240 metres towards the tree to observe the bird closely, thus causing his line of sight to make a $60°$ angle of elevation. How far was Ryan from the tree initially?

A. $\frac{240\sqrt{3}}{\sqrt{3}-1}$ metres.

B. $\frac{240}{\sqrt{3}-1}$ metres.

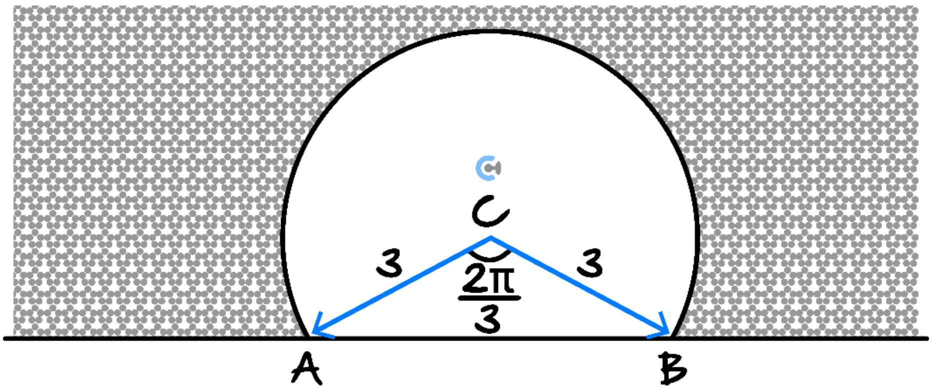
C. $\frac{240}{\sqrt{3}}$ metres.

D. $240\sqrt{3}$ metres.

Question 14

The figure below shows the cross-section of a railway tunnel, modelled as the major segment of a circle, centre at C and radius of $3\ m$.

The angle $\angle ACB$ is $\frac{2\pi}{3}$ radians.



a. Find the exact length of AB.

$$AB = 2 \times 3 \times \sin\left(\frac{\pi}{3}\right) = 3\sqrt{3}$$

b. Determine the area of the triangle ACB.

$$\text{Area} = \frac{1}{2} \times 3 \times 3 \times \sin\left(\frac{2\pi}{3}\right) = \frac{9\sqrt{3}}{4}$$

c. Find the cross-sectional area of the tunnel.

$$\text{Area} = \text{major sector} + \text{ triangle}$$

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 9 \times \frac{4\pi}{3} + \frac{9\sqrt{3}}{4} \\ &= 6\pi + \frac{9\sqrt{3}}{4}\end{aligned}$$

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Question 15

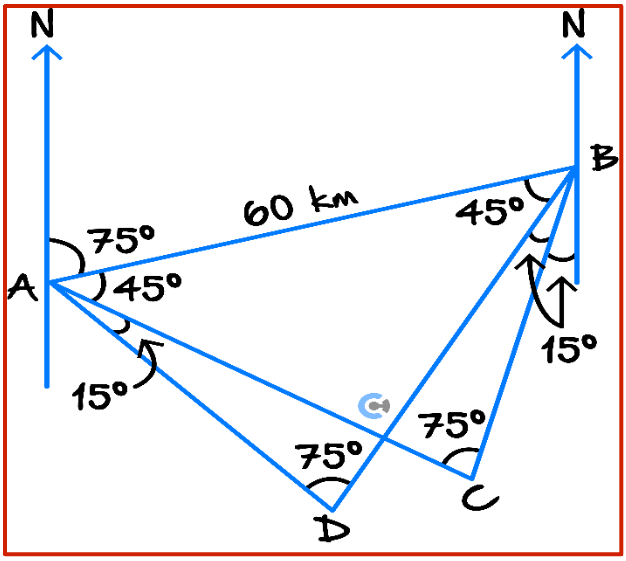
The distance between the town of Alphaville (A) and the town of Betaville (B) is 60 km. Betaville is on a bearing of 75° from Alphaville.

The village of Cappal (C) is on a bearing of 120° from Alphaville and on a bearing of 195° from Betaville. The village of Deltan (D) is on a bearing of 135° from Alphaville and on a bearing of 210° from Betaville.

a. Find, correct to one decimal place where appropriate, the distance between:

i. Betaville and Cappal.

Our first step should be to draw a diagram and fill in as much information as we can.



Then $\frac{60}{\sin(75)} = \frac{BC}{\sin(45)} \implies BC = \frac{120}{1+\sqrt{3}} \approx 43.9$ km.

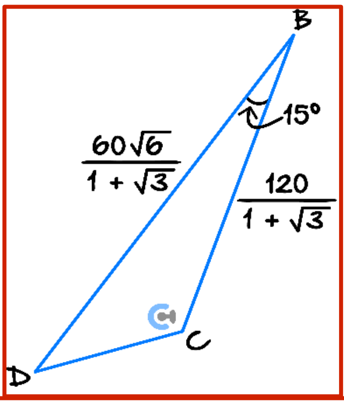
ii. Betaville and Deltan.

$$\frac{60}{\sin(75)} = \frac{BD}{\sin(60)} \implies BD = \frac{60\sqrt{6}}{1+\sqrt{3}} \approx 53.8 \text{ km}$$

iii. Cappal and Deltan.

Draw a diagram and use the cosine rule.

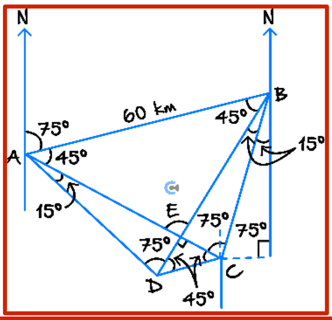
$$|DC| = \sqrt{|DB|^2 + |BC|^2 - 2|DB||BC|\cos(15^\circ)} = 120 - 60\sqrt{3} \approx 16.1$$



b. Find the bearing of Deltan from Cappal.

A diagram and noting that DEC is a right angled isosceles triangle will help us get the bearing as

$$360 - (60 + 45) = 255^\circ$$



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Question 16

The population of koalas in a particular location varies according to the rule:

$$n(t) = 1500 + 500\cos\left(\frac{\pi t}{4}\right)$$

where n is the number of koalas and t is the number of months after 1 March 2015.

a. Find the period and amplitude of the function n.

$\text{Period} = 8 \text{ and amplitude} = 500$

b. Find the maximum and minimum populations of koalas in this location.

$\text{Max} = 2000 \text{ and Min} = 1000$

c. Find $n(2)$.

$n(2) = 1500$

d. Over the 10 months from 1 March 2015, find the fraction of time when the population of koalas in this location was less than $n(2)$.

We find the population is greater than or equal to $n(2) = 1500$ for $t \in [0, 2] \cup [6, 10]$.
So the fraction of time the population is less than $n(2)$ is $\frac{4}{10} = \frac{2}{5}$.

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