Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
Trigonometry II [3.2]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 - Pg 17

Section A: Compulsory Questions

Sub-Section [3.2.1]: Find Trig Ratios of Supplementary Relationships

Question 1

Simplify the following expressions:

a. $\sin(\pi - x)$

$\sin(x)$

b. $\cos\left(\frac{\pi}{2} + x\right)$

$-\sin(x)$

c. $\tan(\pi - x)$

$-\tan(x)$

Space for Personal Notes

Question 2

If $\sin(x) = \frac{3}{5}$, where x is an angle in the first quadrant, evaluate the following:

a. $\sin(\pi + x)$

$$\sin(\pi + x) = -\sin(x) = -\frac{3}{5}$$

b. $\cos(x)$

$$\sin^2(x) + \cos^2(x) = 1 \implies \cos(x) = \frac{4}{5}.$$

c. $\tan(2\pi - x)$

$$\tan(2\pi - x) = -\tan(x) = -\frac{3}{4}$$

Space for Personal Notes

Question 3

If $\cos(x) = -\frac{3}{10}$, where $\pi \leq x \leq \frac{3\pi}{2}$, evaluate the following:

a. $\cos(\pi + x)$

$$\cos(\pi + x) = -\cos(x) = \frac{3}{10}$$

b. $\sin(\pi + x)$

$$\sin(\pi + x) = -\sin(x) = \frac{\sqrt{91}}{10} \text{ (using the Pythagorean identity)}$$

c. $\tan(\pi - x)$

Note that $\tan(x) = \frac{\sqrt{91}}{3}$, so $\tan(\pi - x) = -\frac{\sqrt{91}}{3}$

Question 4 Tech-Active.

If $\sin(x) = -\frac{9}{20}$, where x is a third quadrant angle, evalute $\cos(\pi + x)$.

$$\cos(\pi + x) = -\cos(x) = \sqrt{1 - \left(\frac{9}{20}\right)^2} = \frac{\sqrt{319}}{20}$$

Sub-Section [3.2.2]: Find Particular and General Solutions

Question 5

Solve the following trigonometric equations over the specified domain:

a. $2\cos(x) = \sqrt{3}, x \in [0, 2\pi]$

$$\cos(x) = \frac{\sqrt{3}}{2} \implies x = \frac{\pi}{6}, \frac{11\pi}{6}$$

b. $4\sin(3x) = 2, x \in [0, \pi]$

$$\sin(3x) = \frac{1}{2} \implies 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$
$$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$$

c. $8\tan(2x) - 5 = 3, x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$$\tan(2x) = 1 \implies 2x = -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$$
$$x = -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$$

Question 6

Find the general solution to the following trigonometric equations:

a. $2\sin\left(-4x + \frac{\pi}{6}\right) = 1$

$$\sin\left(-4x + \frac{\pi}{6}\right) = \frac{1}{2}$$
$$-4x + \frac{\pi}{6} = \frac{\pi}{6} + 2n\pi \text{ or } -4x + \frac{\pi}{6} = \frac{5\pi}{6}$$
$$-4x = 2n\pi \text{ or } -4x = \frac{2\pi}{3} + 2n\pi$$
$$x = \frac{n\pi}{2} \text{ or } x = \frac{n\pi}{2} - \frac{\pi}{6}, \quad n \in \mathbb{Z}.$$

b. $\sqrt{2}\cos\left(3x-\frac{\pi}{2}\right)=1$

Equivalent to $\sin(3x)=\frac{1}{\sqrt{2}}$.

$$3x=\frac{\pi}{4}+2n\pi \text{ or } 3x=\frac{3\pi}{4}+2n\pi$$
$$x=\frac{\pi}{12}+\frac{2n\pi}{3} \text{ or } x=\frac{\pi}{4}+\frac{2n\pi}{3}, \quad n\in\mathbb{Z}.$$

c. $\tan\left(\frac{\pi}{2}x+\frac{\pi}{3}\right)-1=0$

$$\tan\left(\frac{\pi}{2}x+\frac{\pi}{3}\right)=1$$
$$\frac{\pi}{2}x+\frac{\pi}{3}=\frac{\pi}{4}+n\pi$$
$$\frac{\pi}{2}x=-\frac{\pi}{12}+n\pi$$
$$x=-\frac{1}{6}+2n, \quad n\in\mathbb{Z}.$$

Space for Personal Notes

Question 7

Consider the function $f(x) = 2\tan\left(3x + \frac{\pi}{3}\right) + 2$.

a. Find the general solution to $f(x) = 0$.

$$\tan\left(3x + \frac{\pi}{3}\right) = -1$$

$$3x + \frac{\pi}{3} = \frac{3\pi}{4} + n\pi$$

$$3x = \frac{5\pi}{12} + n\pi$$

$$x = \frac{5\pi}{36} + \frac{n\pi}{3}, \quad n \in \mathbb{Z}$$

b. Hence, solve $f(x) = 0$ for $x \in [0, \pi]$.

$$x = \frac{5\pi}{36}, \frac{17\pi}{36}, \frac{29\pi}{36}$$

Question 8 Tech-Active.

Find the general solution to $2\sin(\pi(x - 2)) = 1$.

$$x = \frac{13}{6} + 2n \text{ or } x = \frac{17}{6} + 2n, \quad n \in \mathbb{Z}$$

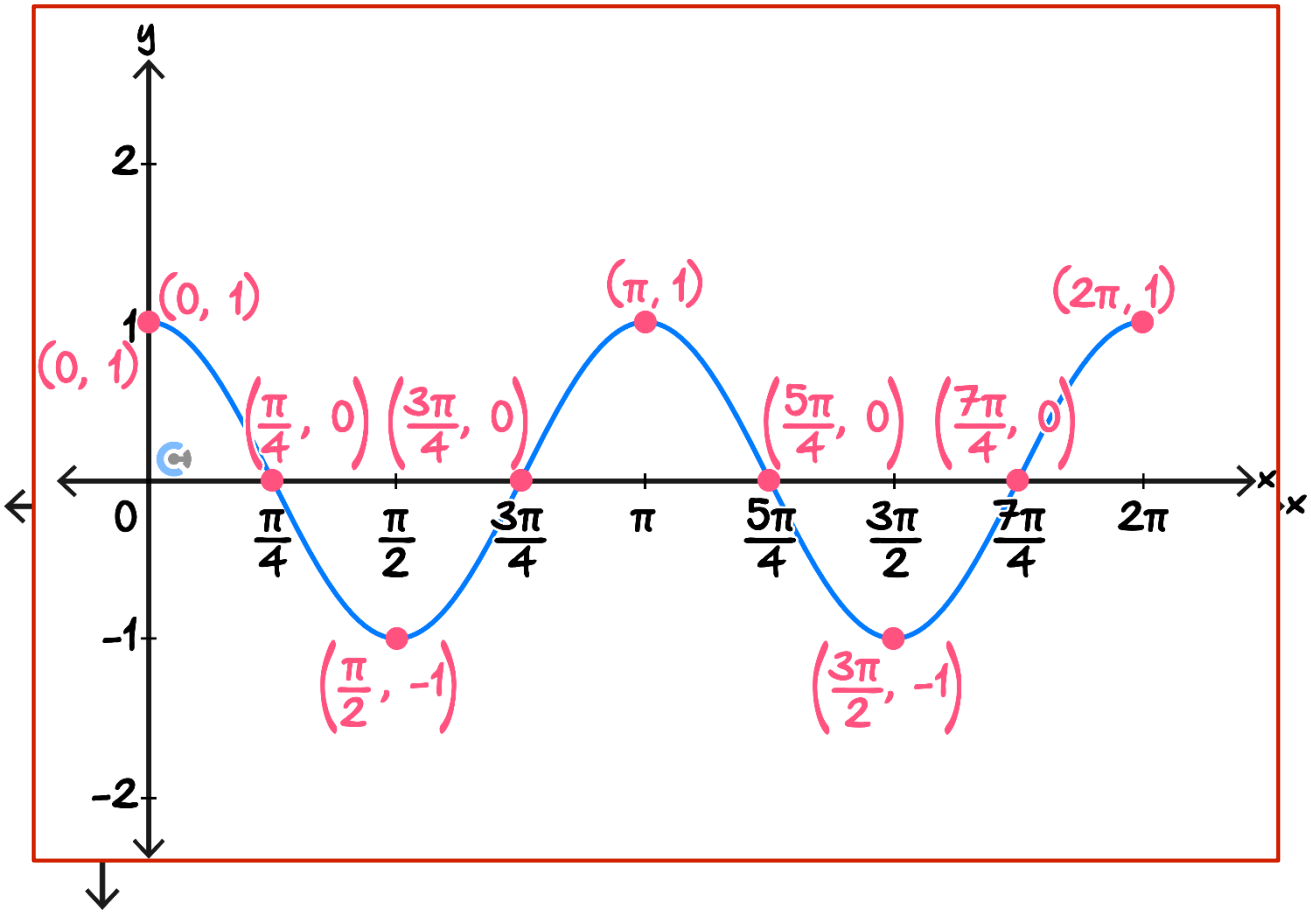
```
In[196]:= Solve[2 Sin[Pi (x - 2)] == 1] // Expand
Out[196]= {{x → 13/6 + 2 c1 if c1 ∈ Z}, {x → 17/6 + 2 c1 if c1 ∈ Z}}
```

Sub-Section [3.2.3] : Graph Sine, Cosine and Tangent Functions

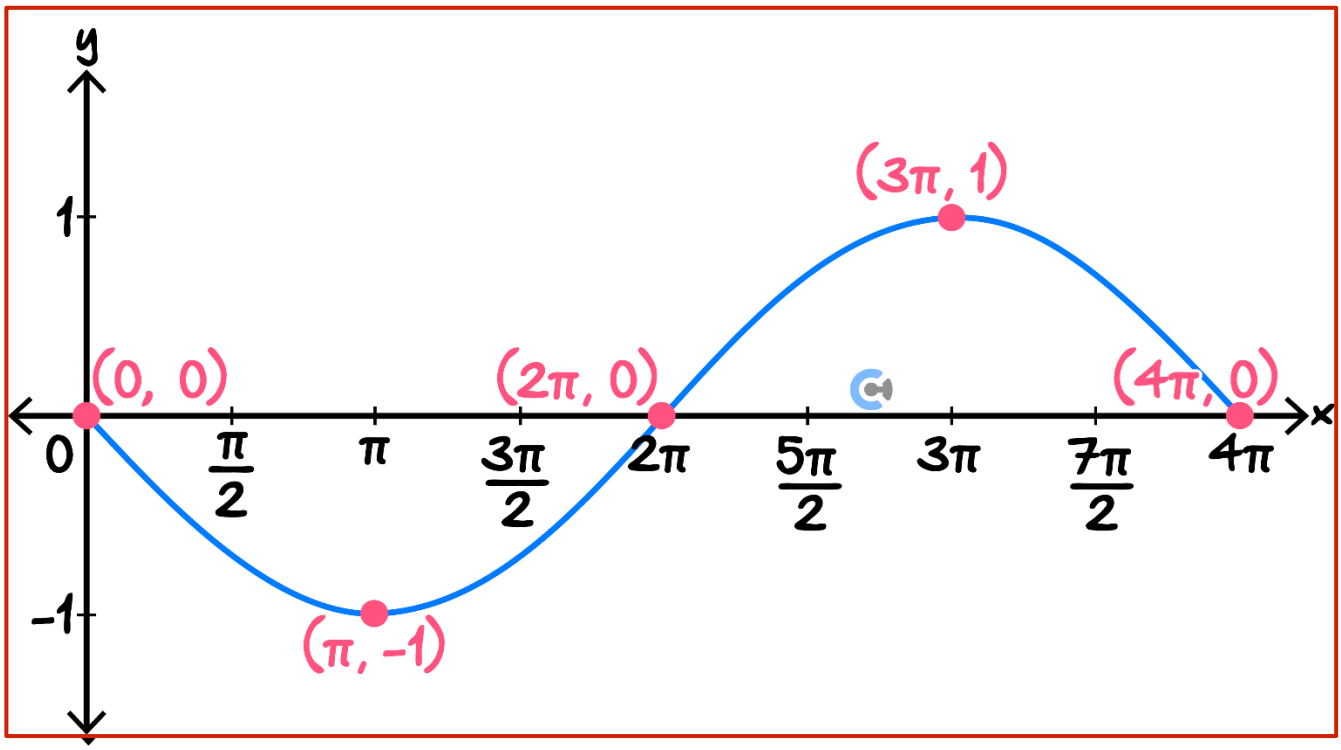
Question 9

Sketch the the graphs of the following functions over the indicated domain. Label all axes intercepts and endpoints with coordinates, and label asymptotes with equations.

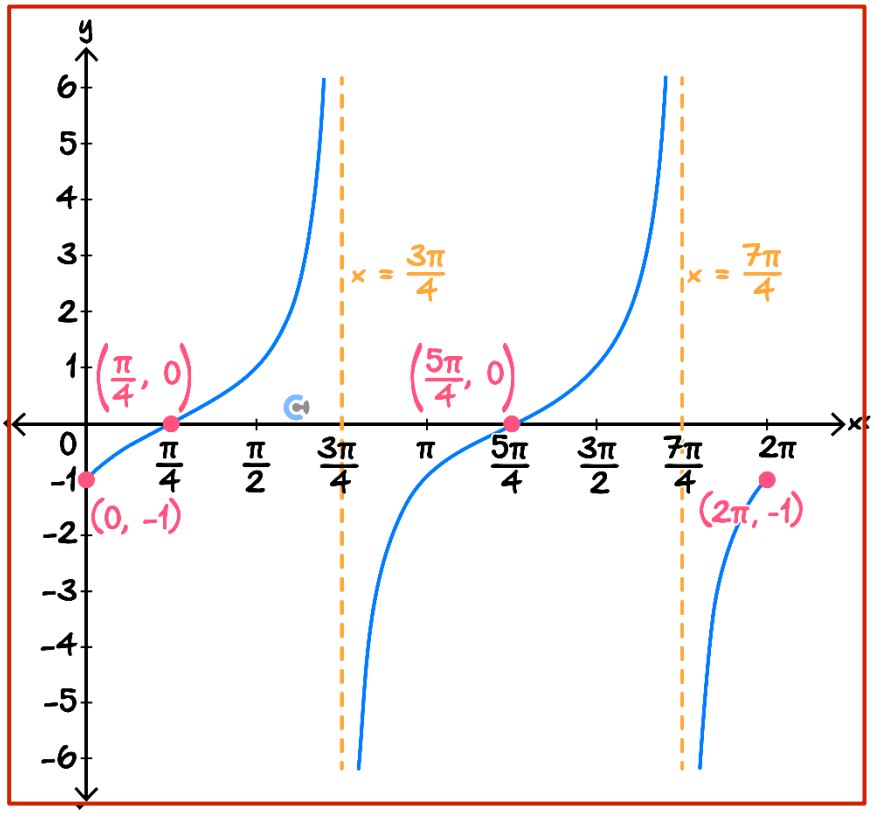
a. $y = \cos(2x), x \in [0, 2\pi]$



b. $y = -\sin\left(\frac{x}{2}\right), x \in [0,4\pi]$



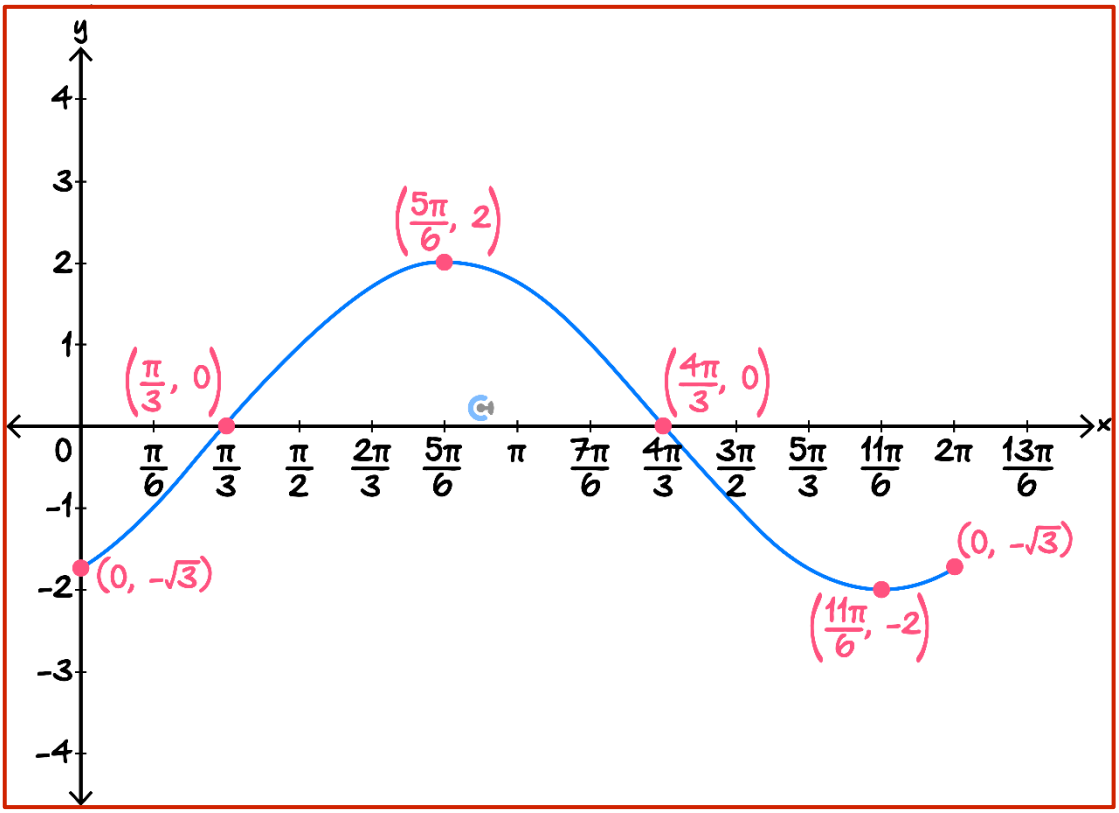
c. $y = \tan\left(x - \frac{\pi}{4}\right), x \in [0,2\pi]$



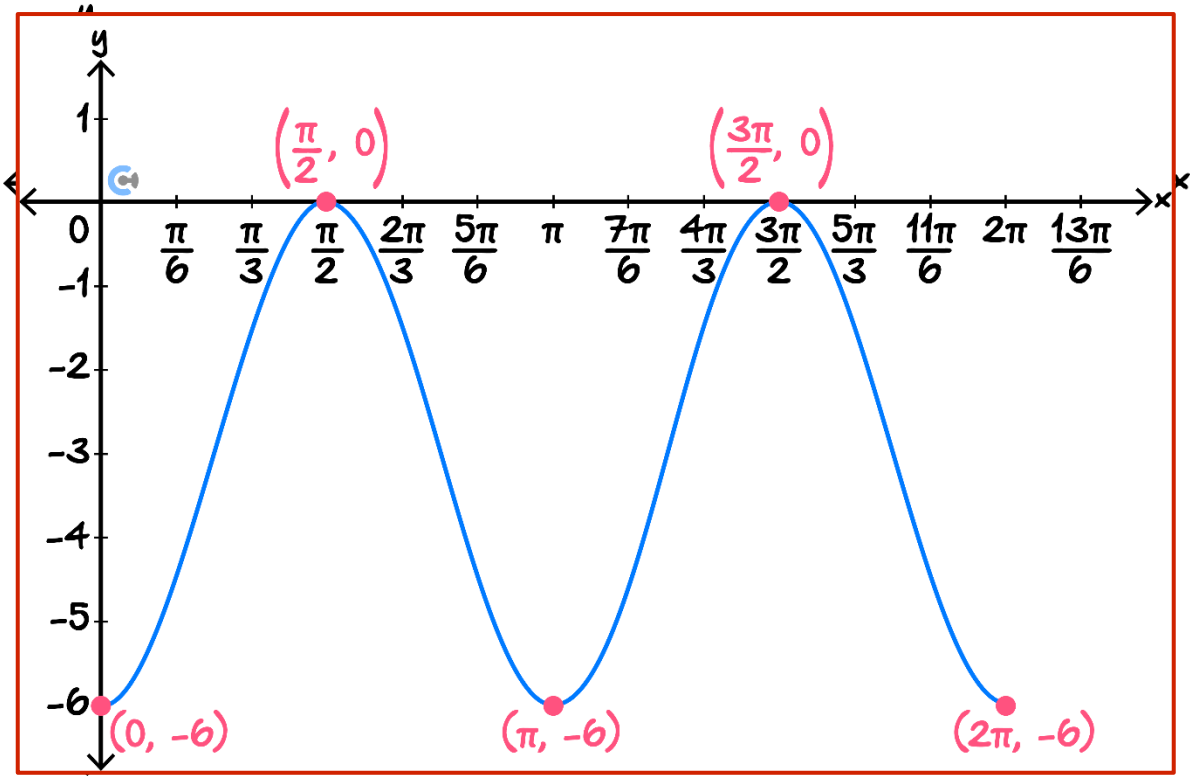
Question 10

Sketch the graphs of the following functions over the indicated domain. Label all axes intercepts, turning points and endpoints with coordinates, and label asymptotes with equations.

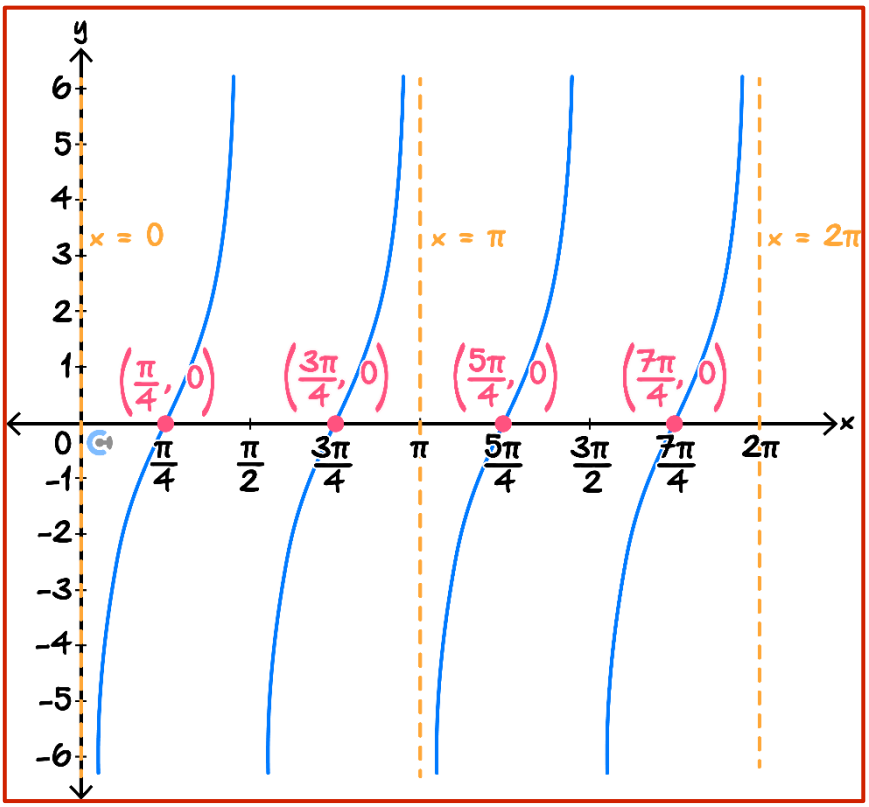
a. $y = 2\sin\left(x - \frac{\pi}{3}\right), x \in [0, 2\pi]$



b. $y = -3\cos(2x) - 3, x \in [0, 4\pi]$



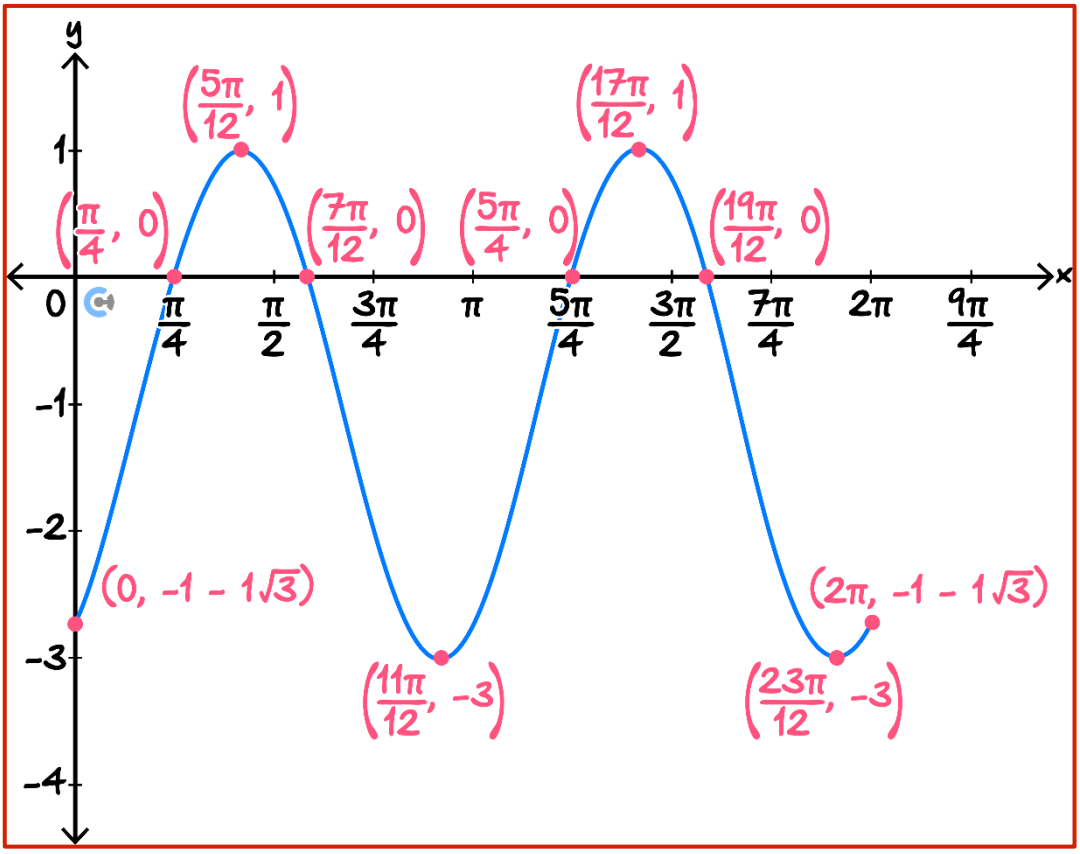
c. $y = 2\tan\left(2x - \frac{\pi}{2}\right), x \in [0, 2\pi]$



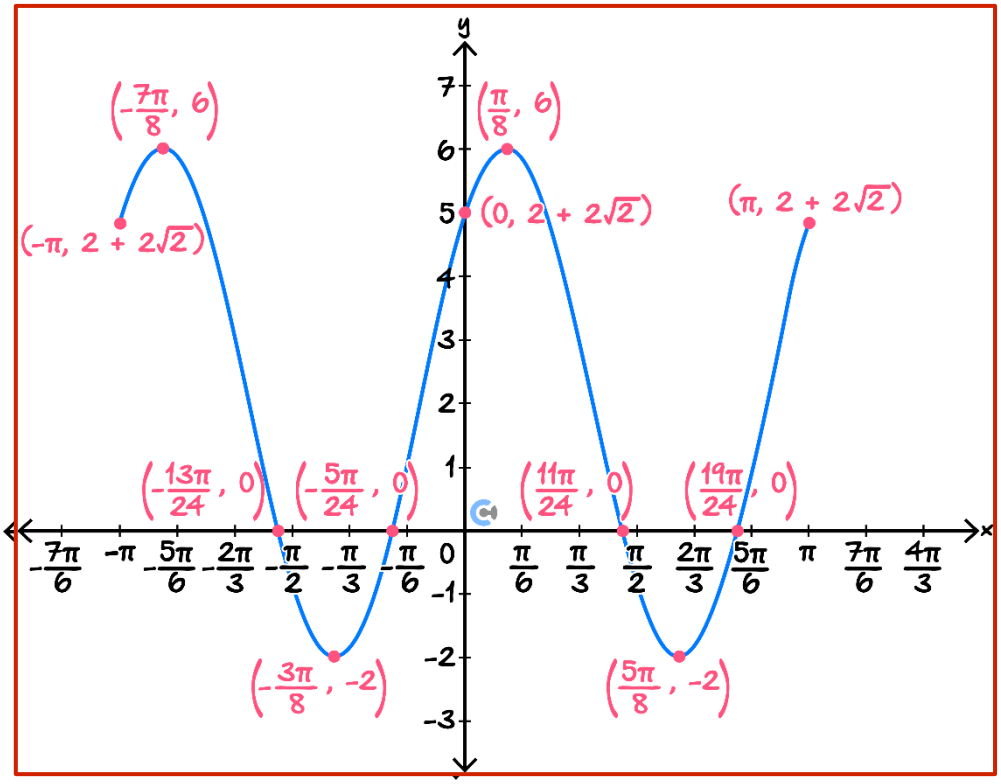
Question 11

Sketch the graphs of the following functions over the indicated domain. Label all axes intercepts, turning points and endpoints with coordinates, and label asymptotes with equations.

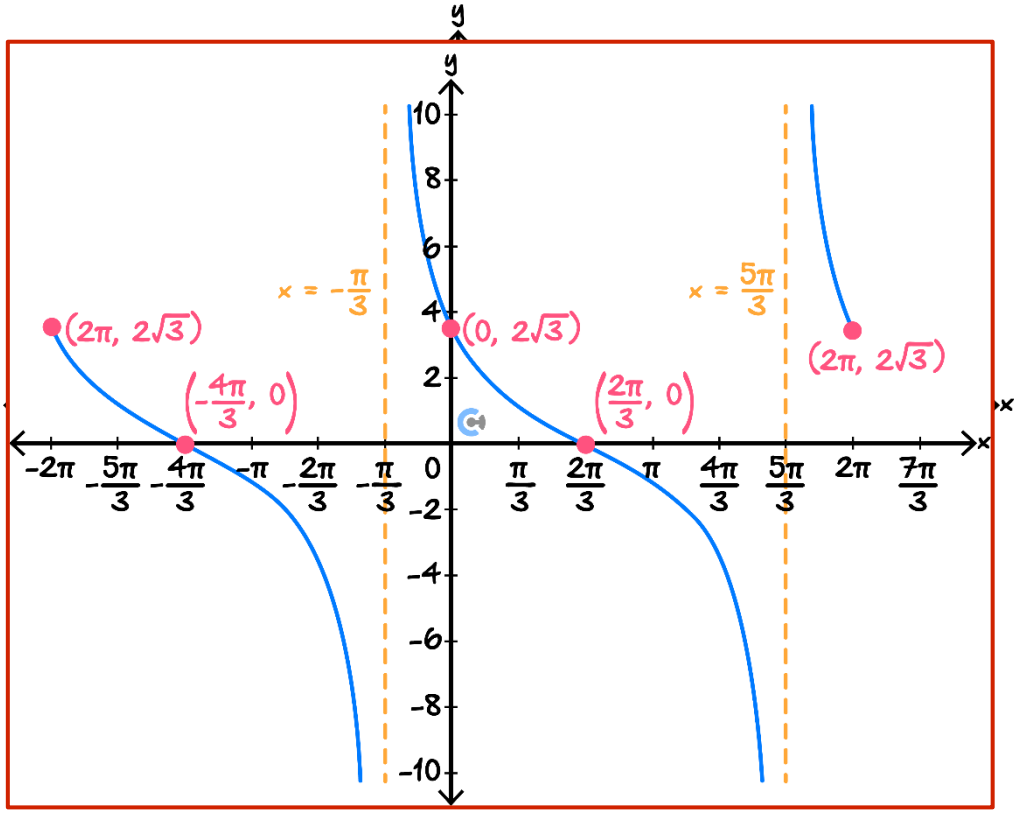
a. $y = 2\sin\left(2x - \frac{\pi}{3}\right) - 1, x \in [0,2\pi]$



b. $y = 4\cos\left(\frac{\pi}{4} - 2x\right) + 2, x \in [-\pi, \pi]$

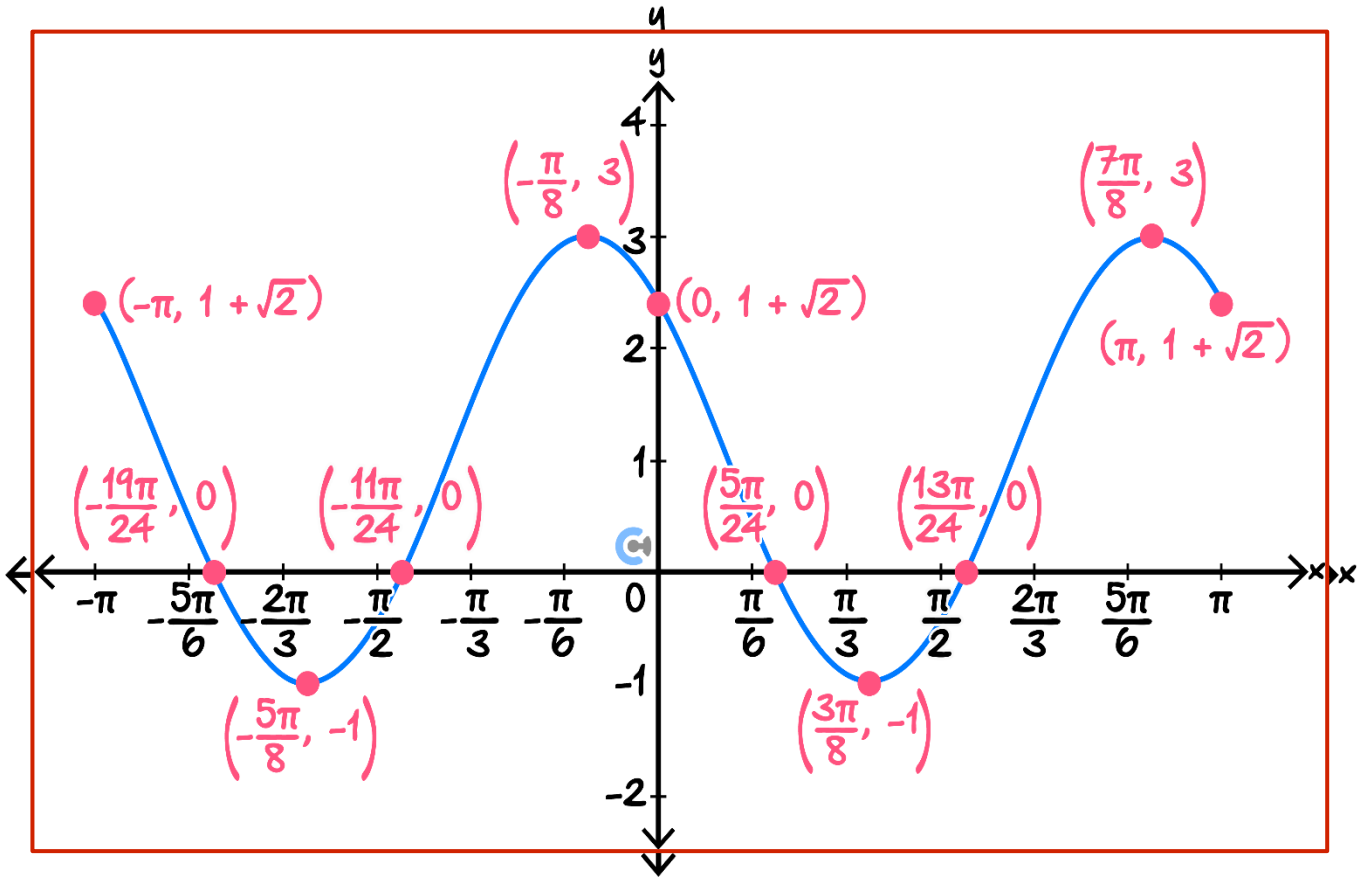


c. $y = 2\tan\left(\frac{\pi}{3} - \frac{x}{2}\right), x \in [-2\pi, 2\pi]$



Question 12 Tech-Active.

Sketch the graph of $y = 2\cos\left(2x + \frac{\pi}{4}\right)$. Label all axes intercepts, turning points and endpoints with coordinates.



Space for Personal Notes

Sub-Section: The 'Final Boss'

Question 13

Consider the function $f(x) = 3\sin\left(2x + \frac{\pi}{3}\right) + \cos\left(2x + \frac{5\pi}{6}\right) - 1$.

a. Express $f(x)$ in the form $f(x) = a\sin(2x + b) - 1$.

Note that $\cos\left(2x + \frac{5\pi}{6}\right) = \cos\left(2x + \frac{\pi}{3} + \frac{\pi}{2}\right) = -\sin\left(2x + \frac{\pi}{3}\right)$.

Therefore $f(x) = 2\sin\left(2x + \frac{\pi}{3}\right) - 1$

b. Find the general solution to $f(x) = 0$.

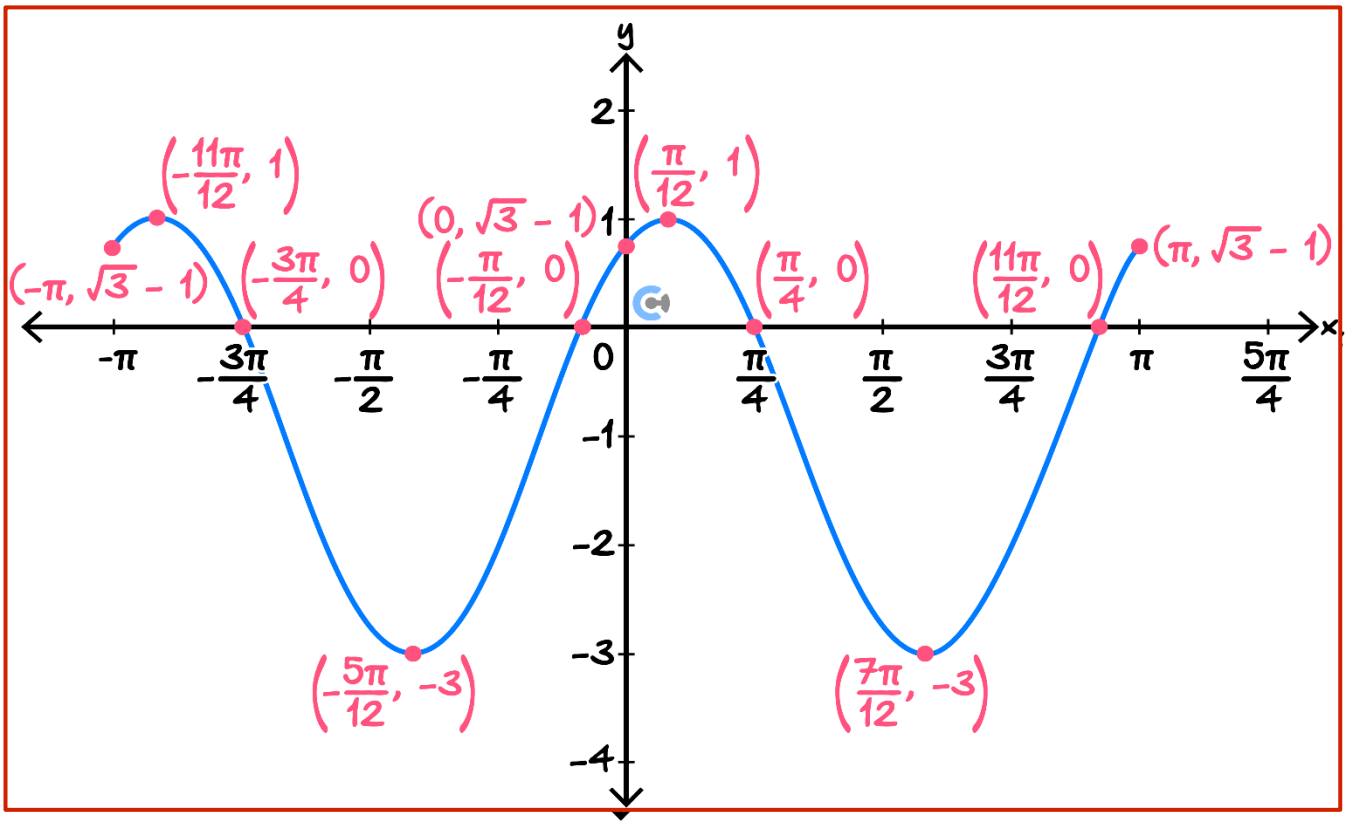
$f(x) = 0 \implies \sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$. Then

$$2x + \frac{\pi}{3} = \frac{\pi}{6} + 2n\pi, \text{ or } 2x + \frac{\pi}{3} = \frac{5\pi}{6} + 2n\pi$$
$$2x = -\frac{\pi}{6} + 2n\pi, \text{ or } 2x = \frac{\pi}{2} + 2n\pi$$
$$x = -\frac{\pi}{12} + n\pi, \text{ or } x = \frac{\pi}{4} + n\pi, n \in \mathbb{Z}.$$

c. Find all solutions to $f(x) = 0$ for $x \in [-\pi, \pi]$.

$$x = -\frac{\pi}{12}, \frac{11\pi}{12}, \frac{\pi}{4}, -\frac{3\pi}{4}$$

d. Sketch the graph of $y = f(x)$ on the axes below. Labell all axes intercepts, turning points and endpoints with coordinates.



Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Specialist Mathematics ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45 + raw scores and 99 + ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after school weekdays, and all day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-specialist-consult-2025