Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
Trigonometry I [3.1]
Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 – Pg 17

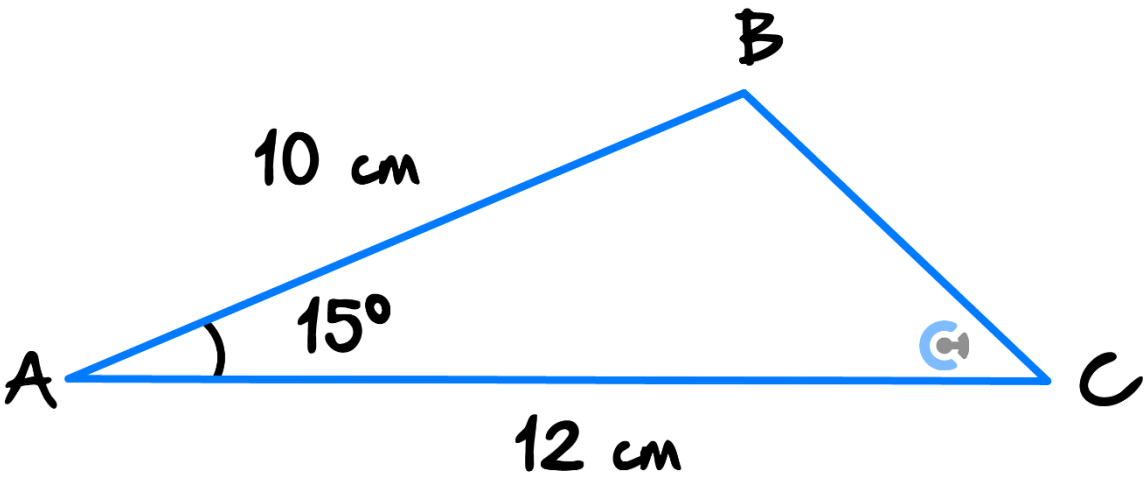
Section A: Compulsory Questions

Sub-Section [3.1.1]: Find Lengths, Angles and Area of Triangles Using Sine and Cosine Rule

Question 1

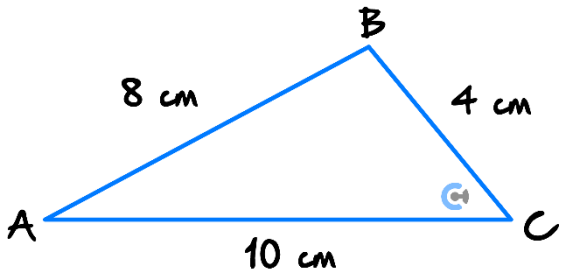
You may use a CAS for the following questions. Give your answers correct to two decimal places.

a. Find the length of BC in the following triangle.



$$BC^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos(15) \implies BC = 3.49\text{cm}.$$

b. Find the angle BAC in the following triangle.

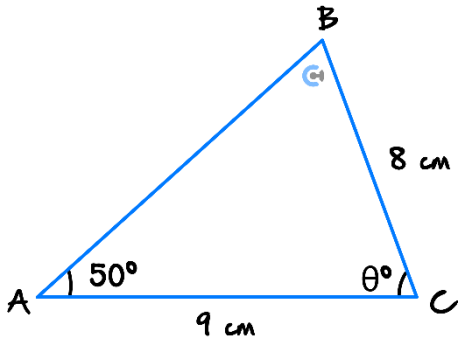


Let $\theta = \angle BAC$. Then

$$\cos(\theta) = \frac{8^2 + 10^2 - 4^2}{2 \times 8 \times 10}$$

$$\theta \approx 22.33$$

c. Find the angle θ in the following triangle given that $\angle ABC$ is acute.



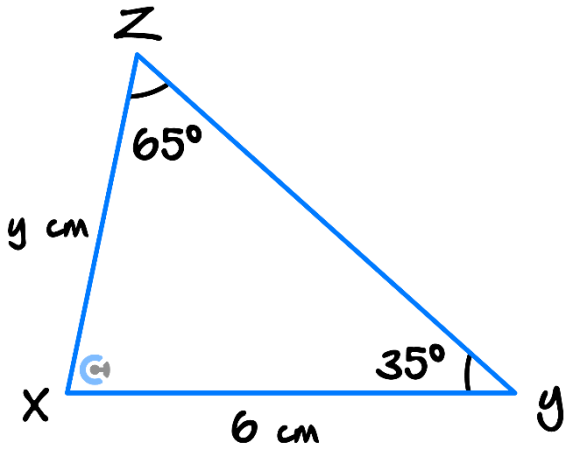
Let $\angle ABC = \phi$ then using sin law

$$\frac{\sin(50)}{8} = \frac{\sin(\phi)}{9} \implies \phi = 59.5193^\circ$$

Thus $\theta = 180 - 59.5193 - 50 = 70.48^\circ$

Question 2

Find the area of the following triangle. Give your answer correct to two decimal places.



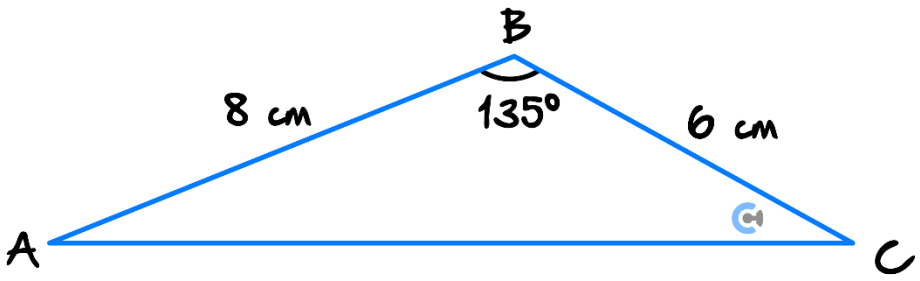
$$\frac{y}{\sin(35)} = \frac{6}{\sin(65)} \implies y = 3.79723$$

$$\text{Area} = \frac{1}{2} \times 3.79723 \times 6 \times \sin(80) = 11.22$$

Space for Personal Notes

Question 3

Find all side lengths and angles for the following triangle. Give your answers correct to two decimal places.



$AC^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos(135) = 100 + 48\sqrt{2} \implies AC = 12.96.$

Then sin law $\dfrac{\sin(A)}{6} = \dfrac{\sin(135)}{\sqrt{100 + 48\sqrt{2}}} \implies A = 19.11°$

Thus $C = 25.89°$

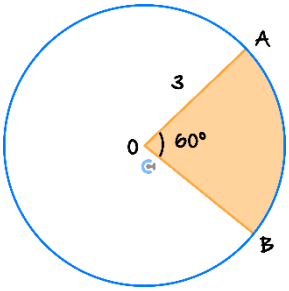
Space for Personal Notes

Sub-Section [3.1.2]: Find Arc Lengths, Chord Lengths, Sector and Segment Areas

Question 4

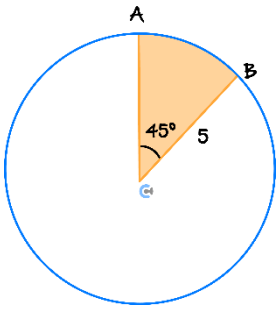
Consider the following circles:

a. Find the area of the shaded sector.



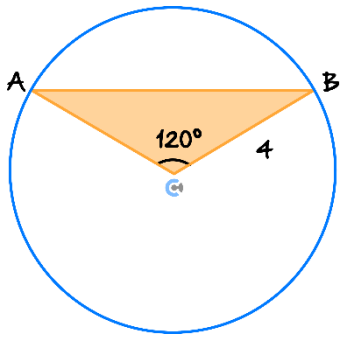
$$\text{Area} = \frac{60}{360} \times 3^2\pi = \frac{3\pi}{2}$$

b. Find the length of the arc AB.



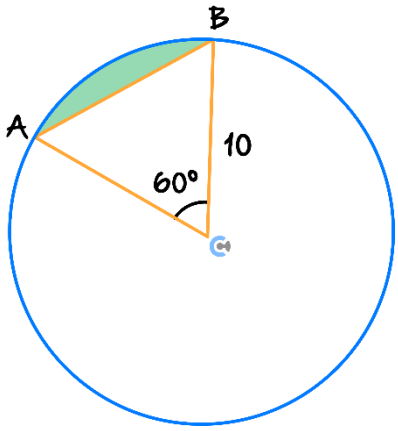
$$\text{Arc } AB = \frac{45}{360} \times 2 \times \pi \times 5 = \frac{5\pi}{4}$$

c. Find the length of the chord AB.



$$\text{Chord } AB = 2 \times 4 \times \sin(60) = 4\sqrt{3}$$

d. Find the area of the shaded segment.

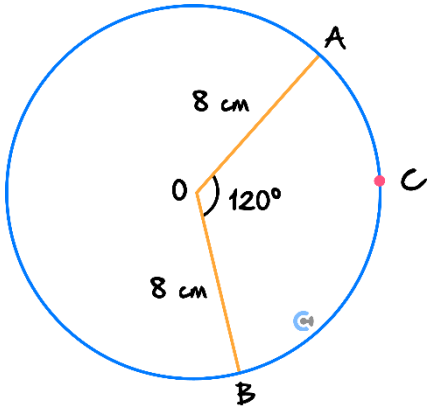


$60° = \frac{\pi}{3}^c$. Then

$$\text{Segment area} = \frac{1}{2} \times 10^2 \left(\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right)\right)$$
$$= \frac{50\pi}{3} - 25\sqrt{3}$$

Question 5

The circle shown has a centre O and radius length 8 cm. The angle subtended at O by arc ACB has magnitude 120^o.



a. Find the exact length of the chord AB.

$$AB = 2 \times 8 \times \sin(60) = 8\sqrt{3}$$

b. Find the exact length of the arc ACB.

Note $120° = \frac{2\pi}{3}^c$.

$$\text{Arc length} = 8 \times \frac{2\pi}{3} = \frac{16\pi}{3}$$

c. Find the exact area of the minor sector AOB.

$$A = \frac{1}{2} \times 8^2 \times \frac{2\pi}{3} = \frac{64\pi}{3}$$

d. Find the magnitude of the angle AOC, in degrees, if the minor arc has a length 3 cm. Give your answer in degrees correct to two decimal places.

Let $\angle AOC = \theta$ in radians. Then $8\theta = 3 \implies \theta = \dfrac{3}{8}^c$.

In degrees this is $\dfrac{3}{8} \times \dfrac{180}{\pi} = \dfrac{540}{8\pi} = \dfrac{135}{2\pi} = 21.49°$

Question 6

Two circles both with a radius of 6 cm have their centres 10 cm apart. Calculate the exact area of the region common to both circles and then round this result to two decimal places.

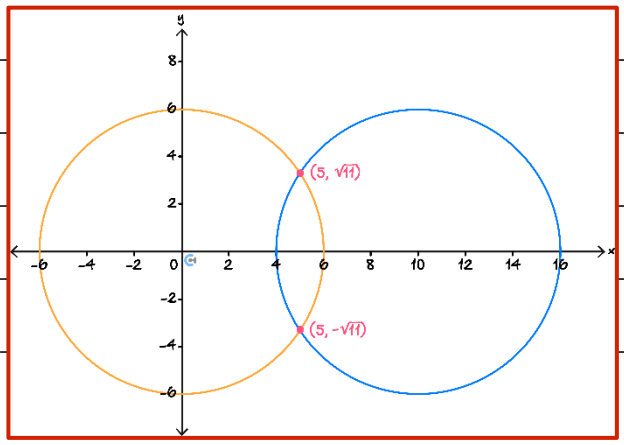
By symmetry the required area is double the area of one of the segments and we find that the circles intersect each other at $(5, \sqrt{11})$ and $(5, -\sqrt{11})$.
Isosceles triangle with side lengths 6, 6 and $2\sqrt{11}$. Then

$$\cos(\theta) = \frac{36 + 36 - (2\sqrt{11})^2}{2 \times 36} = \frac{7}{18}$$

then $\cos^2(\theta) + \sin^2(\theta) = 1 \implies \sin(\theta) = \dfrac{5\sqrt{11}}{18}$.

Common area $= 2 \times \dfrac{1}{2} \times 6^2 \left(\cos^{-1}\left(\dfrac{7}{18}\right) - \dfrac{5\sqrt{11}}{18} \right) = 36\cos^{-1}\left(\dfrac{7}{18}\right) - 10\sqrt{11} \approx 9.00\text{cm}^2$

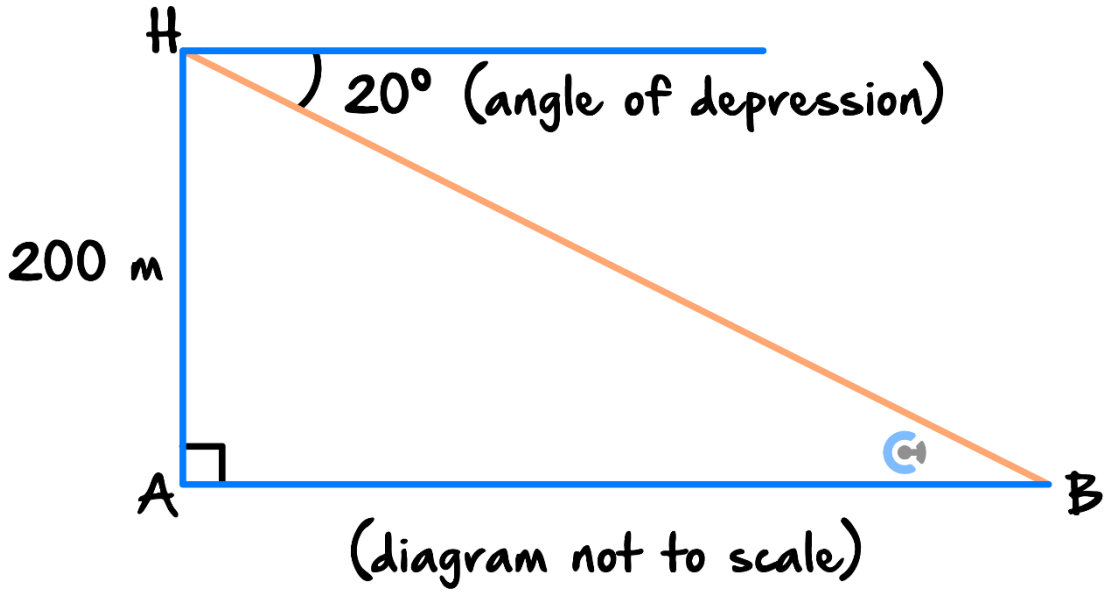
Alternatively we could have found $\theta = 2\tan^{-1}\left(\dfrac{\sqrt{11}}{5}\right)$



Sub-Section [3.1.3]: Apply Angle of Elevation/Depression and Bearing to Solve Geometric Problems (Only 2D)

Question 7

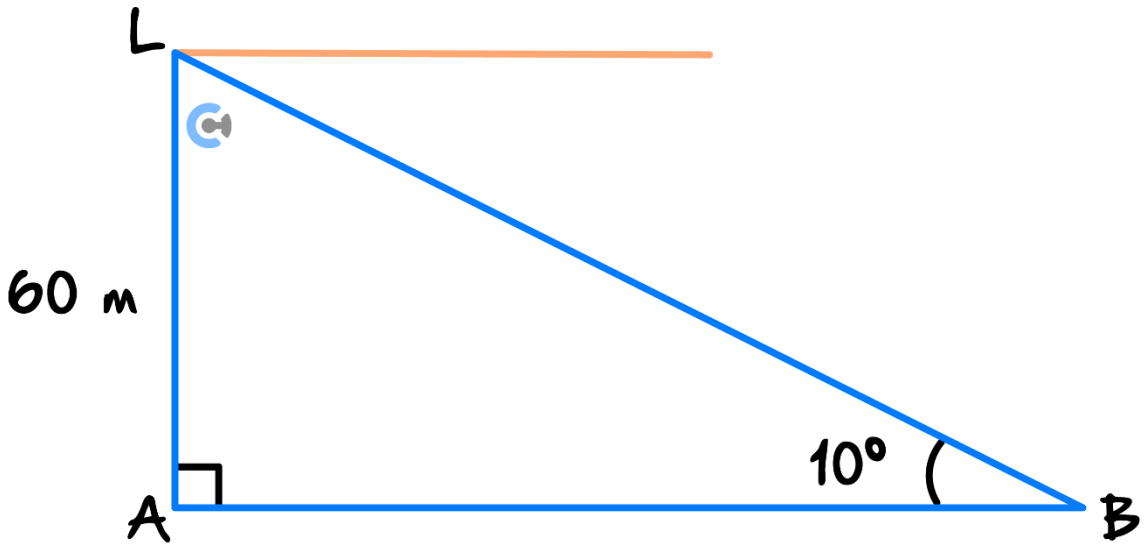
a. A passenger in a hot air balloon flying at 200 m observes a boat at an angle of depression of 20°, as shown in the diagram below. Calculate the horizontal distance between the balloon and the boat correctly to the nearest metre.



Note that $\angle ABH = 20^{\circ}$.

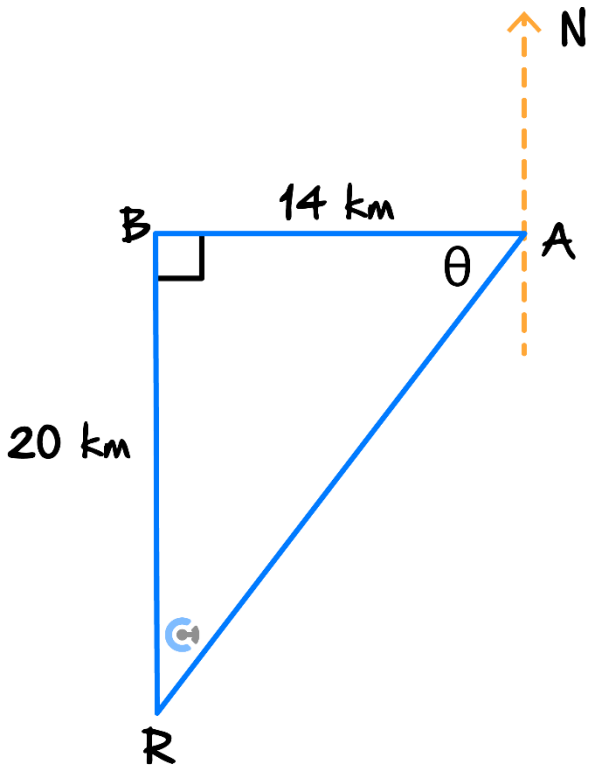
$$\frac{AH}{AB} = \tan(20^{\circ}) \implies AB = \frac{400}{\tan(20^{\circ})} = 1099 \text{ metres.}$$

b. The light shining at the top of a lighthouse is known to be 60 m above sea level. The light is observed from a boat at an angle of elevation of 10^o. The situation is shown in the diagram below. Calculate the horizontal distance from the boat to the bottom of the lighthouse, correct to the nearest metre.



$$\tan(10°) = \frac{60}{AB} \implies AB = \frac{60}{\tan(10°)} = 340 \text{ metres.}$$

c. The road from Alphaville runs due west for 14 km to Betaville. A monument to Bertrand Russell is located 20 km due south of Betaville. The situation is shown in the diagram below. Calculate the distance and bearing of the monument from the centre of Alphaville give your answers correct to two decimal places.

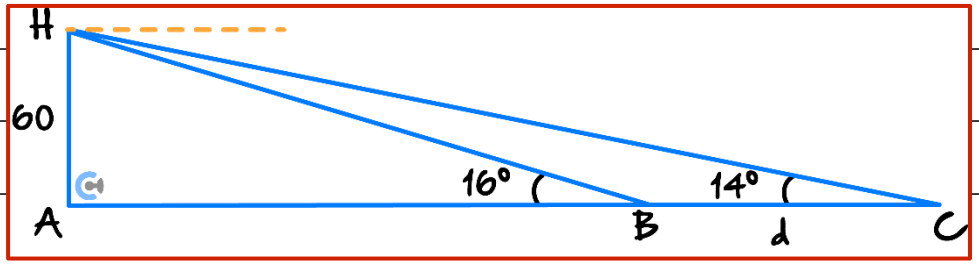


$\tan\theta = \dfrac{20}{14} \implies \theta = 55.00°.$
Thus the bearing is $180 + (90 - 55) = 180 + 35 = 215.00°$.
Then $AR^2 = 20^2 + 14^2 \implies AR = 2\sqrt{149} = 24.41$ km.

Space for Personal Notes

Question 8

A person standing on top of a sea cliff that is 60 m high is in line with two buoys whose angles of depression are 14^o and 16^o. Calculate the distance between the buoys. Give your answer correct to two decimal places.



$$\tan(16°) = \frac{60}{AB} \implies AB \approx 209.245$$

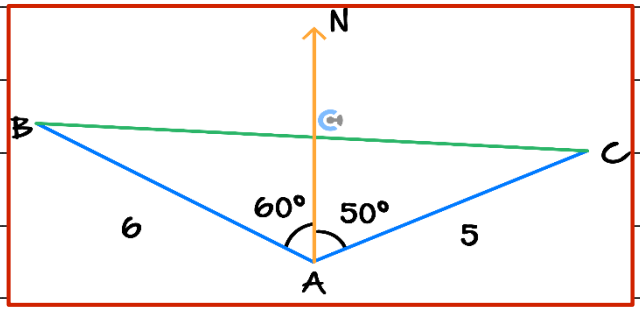
Similarly $\tan(14°) = \frac{60}{AC} \implies AC \approx 240.647.$

Thus $d = 240.647 - 209.245 = 31.40$ metres.

Space for Personal Notes

Question 9

From a ship A, two other ships B and C are on bearings of 300^o and 050^o respectively. The distance BA is 6 km and the distance CA is 5 km. Find the distance BC. Give your answer correct to two decimal places.



From our diagram it is clear that we should use the cosine rule to find BC with $\theta = 60 + 50 = 110°$.

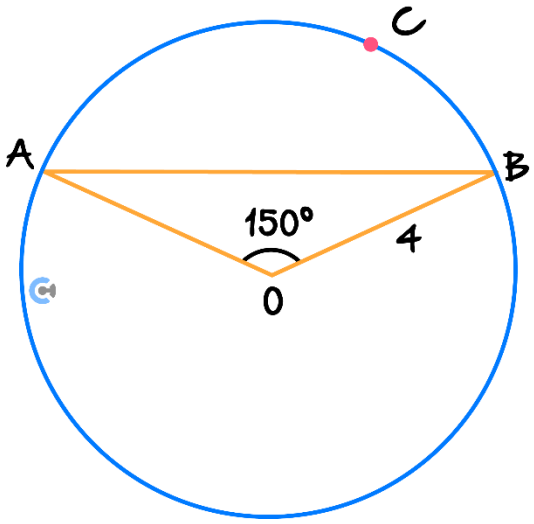
$$BC^2 = 6^2 + 5^2 - 2 \times 5 \times 6\cos(110°)$$
$$BC \approx 9.03 \text{ km}$$

Space for Personal Notes

Sub-Section: The 'Final Boss'

Question 10

Consider a circle of radius 4, and centre O. The angle subtended at O by the arc AB has a magnitude of 150^o. Exact answers are required for all parts and no CAS allowed.



a.

i. Find the length of the chord AB.

We can't compute $\sin(75°)$ exactly without CAS (yet!). So use the cosine rule.

$$\begin{aligned} AB^2 &= 4^2 + 4^2 - 2 \times 16 \times \cos(150°) \\ &= 32 - 32\left(-\frac{\sqrt{3}}{2}\right) \\ &= 32 + 16\sqrt{3} \end{aligned}$$

Thus $AB = \sqrt{32 + 16\sqrt{3}}$

ii. Find the length of the arc AB.

$$\frac{150}{360} \times 2 \times \pi \times 4 = 8\pi \times \frac{5}{12} = \frac{10\pi}{3}$$

b. Find the area of the minor segment formed by the chord AB.

$$\text{Area} = \frac{1}{2} \times 4^2 \times \left(\frac{5\pi}{6} - \sin\left(\frac{5\pi}{6}\right)\right) = 8 \times \frac{5\pi}{6} - 8 \times \frac{1}{2} = \frac{20\pi}{3} - 4$$

c. The point C is located between A and C such that it divides the arc AB in a $2:1$ ratio. Find the length of the arc CB and the angle BOC in degrees.

$AB = 4 \times \dfrac{5\pi}{6} = \dfrac{10\pi}{3}$

Thus $AC = \dfrac{20\pi}{9}$ and $CB = \dfrac{10\pi}{9}$.

Let $\theta = \angle BOC$. Then $4 \times \theta = \dfrac{10\pi}{9} \implies \theta = \dfrac{5\pi}{18}$.

In degrees $\theta = \dfrac{5\pi}{18} \times \dfrac{180}{\pi} = 5 \times 10 = 50°$

Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Specialist Mathematics ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45 + raw scores and 99 + ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after-school weekdays, and all-day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-specialist-consult-2025