Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
AOS 3 Revision [3.0]
Contour Check

Contour Check

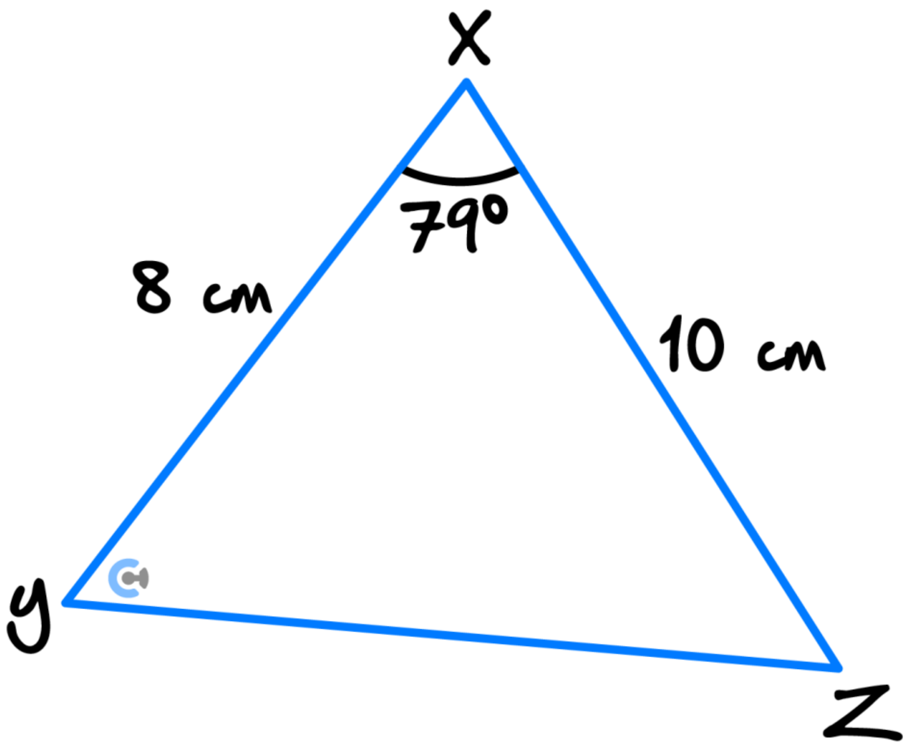
Section A: [3.1] - Trigonometry I (Checkpoints)

Sub-Section: [3.1.1] - Find Lengths, Angles and Area of Triangles Using Sine and Cosine Rule

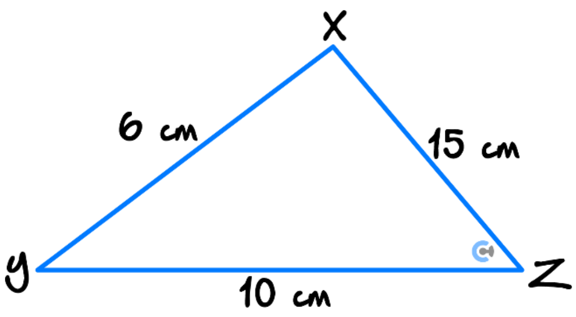
Question 1

You may use a CAS for the following questions. Give your answers correct to two decimal places.

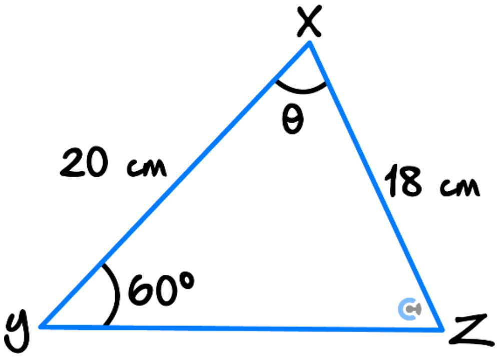
a. Find the length of YZ in the following triangle.



b. Find the angle YXZ in the following triangle.

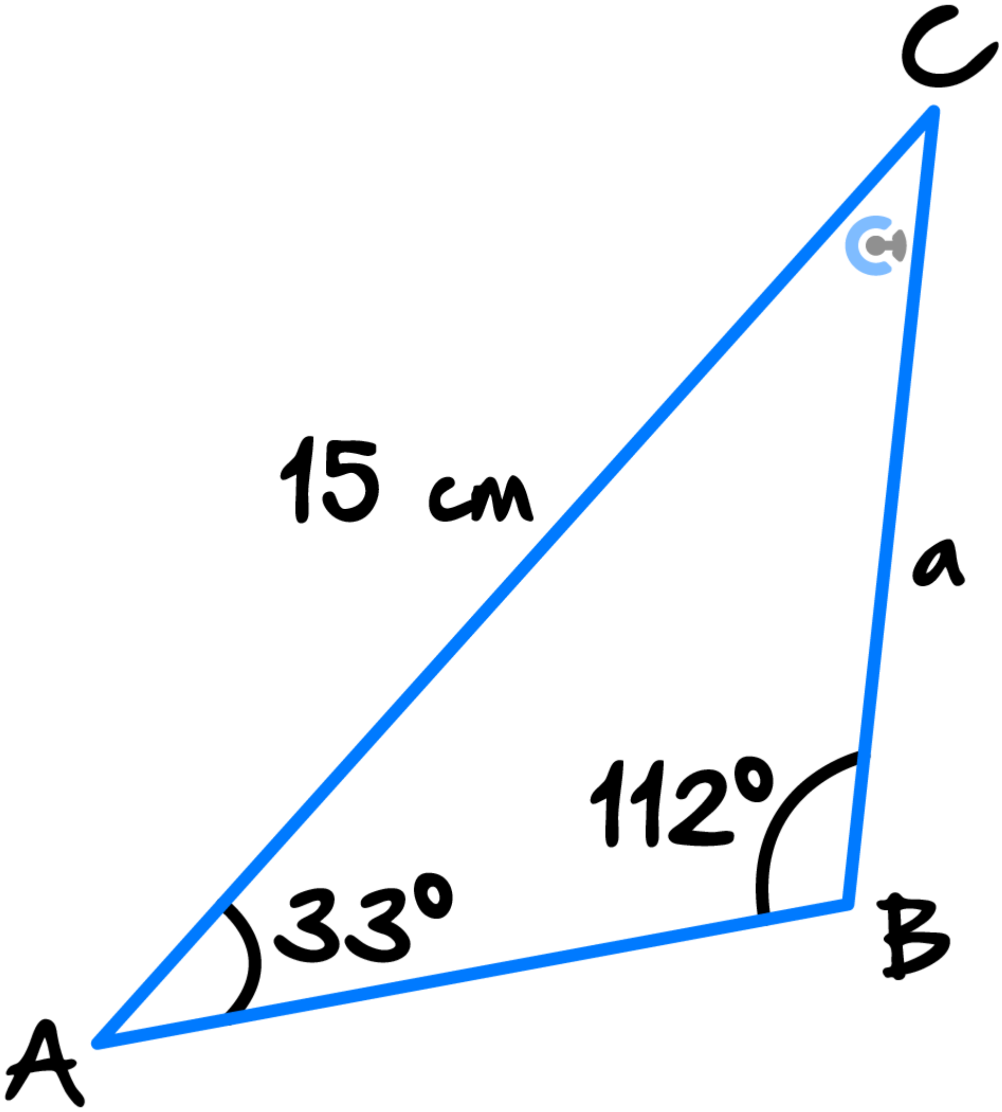


c. Find the angle θ in the following triangle given that $\angle XZY$ is acute.



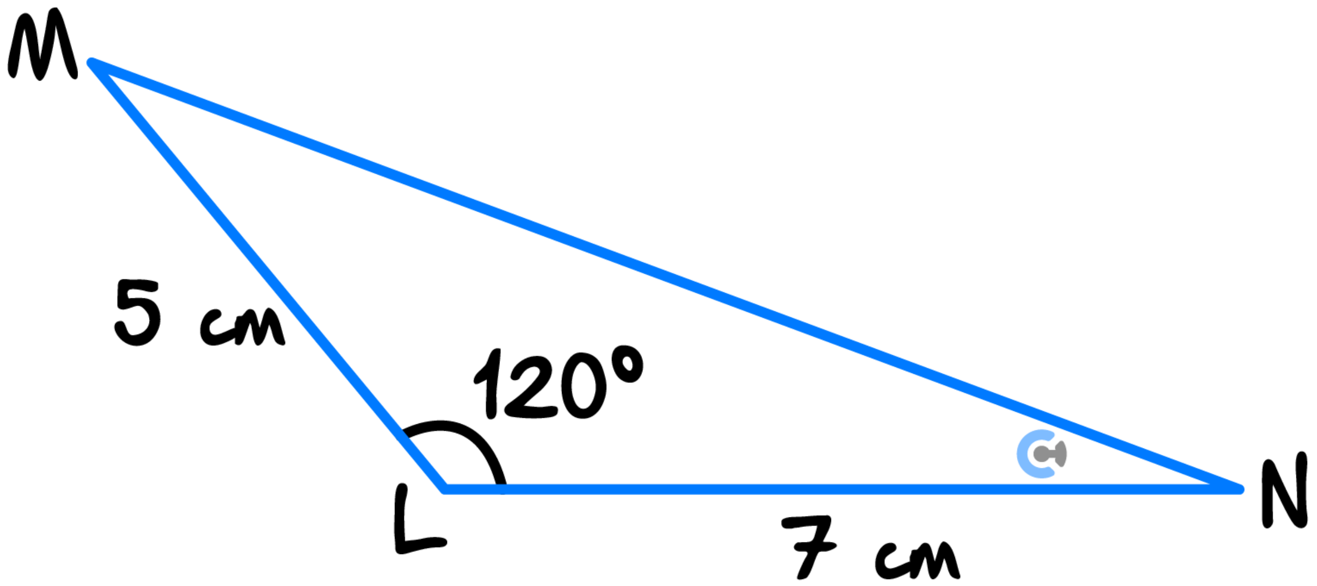
Question 2 Tech-Active.

Find the area of the following triangle. Give your answer correct to two decimal places.



Question 3 Tech-Active.

Find all side lengths and angles for the following triangle. Give your answers correct to two decimal places.



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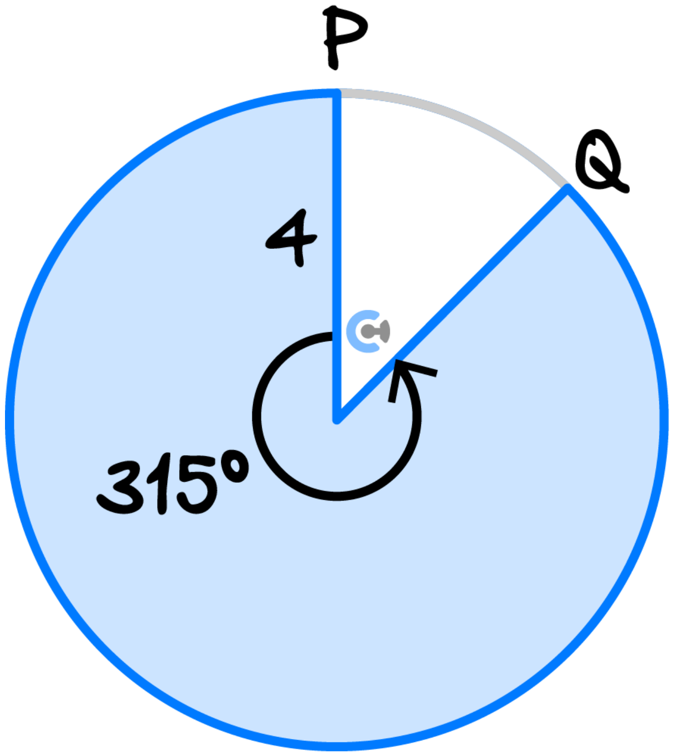
Sub-Section: [3.1.2] - Find Arc Lengths, Chord Lengths, Sector and Segment Areas

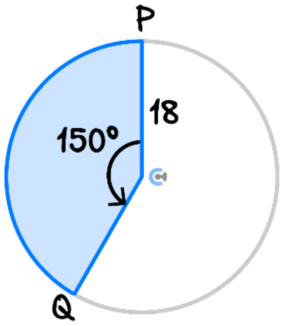
Question 4

Consider the following circles:

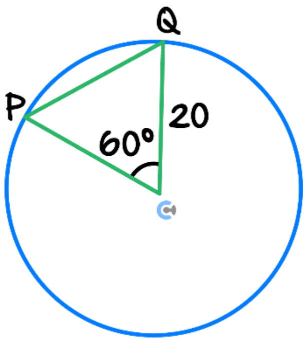
a. Find the area of the shaded sector.



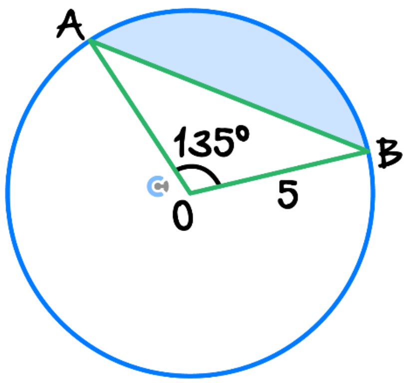
b. Find the length of the arc PQ.



c. Find the length of the chord PQ.

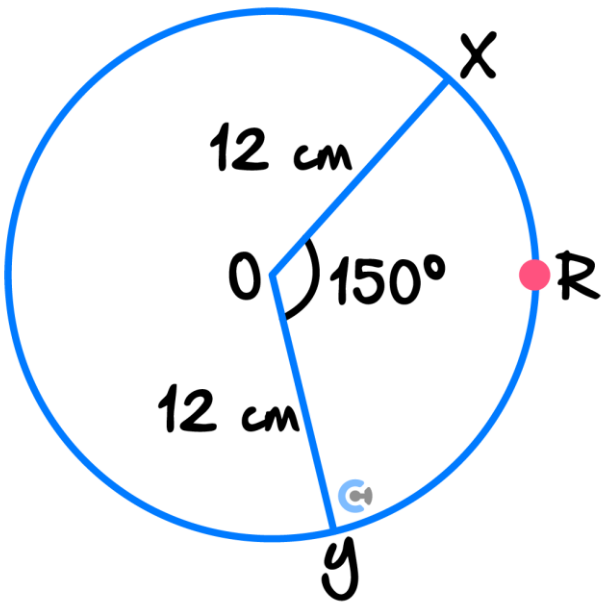


d. Find the area of the shaded segment.



Question 5 Tech-Active.

A circle has a centre O and a radius of 12 cm. The angle subtended at O by arc XY has a magnitude of 150°.



a. Find the exact length of the chord XY. [USE: $\sin(75°) = \frac{\sqrt{3}+1}{2\sqrt{2}}$]

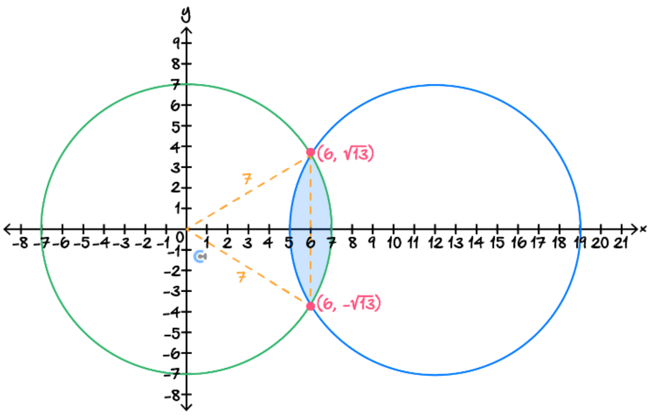
b. Find the exact length of the arc XY.

c. Find the exact area of the minor sector XOY.

d. Find the magnitude of the angle XOR, in degrees, if the minor arc has a length of $5\ cm$. Give your answer correct to two decimal places.

Question 6 Tech-Active.

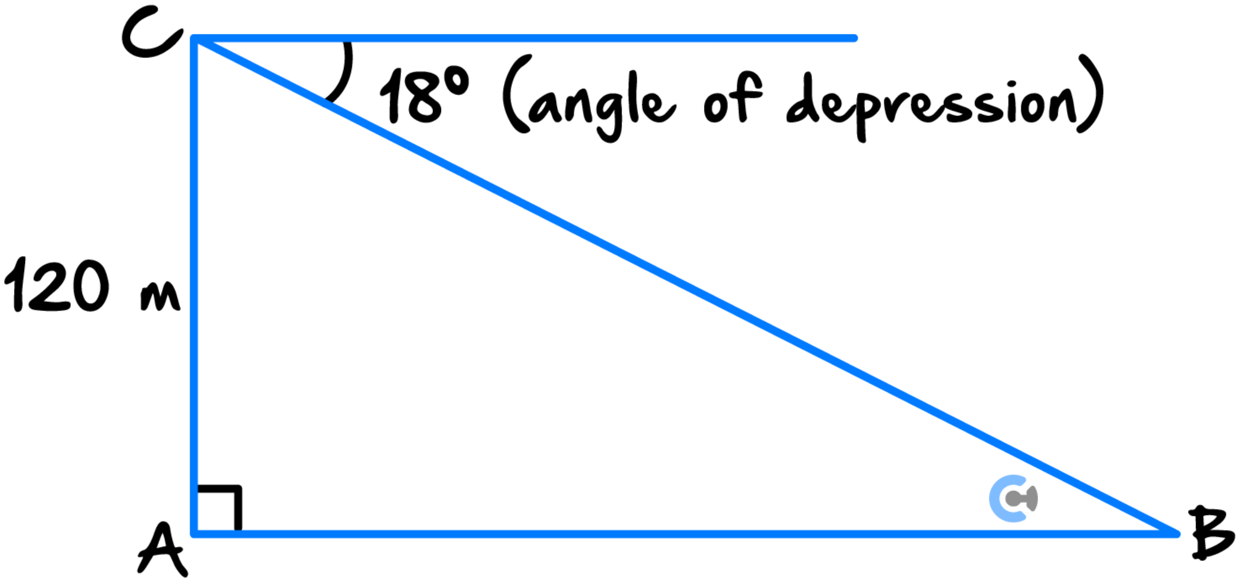
Two circles, each with a radius of $7\ cm$, have their centres $12\ cm$ apart. Calculate the exact area of the region common to both circles and then round this result to two decimal places.



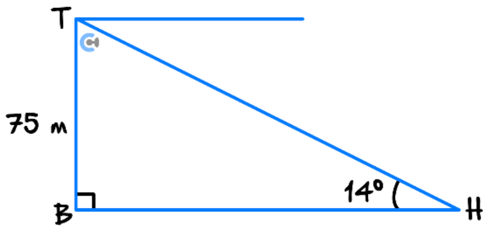
Sub-Section: [3.1.3] - Apply Angle of Elevation/Depression and Bearing to Solve Geometric Problems (Only $2D$)

Question 7 Tech-Active.

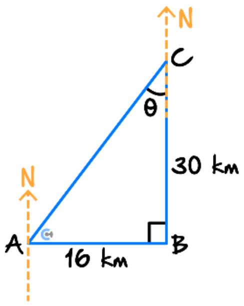
a. A cliff is 120 m high. A person standing at the top of the cliff observes a boat in the ocean at an angle of depression of 18°. Calculate the horizontal distance between the boat and the base of the cliff, correct to the nearest metre.



b. A tree stands 75 m tall. A hiker on the ground observes the top of the tree at an angle of elevation of 14°. Calculate the horizontal distance from the hiker to the base of the tree, correct to the nearest metre.

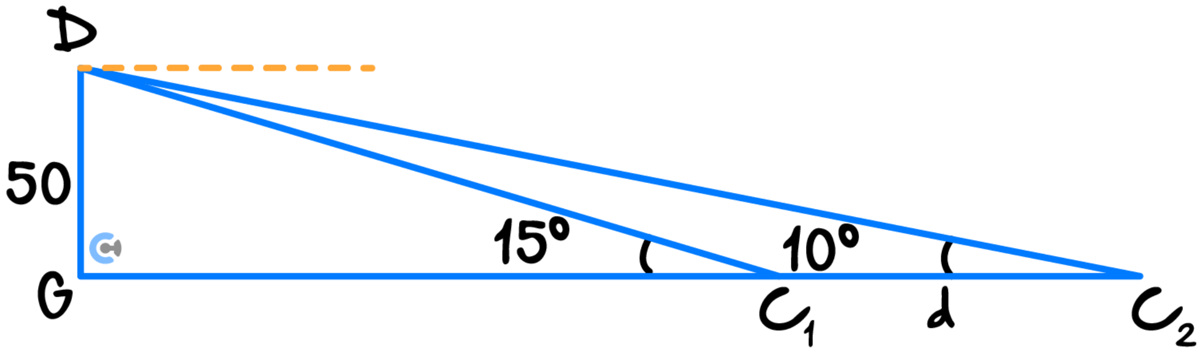


c. An airplane flies 16 km due east from Point A to Point B. It then changes direction and flies 30 km due north to Point C. Calculate the distance and bearing of Point C from Point A, giving your answers correct to two decimal places.



Question 8 Tech-Active.

A drone is hovering at a constant height of 50 m above a straight road. It tracks two cars moving along the road, both in line with each other. The angles of depression to the cars are 10° and 15°. Calculate the distance between the two cars, giving your answer correct to two decimal places.



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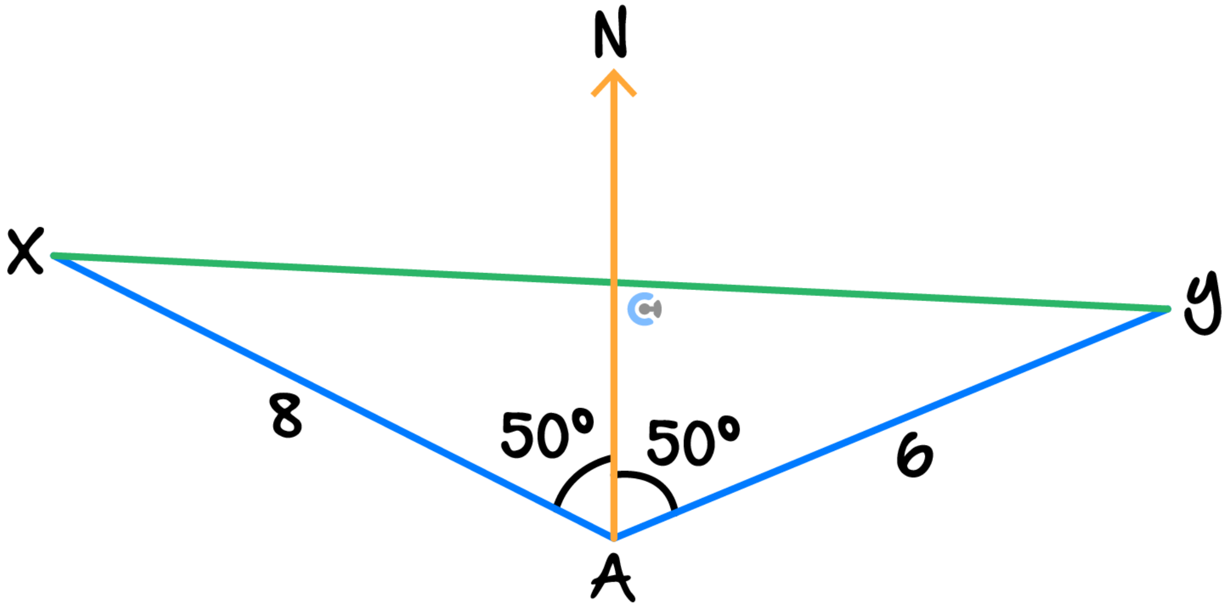
Question 9 Tech-Active.

From an air traffic control tower, two airplanes X and Y are on bearings of $310°$ and $050°$, respectively. The distance XA (from the tower to airplane X) is $8\ km$, and the distance YA (from the tower to airplane Y) is $6\ km$. Find the distance XY, giving your answer correct to two decimal places.



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Section B: [3.2] - Trigonometry II (Checkpoints)

Question 10

Simplify the following expressions:

a. $\cos(\pi - x)$

b. $\tan(\pi + x)$

c. $\sin\left(x - \frac{\pi}{2}\right)$

Question 11

If $\cos(x) = \frac{5}{13}$, where x is an angle in the first quadrant, evaluate the following:

a. $\cos(\pi - x)$

b. $\sin(\pi + x)$

c. $\tan\left(\frac{\pi}{2} - x\right)$

Question 12

If $\sin(x) = \frac{6}{11}$, where $\frac{\pi}{2} \leq x \leq \pi$, evaluate the following:

a. $\sin(\pi + x)$

b. $-\tan\left(\frac{\pi}{2}+x\right)$

c. $\cos(\pi - x)$

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Question 13 Tech-Active.

If $\cos(x) = -\frac{8}{15}$, where x is an angle which lies in the third quadrant, evaluate $\sin(\pi + x)$.

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Sub-Section: [3.2.2] - Find Particular and General Solutions

Question 14

Solve the following trigonometric equations over the specified domain:

a. $2\cos(3x) = -\sqrt{3}$, for $x \in [0, \pi]$.

b. $\sqrt{2}\sin(2x) = 1$, for $x \in [0, 2\pi]$.

c. $4\tan(x) - 2 = 2, x \in [-\pi, \pi]$.

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Question 15

Find the general solution to the following trigonometric equations:

a. $\sin\left(-3x + \frac{\pi}{4}\right) = \frac{1}{2}$

b. $2\cos\left(2x - \frac{\pi}{3}\right) = 1$

c. $\tan\left(\frac{\pi}{3}x + \frac{\pi}{6}\right) = 1$

Question 16

Consider the function $f(x) = \sqrt{3}\tan\left(2x + \frac{\pi}{4}\right) - 1$.

a. Find the general solution to $f(x) = 0$.

b. Hence, solve $f(x) = 0$ for $x \in \left[0, \frac{3\pi}{2}\right]$.

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Question 17 Tech-Active.

Find the general solution to $\sqrt{2}\cos\left(\pi(x+1)\right) = 1$.

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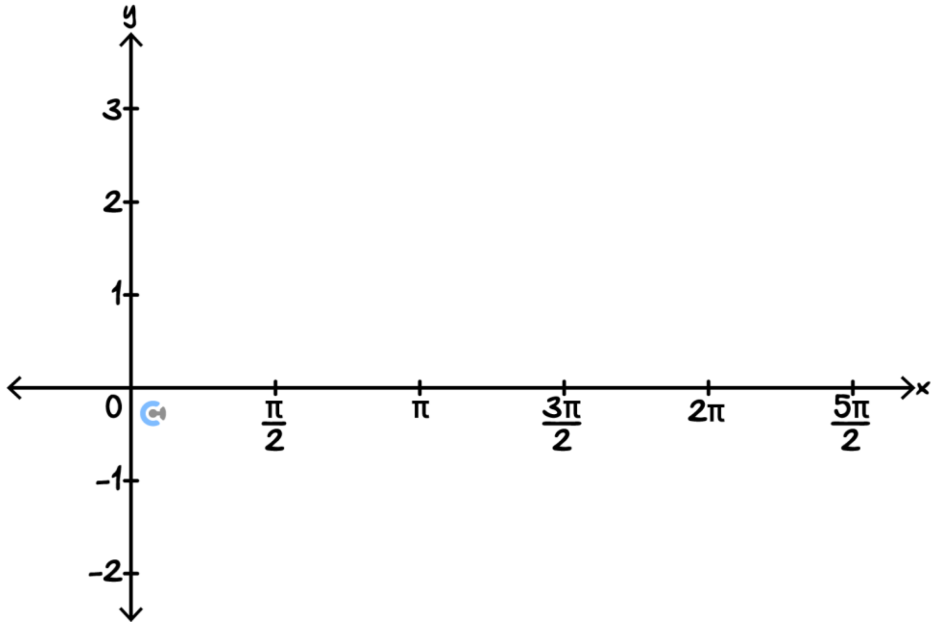
Sub-Section [3.2.3]: Graph Sine, Cosine and Tangent Functions

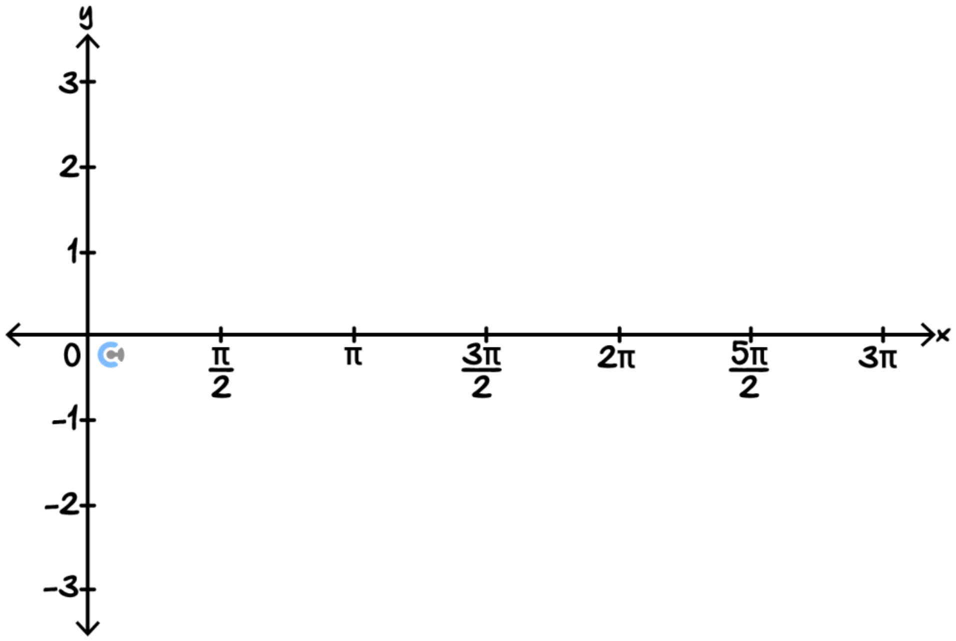
Question 18

Sketch the graphs of the following functions over the indicated domain. Label all axes intercepts and endpoints with coordinates, and label asymptotes with equations.

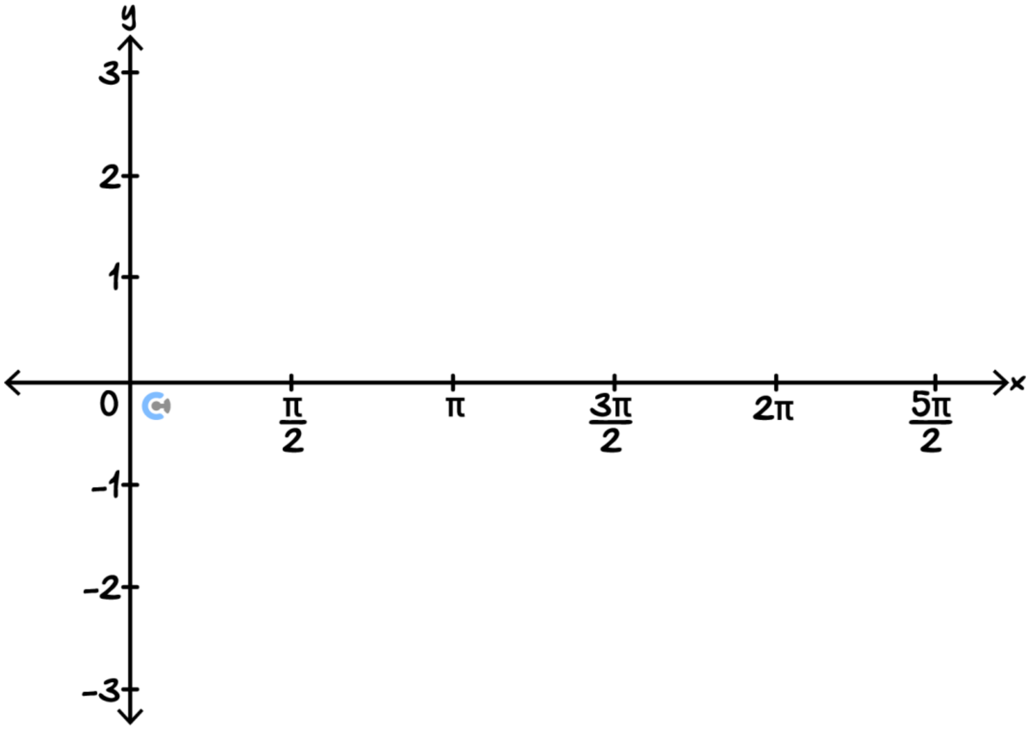
a. $y = \sin(2x), x \in [0,2\pi]$.



b. $y = -\cos\left(\frac{3x}{2}\right), x \in [0,3\pi]$.



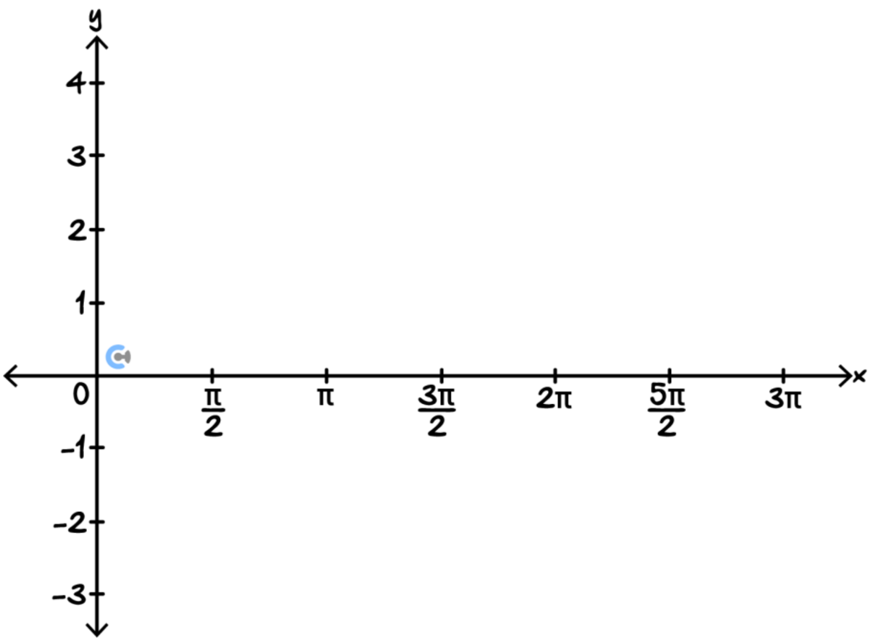
c. $y = \tan(x - \pi), x \in [0,2\pi]$.



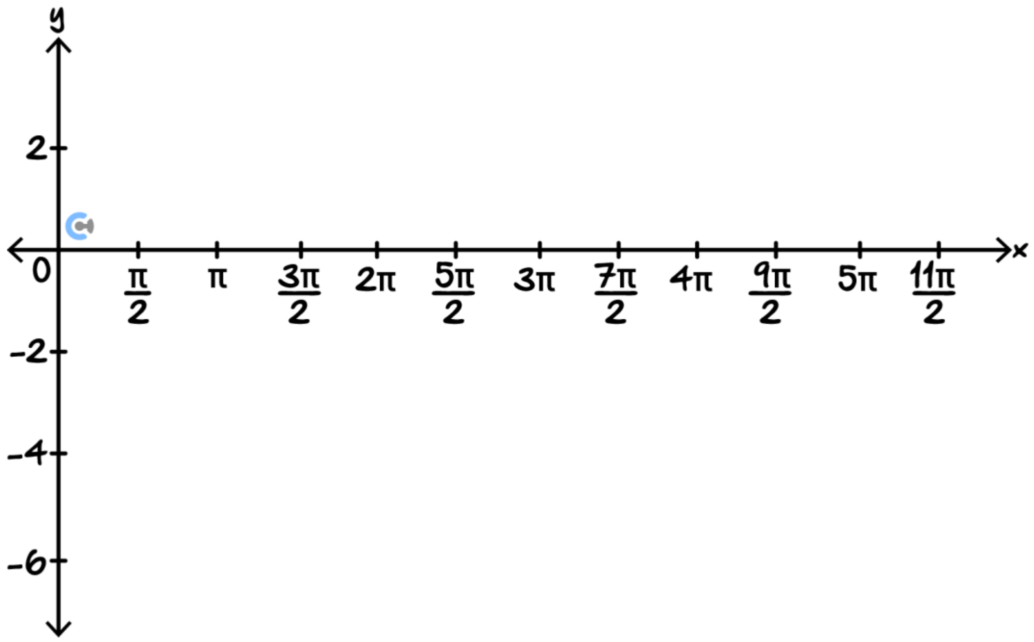
Question 19

Sketch the graphs of the following functions over the indicated domain. Label all axes intercepts, turning points and endpoints with coordinates, and label asymptotes with equations.

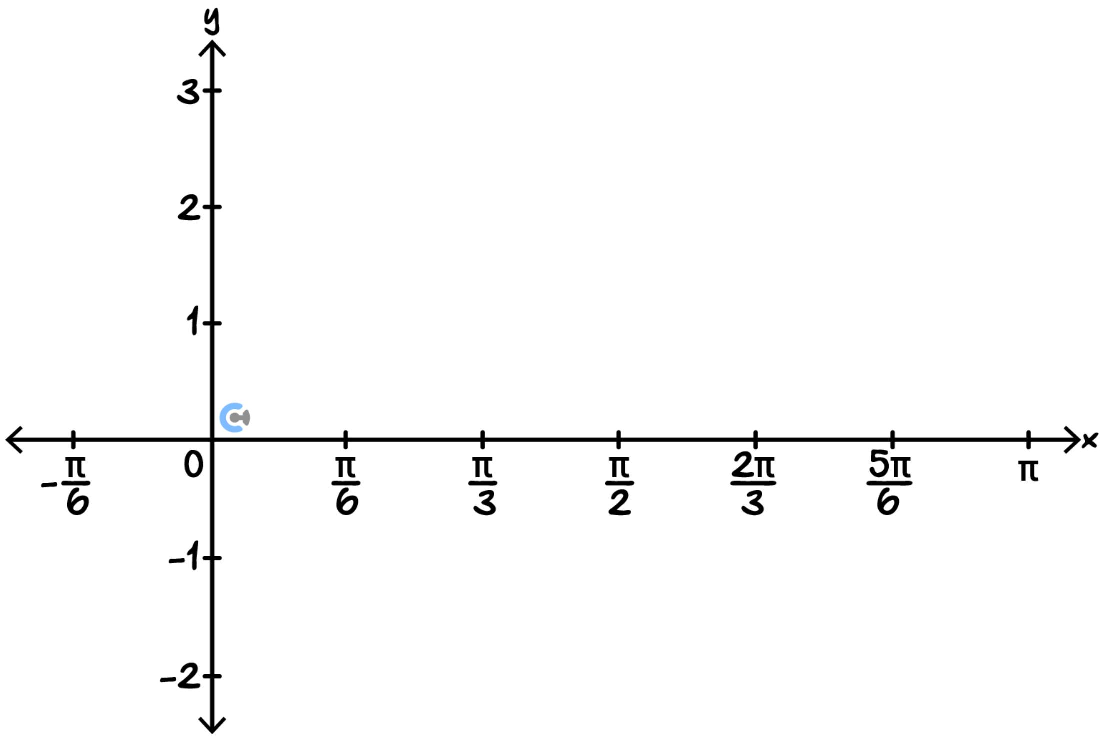
a. $y = 2\cos\left(x + \frac{\pi}{2}\right), x \in [0,2\pi]$.



b. $y = -2\sin(x) - 2, x \in [0,5\pi]$.



c. $y = -2\tan\left(3x + \frac{\pi}{2}\right),\ x \in [0,\pi]$.



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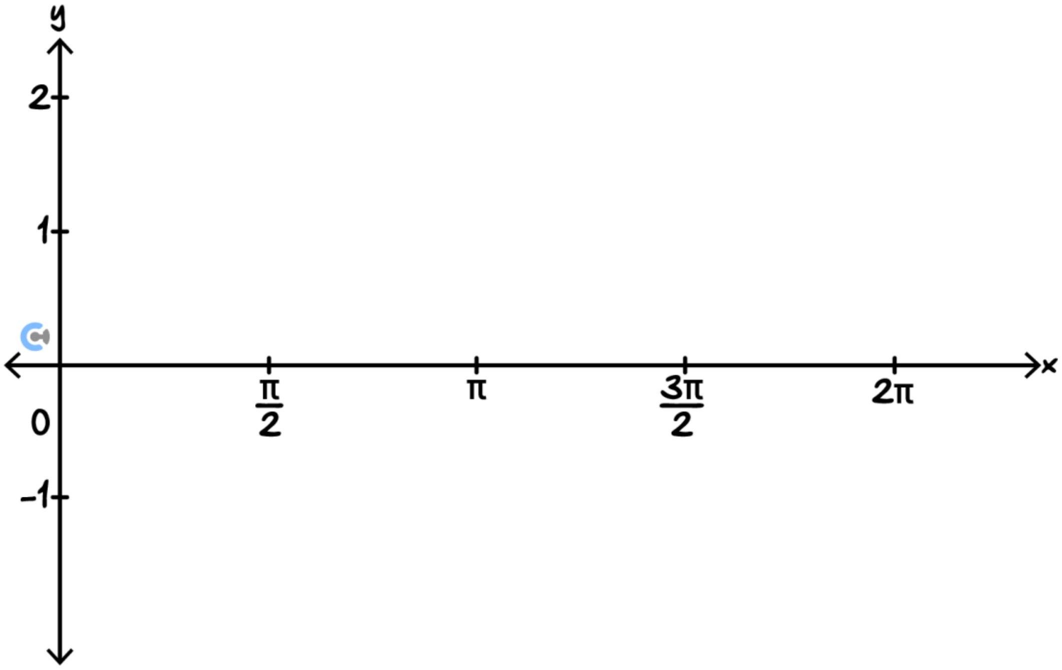
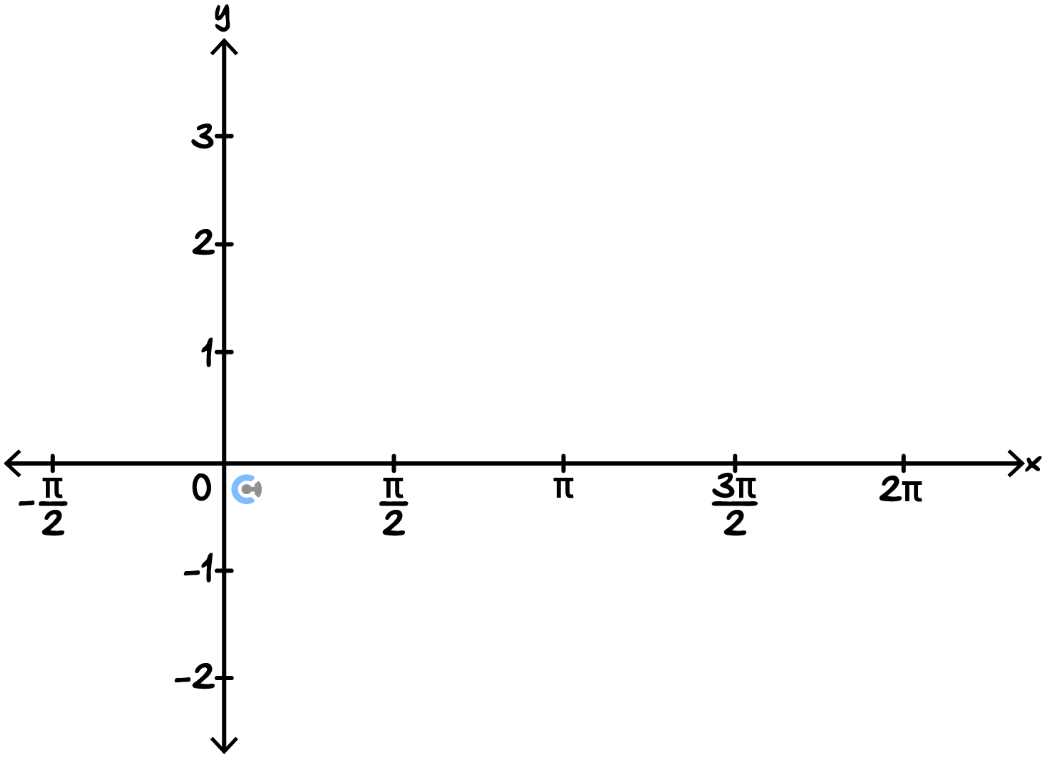
Question 20

Sketch the graphs of the following functions over the indicated domain. Label all axes intercepts, turning points and endpoints with coordinates, and label asymptotes with equations.

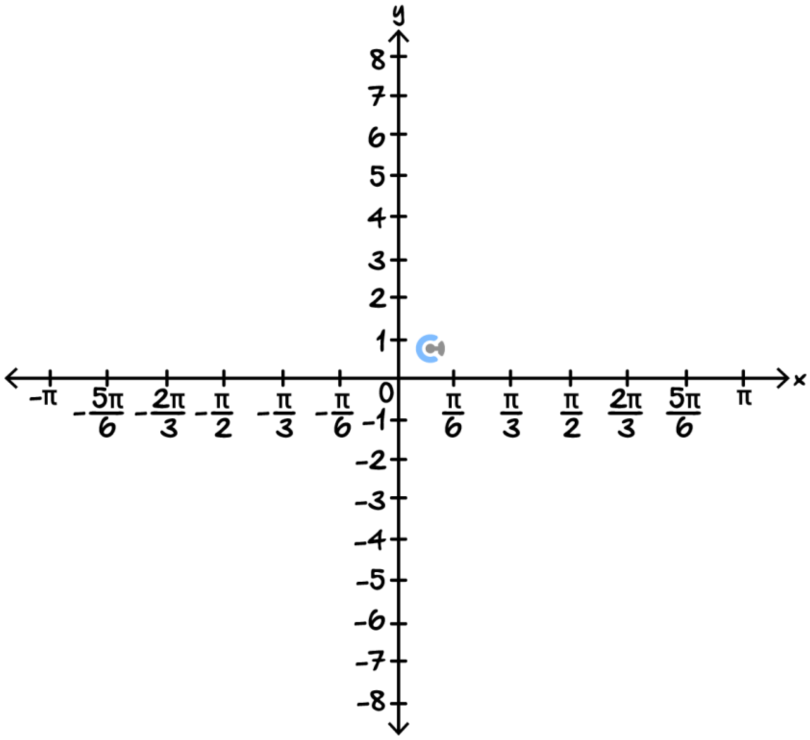
a. $y = \sin\left(3x - \frac{\pi}{6}\right), x \in [0, 2\pi]$.



b. $y = 2\sin\left(\frac{\pi}{3} - 3x\right) + 1, x \in [0, 2\pi]$.

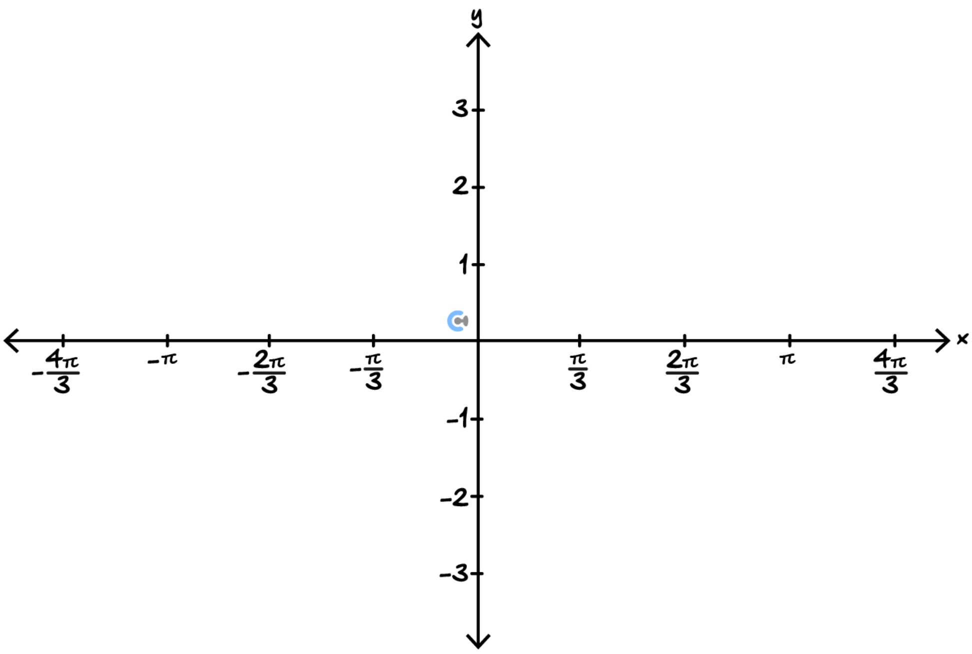


c. $y = 2\tan\left(\frac{\pi}{3} - x\right), x \in [-\pi, \pi]$.



Question 21 Tech-Active.

Sketch the graph of the equation $y = 2\cos\left(2x - \frac{\pi}{4}\right)$. Label all axes intercepts, turning points and endpoints with coordinates.

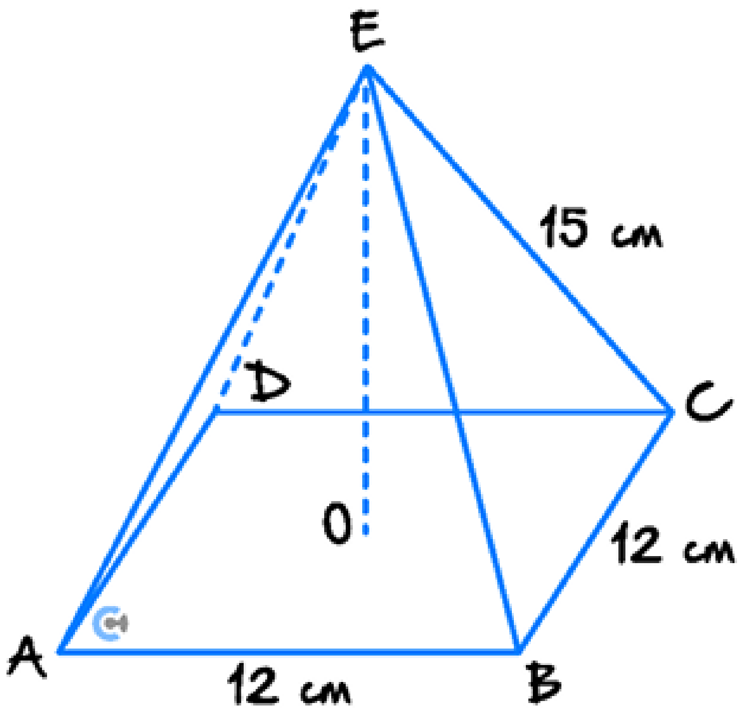


Section C: [3.3] – Trigonometry Exam Skills (Checkpoints)

Sub-Section [3.3.1] and [3.3.2]: Apply Trigonometry to Solve Problems in 3D and Find the Angle between Planes

Question 22

A square pyramid $PQRST$ stands on level horizontal ground. The vertex of the pyramid is at T. The points P, Q, R, S are the corners of a square of side 18 cm, whose diagonals intersect at the point O. Each of the sloping edges of the pyramid has a length of 24 cm.



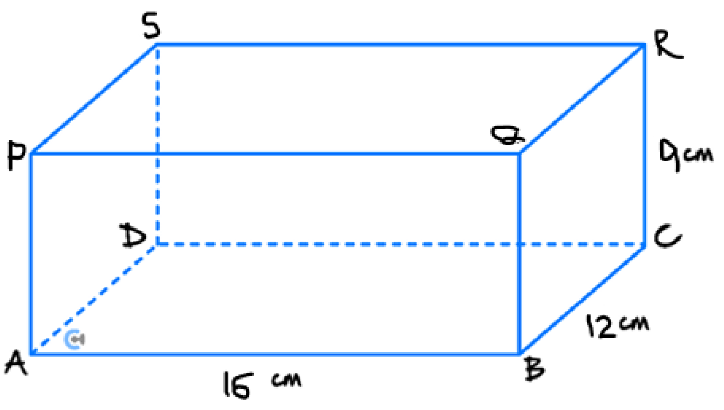
a. Calculate the length OR.

b. Calculate the volume of the pyramid. (Recall: $V = \frac{1}{3} \times base \times height$)

c. Calculate the total surface area of the pyramid.

Question 23

The figure shows a cuboid $ABCDPQRS$ standing on level horizontal ground. The lengths of AB, BC and CR are 16 cm, 12 cm and 9 cm, respectively.



a. Find the length of AR.

b. Calculate the angle AR makes with the ground, correct to two decimal places.

c. Determine the area of the triangle ABY.

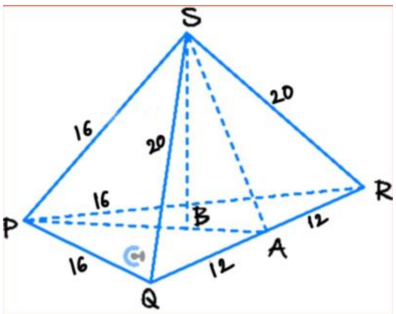
The point M is the midpoint of AB and the point N lies on AR.

d. The point M is the midpoint of AB and the point N lies on AR. Calculate the length of MN, given that MN is perpendicular to AR. Give your answer correct to two decimal places.

Question 24

A pyramid $PQRS$ has a triangular horizontal base PQR, where $PQ = PR = 16\ m$ and $RQ = 24\ m$. The vertex of the pyramid S lies directly above the level of PQR so that $SQ = SR = 20\ m$ and $SP = 16\ m$.

a. Show that the shortest distance of S from the base PQR is $2\sqrt{57}\ m$.



b. Calculate, in degrees correct to two decimal places, the acute angle between:

i. The plane SQR and the plane PQR.

ii. The edge SQ and the plane PQR.

c. Determine, as an exact surd, the shortest distance of P from the plane SQR.

HINT: Compute the volume of the pyramid in two different ways.

Section D: [3.4] - Advanced Trigonometric Functions (Checkpoints)

Sub-Section [3.4.1]: Trigonometric Identities and Solving Exact Values of Reciprocal Functions

Question 25

Evaluate the following:

a. $\sec\left(\frac{\pi}{4}\right)$

b. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

c. $\tan^{-1}(1)$

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Question 26

Evaluate the following:

a. $\cot\left(\frac{11\pi}{6}\right)$

b. $\operatorname{cosec}\left(\frac{7\pi}{3}\right)$

c. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

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Question 27

Prove the identity $(\cot x + \operatorname{cosec} x)^2 = \frac{1+\cos x}{1-\cos x}$.

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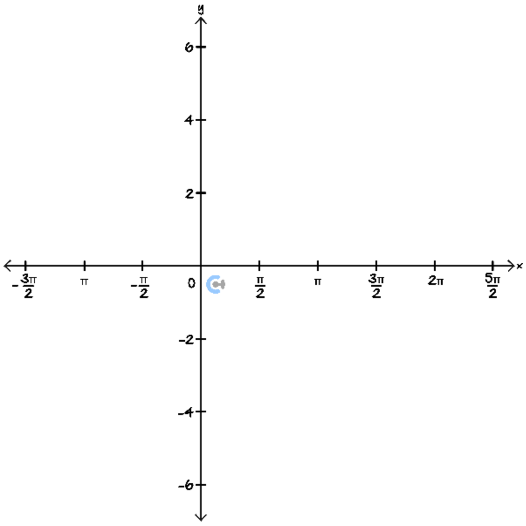
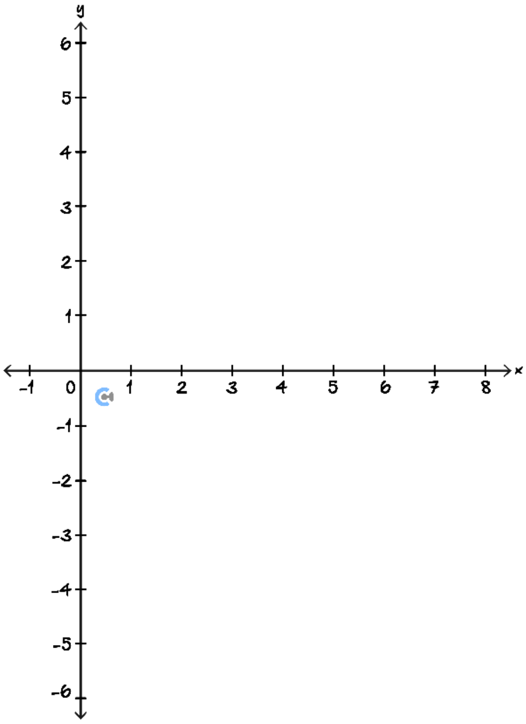
Sub-Section [3.4.2]: Graph Reciprocal Trigonometric Functions

Question 28

a. Sketch the graph of $y = 2\sec\left(x - \frac{\pi}{2}\right)$ for $-\pi < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.

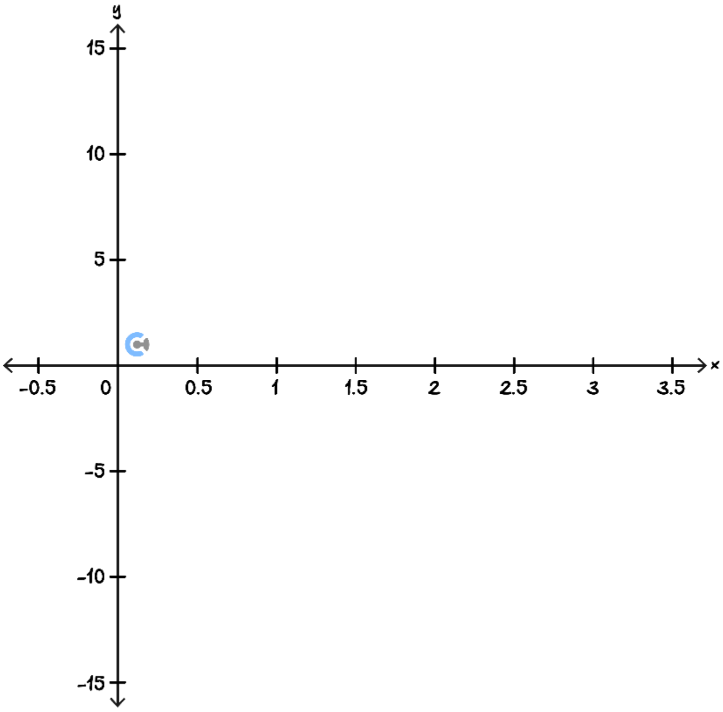


b. Sketch the graph of $\frac{\text{cosec}(x)}{2} - \frac{1}{2}$ for $0 < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.

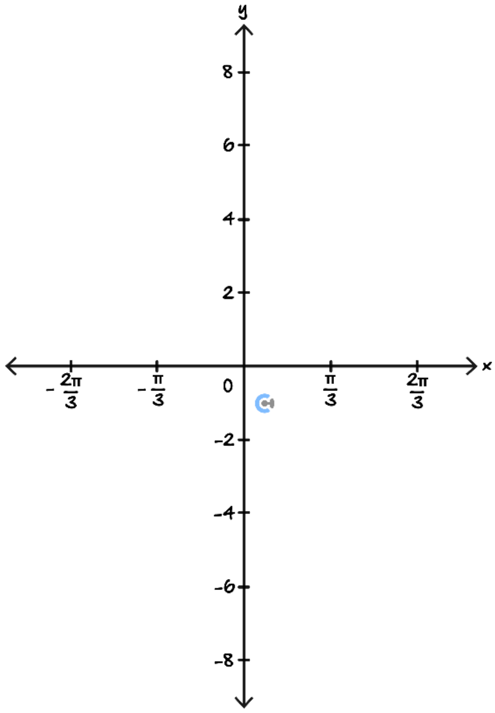


Question 29

a. Sketch the graph of $y = 4\text{cosec}\left(7\pi x - \frac{2\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.

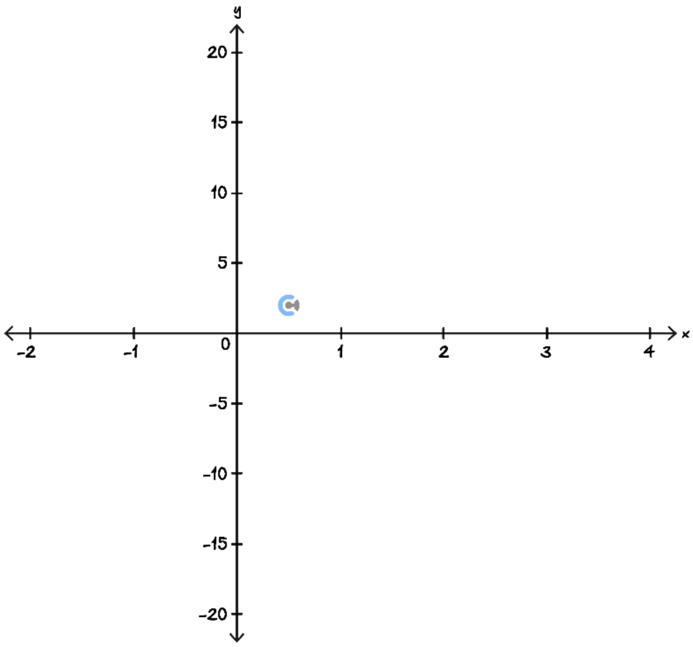


b. Sketch the graph of $y = -\cot(\pi - 3x)$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, labelling all stationary points, axes intercepts and asymptotes with their equations.

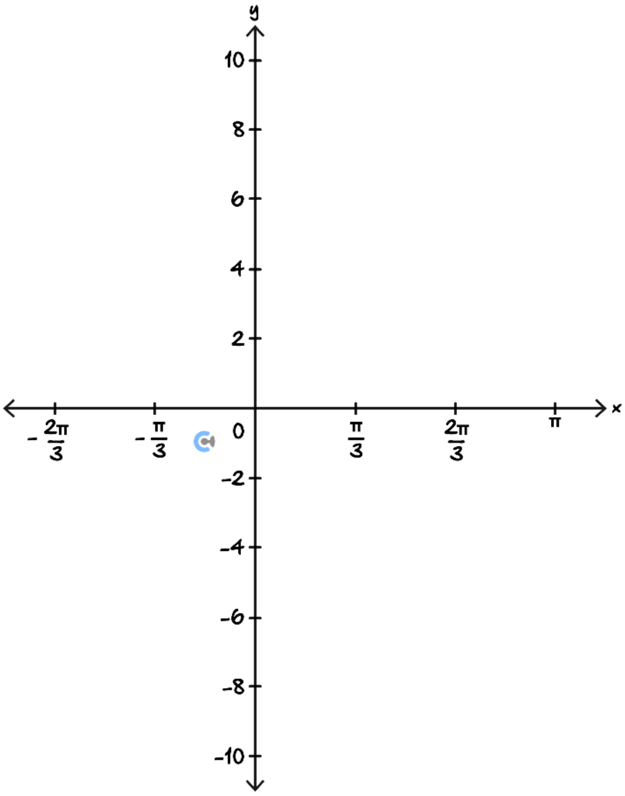


Question 30

a. Sketch the graph of $y = 1 - \sqrt{3}\cot\left(\pi x - \frac{\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.



b. Sketch the graph of $y = \cot\left(2x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$, labelling all stationary points, axes intercepts and asymptotes with their equations.



Sub-Section [3.4.3]: Apply Compound and Double Angle Formula to Solve Exact Values

Question 31

Use a compound angle formula to evaluate $\sin\left(\frac{5\pi}{12}\right)$.

Question 32

Use a double-angle formula to evaluate $\tan\left(-\frac{\pi}{8}\right)$.

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Question 33

Use a compound angle formula to evaluate $\cos\left(\frac{19\pi}{12}\right)$.

Question 34

Given that $\cos(x-y)=\frac{7}{25}$ and $\cot(x)\cot(y)=\frac{4}{3}$, find $\cos(x+y)$.

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Sub-Section [3.4.4]: Find Domain, Range and Rule of the Inverse Trigonometric Function

Question 35

Consider the function $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R : f(x) = \frac{\tan(x)}{3}$.

a. State the domain of $f^{-1}(x)$.

b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

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Question 36

Consider the function $f : \left[-\frac{9\pi}{4}, \frac{3\pi}{4}\right] \to R : f(x) = 2\sin\left(\frac{x}{3} + \frac{\pi}{4}\right) - \sqrt{2}$.

a. State the domain of $f^{-1}(x)$.

b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Question 37

Consider the function $f : \left[\frac{5\pi}{3}, \frac{8\pi}{3}\right] \to R : f(x) = \sqrt{5}\cos\left(x + \frac{\pi}{3}\right)$.

a. State the domain of $f^{-1}(x)$.

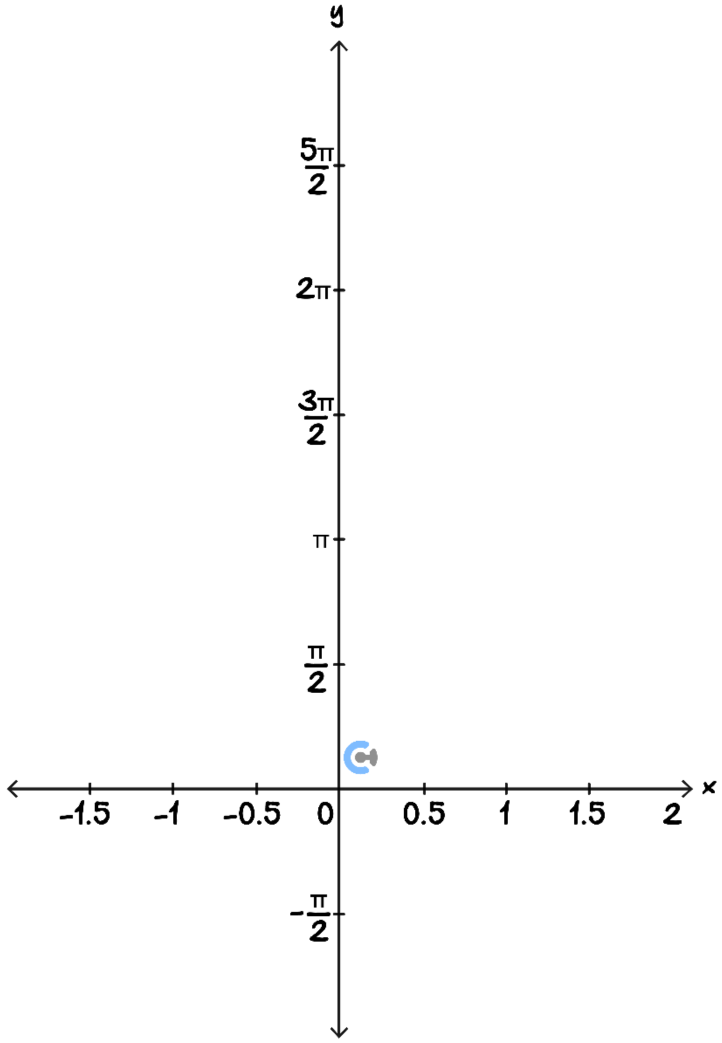
b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Sub-Section [3.4.5]: Graphing Inverse Trigonometric Functions

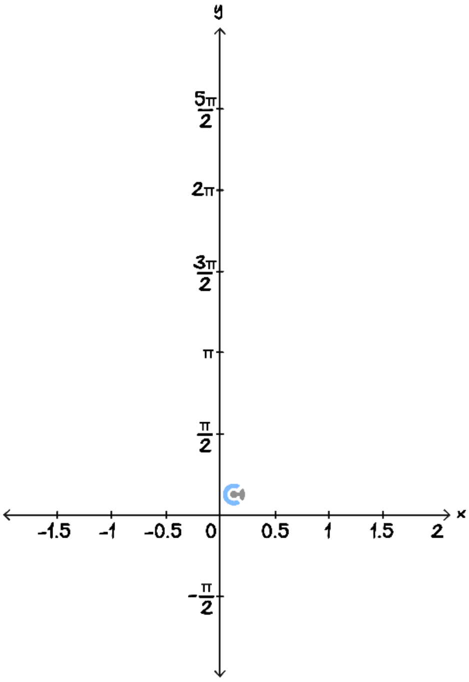
Question 38

a. Sketch the graph of $y = 2\sin^{-1}(x) + \pi$ on the axes below. Label all endpoints and axes intercepts.



b.

i. Sketch the graph of $y = 2\cos^{-1}(-x)$ below.



ii. What do you notice?

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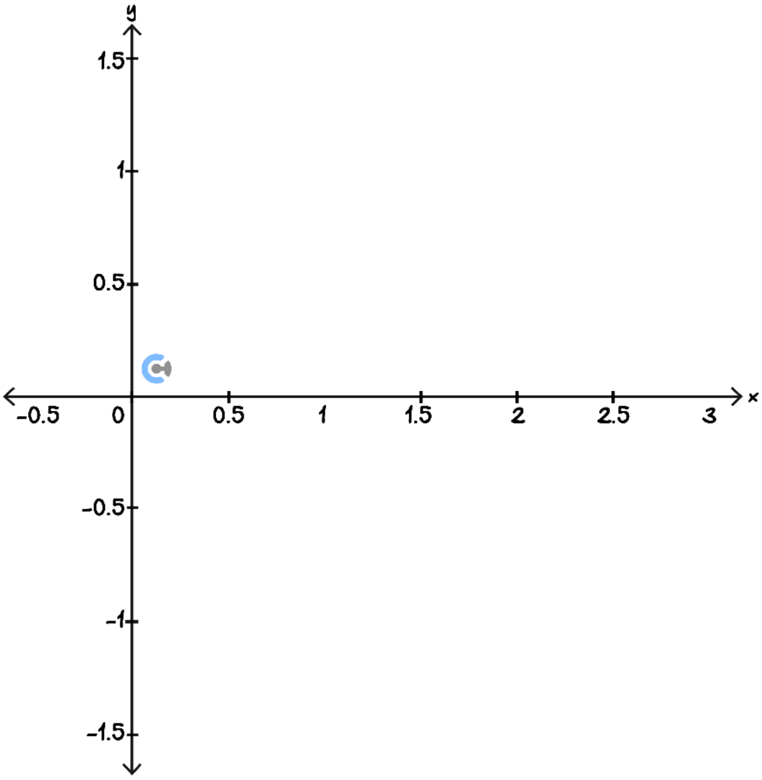
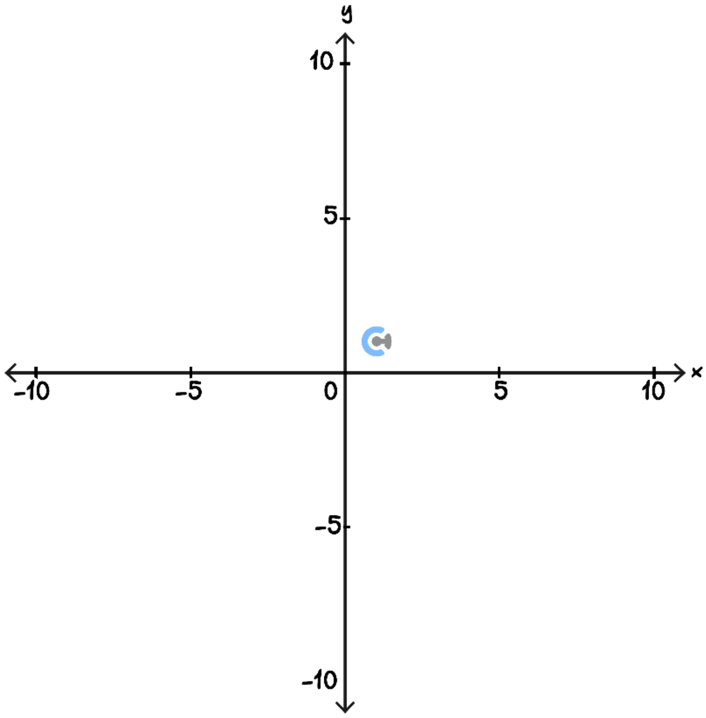
Question 39

a. Sketch the graph of $y = -\frac{2}{\pi}\cos^{-1}(4 - 2x) + 1$ on the axes below, labelling all endpoints and axes intercepts.

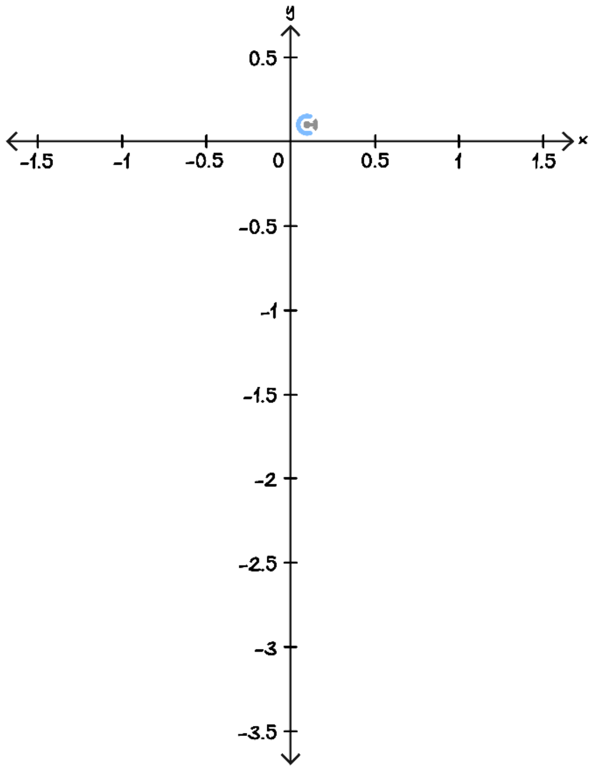


b. Sketch the graph of $y = -3\tan^{-1}(2x + 1)$ below, labelling all key points and asymptotes.

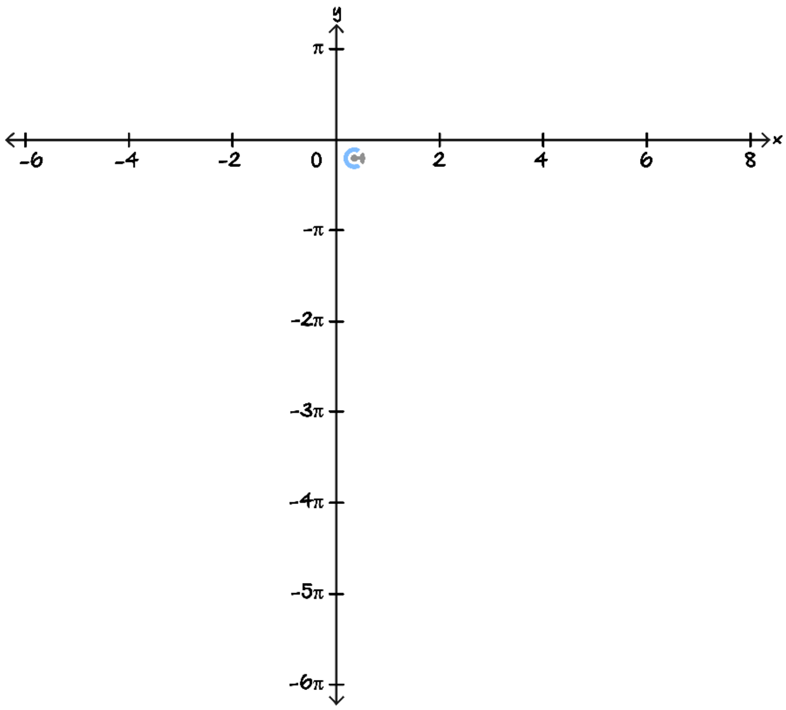


Question 40

a. Sketch the graph of $y = \sin^{-1}(2x) - \sqrt{3}$ on the axes below. Label all endpoints.



b. Sketch the graph of $y = \pi \tan^{-1}\left(\frac{x}{2} - 1\right) - \pi^2$ on the axes below. Label all axes intercepts and asymptotes with their equation.



Section E: [3.5] - Advanced Trigonometric Functions Exam Skills (Checkpoints)

Sub-Section [3.5.1]: Simplify the Composition of Inverse Trigonometric

Question 41

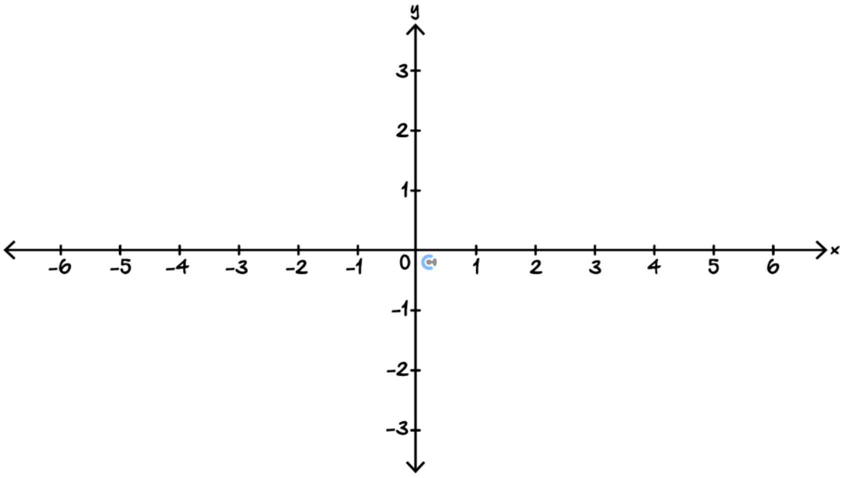
a. Simplify $\tan\left(\arctan\left(\frac{3}{4}\right)\right)$.

b. Simplify $\cos(\arctan(5))$.

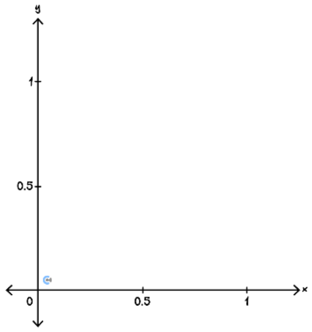
c. Simplify $\sin\left(\arccos\left(\frac{5}{13}\right)\right)$.

Question 42

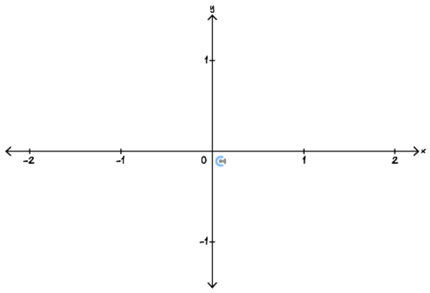
a. Simplify and sketch the graph of $\cos(\arctan(x+2))$.



b. Simplify and sketch the graph of $\tan(\arcsin(3x-1))$.



c. Simplify and sketch the graph of $\sin(\arctan(x))$.



Question 43

a. Simplify and determine the maximal domain of $g(x) = \cos(\arcsin(3x - 2)) + \sin(\arctan(x + 1))$.

b. Simplify and determine the maximal domain of $g(x) = \sin(\arccos(2 - x^2)) + \cos(\arcsin(x + 2))$.

c. Simplify and determine the maximal domain of $g(x) = \cos(\arcsin(3x + 1)) + \sin(\arctan(2x))$.

Sub-Section [3.5.2]: Simplify $a\cos(x) + b\sin(x)$

Question 44

a. Express $3\sin(x) + 3\cos(x)$ in the form of $r\sin(x - \alpha)$.

b. Express $\cos(x) - \sin(x)$ in the form of $r\cos(x - \alpha)$.

c. Express $\sqrt{3}\sin(x) + 3\cos(x)$ in the form of $r\sin(x + \alpha)$.

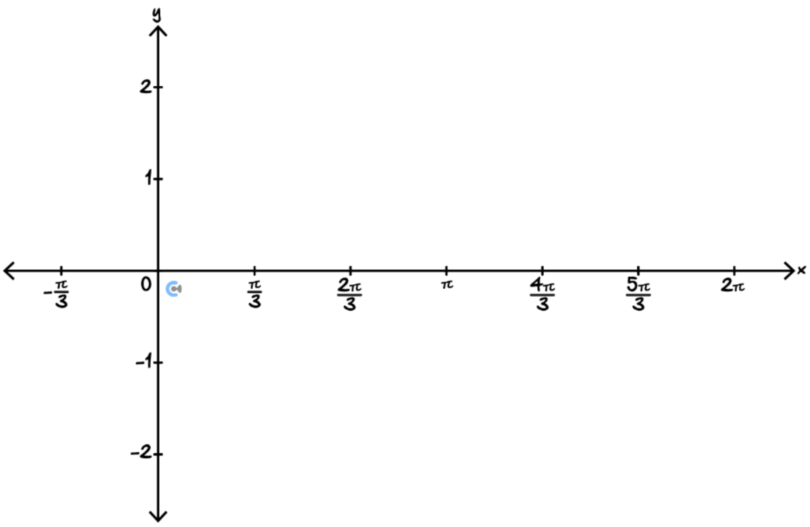
Space for Personal Notes

Question 45

a. Solve $\cos(x) + \sin(x) = 1$ for $0 \leq x \leq 2\pi$.

b. Solve $4\cos(x) + 4\sqrt{3}\sin(x) = 2$ for $0 \leq x \leq 2\pi$.

c. Sketch the graph of $f(x) = \sqrt{3}\sin(x) + \cos(x)$ for $0 \leq x \leq 2\pi$. Label all turning points, endpoints, and axes intercepts with coordinates.



Question 46

a. Find the maximum and minimum values if $h(x) = 7\sin(x) + 24\cos(x)$.

b. Solve $3\sin\left(x - \frac{\pi}{4}\right) + 3\sqrt{3}\cos\left(x - \frac{\pi}{4}\right) = 0$ for $0 \leq x \leq 2\pi$.

c. Show that for $a > 0$, $a\sin(4x) - b\cos^2(2x) = \sqrt{4a^2 + b^2}\cos(2x)\sin(2x - \alpha)$, where $\beta = \arctan\left(\frac{b}{2a}\right)$.

Sub-Section [3.5.3]: Apply Product-to-Sum and Sum-to-Product Identities to Simplify Trigonometric Expressions

Question 47

a. Express $\sin(5\theta)\cos(3\theta)$ as a sum or difference.

b. Express $2\cos(4B)\cos(6B)$ as a sum or difference.

c. Express $\sin(7A)\cos(4A)$ as a sum or difference.

d. Express $\sin(3\alpha) + \sin(4\alpha)$ as a product.

e. Express $\cos(3x) + \cos(3y)$ as a product.

f. Express $\cos(x+k) - \sin(x)$ as a product.

Space for Personal Notes

Question 48

a. Solve $\sin(2\theta) + \sin(4\theta) = 0$ for $0 \leq \theta \leq \pi$.

b. Solve $\cos(3x) - \cos(x) = 0$ for $0 \leq \theta \leq \pi$.

c. Solve $\sin(z) - \sin\left(\frac{\pi}{3} - z\right) = 0$ for $0 \leq \theta \leq 2\pi$.

Question 49

a. Express $|b|\cos(2y) - |b|\cos(4y)$ as a product and hence, determine its minimum value in terms of b.

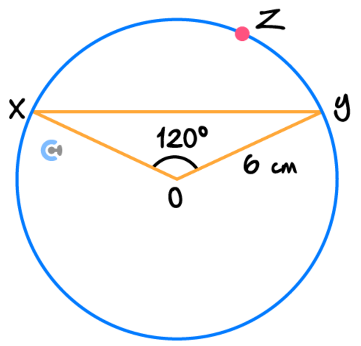
b. If $p + q + r = \pi$, show that $\sin(p) + \sin(q) + \sin(r) = 4\cos\left(\frac{p}{2}\right)\cos\left(\frac{q}{2}\right)\cos\left(\frac{r}{2}\right)$.

c. Solve the equation $\cos(4x) + \cos(2x) - \cos(3x) = 0$ for $x \in [0, 2\pi]$.

Section F: [3.1-3.5] - Exam 1 Overall (Checkpoints) (19 Marks)

Question 50

Consider a circle of radius 6 cm, with centre O. The angle subtended at O by the arc XY has a magnitude of 120°. Exact answers are required for all parts, and no CAS is allowed.



a.

i. Find the length of the chord XY.

ii. Find the length of the arc XY.

b. Find the area of the minor segment formed by the chord XY.

---

c. The point Z is located between X and Y such that it divides the arc XY in a $5:3$ ratio. Find the length of the arc ZY and the angle YOZ in degrees.

Space for Personal Notes

Question 51

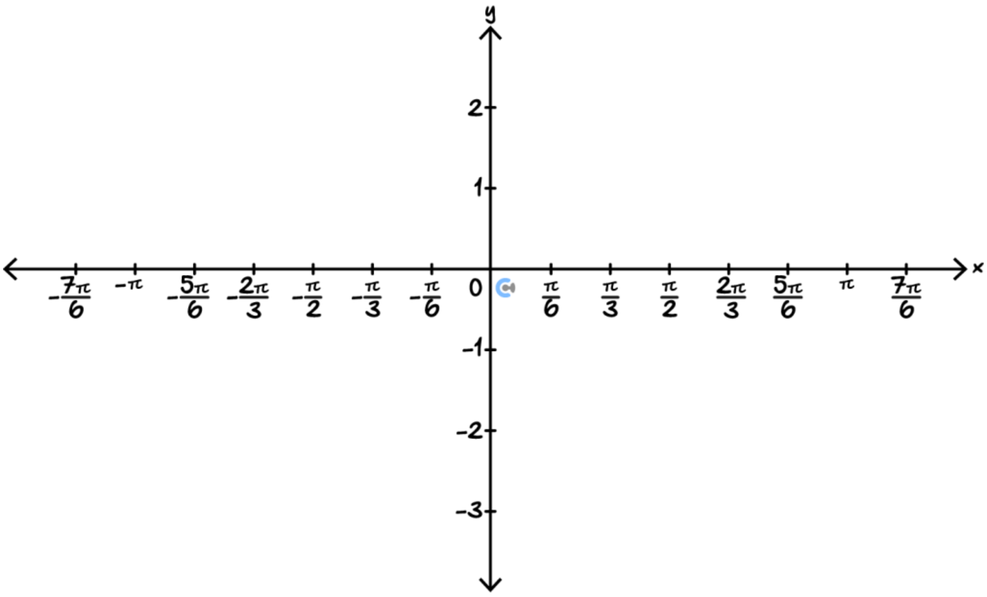
Consider the function $f(x) = 3\sin\left(2x + \frac{\pi}{4}\right) + \cos\left(2x + \frac{3\pi}{4}\right) - 1$.

a. Express $f(x)$ in the form $f(x) = a\sin(2x + b) - 1$.

b. Find the general solution to $f(x) = 0$.

c. Find all solutions to $f(x) = 0$ for $x \in [-\pi, \pi]$.

d. Sketch the graph of $y = f(x)$ on the axes below. Label all axes' intercepts, turning points and endpoints with coordinates.



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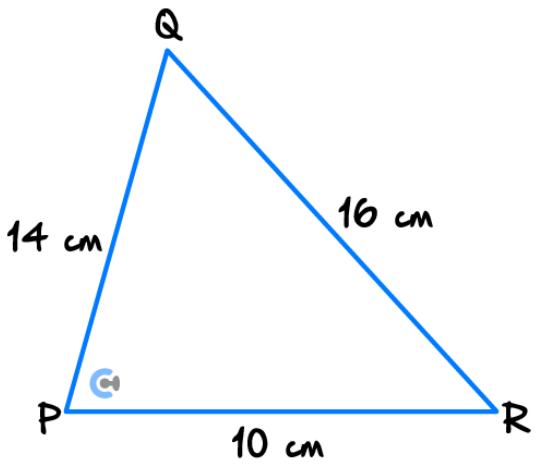
Question 52

The figure below shows a triangle PQR where the following information is given. $|PQ| = 14\ cm, |QR| = 16\ cm, |PR| = 10\ cm$.

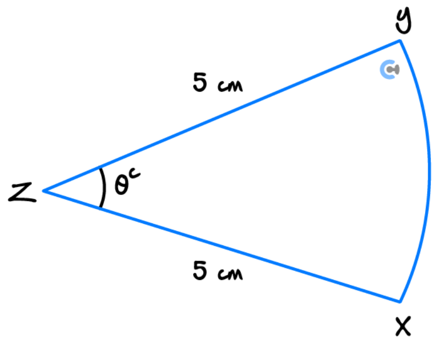


a. Find the size of the angle $\angle PRQ$ in degrees.

b. Hence, determine as an exact surd the area of the triangle PQR.

Question 53

The figure below shows a circular sector XYZ of radius $5\ cm$ subtending an angle θ radians at Z. Given that the perimeter of the sector is equal to the area of the sector, find the value of θ in radians.

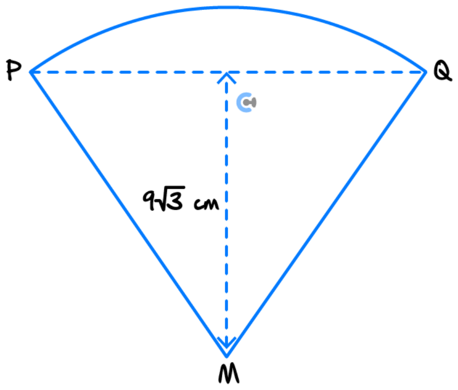


Question 54

Prove the identity: $\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$

Question 55

The figure above shows a badge in the shape of a circular sector MPQ, centred at M. The triangle MPQ is equilateral and its perpendicular height is $9\sqrt{3}\ cm$.



a. Find the length of MP.

b. Determine in terms of π:

i. The area of the badge.

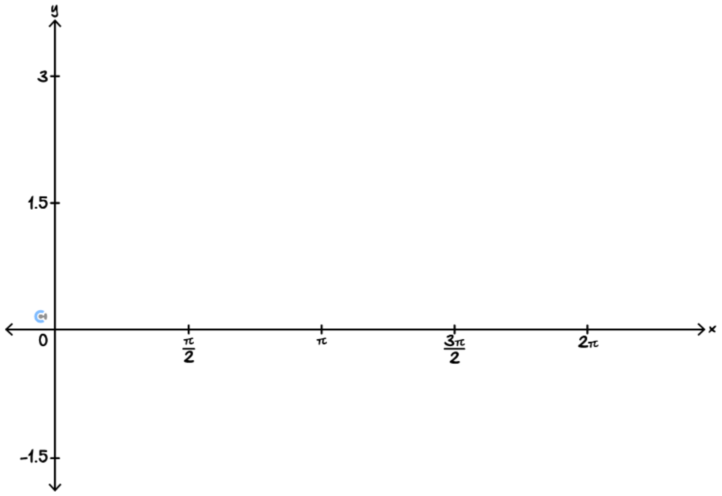
ii. The perimeter of the badge.

Question 56

Consider the function $g(x) = 2\sin\left(3x - \frac{\pi}{4}\right) + 1$.

a. Find the general solution to $f(x) = 0$.

b. Sketch the graph of $y = f(x)$ for $x \in [0, 2\pi]$ on the axes below. Label all axes intercepts, turning points and endpoints with coordinates.



c. Find the values of x for which $f(x) > 2$.

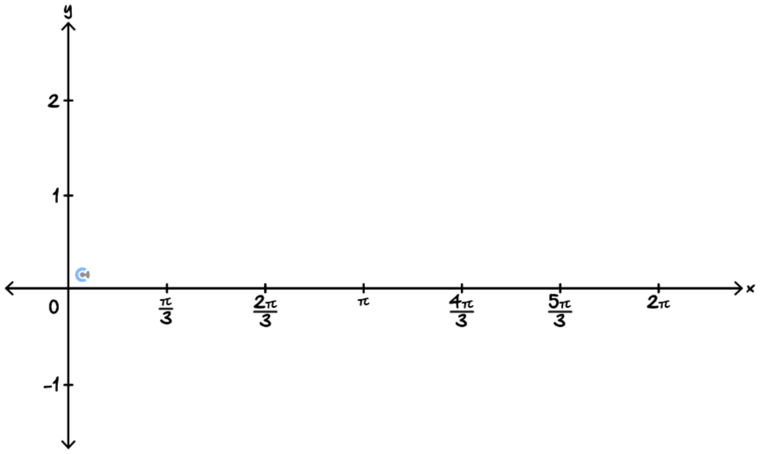
d. The function g(x) has an equivalent expression $g(x) = 2\cos\left(3x + \frac{a\pi}{4}\right) + 1$, where $0 < a < 10$.
State the value of a.

Question 57

a. Solve the equation $\sin(3x) + \sin(5x) = 0$, where $x \in [0, \pi]$.

b. Consider the function $g : [0, 2\pi] \to R$, where $g(x) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{2}\right)$.

i. Sketch the graph of g on the axes below. Label all axes intercepts, turning points, and endpoints with coordinates.



ii. Use your sketch to solve the equation $g(x) = \frac{1}{2}$.

iii. Hence, find $\left\{x : g(x) > \frac{1}{2}\right\}$.

Question 58 (3 marks)

Given that $\cos(x - y) = \frac{3}{5}$ and $\tan(x)\tan(y) = 2$, find $\cos(x + y)$.

Space for Personal Notes

Question 59 (3 marks)

Given that $\cot(2x) + \frac{1}{2}\tan(x) = a\cot(x)$, use a suitable double angle formula to find the value of a, $a \in R$.

Question 60 (3 marks)

Find all real solutions of $\tan(2x) = -\tan(x)$.

Space for Personal Notes

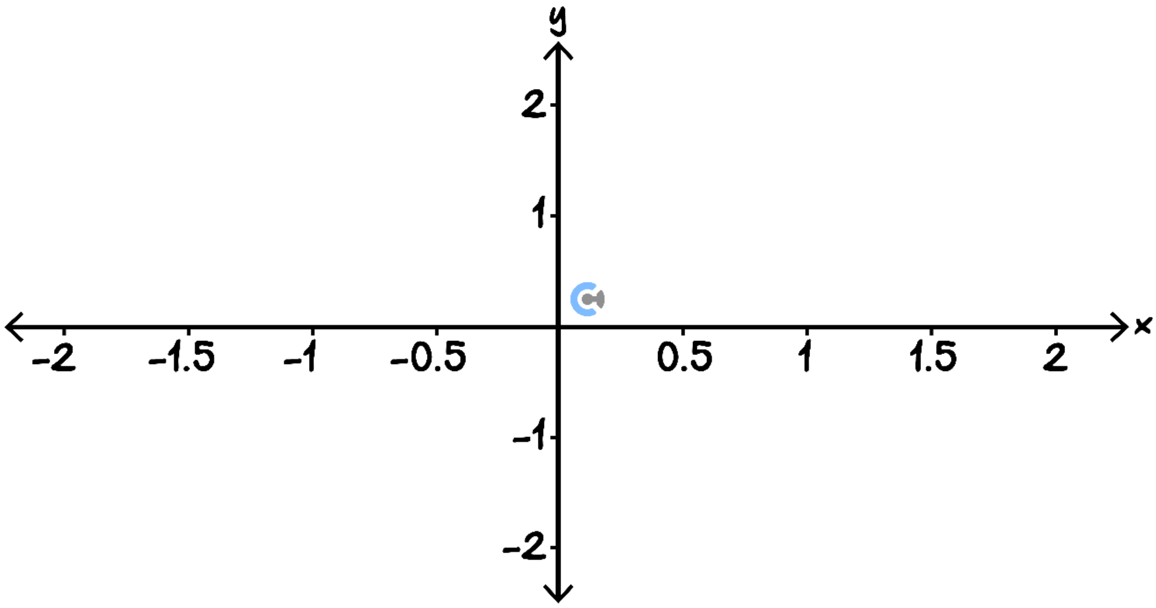
Question 61 (3 marks)

Find $\sin(t)$, given that $t = \arccos\left(\frac{12}{13}\right) + \arctan\left(\frac{3}{4}\right)$.

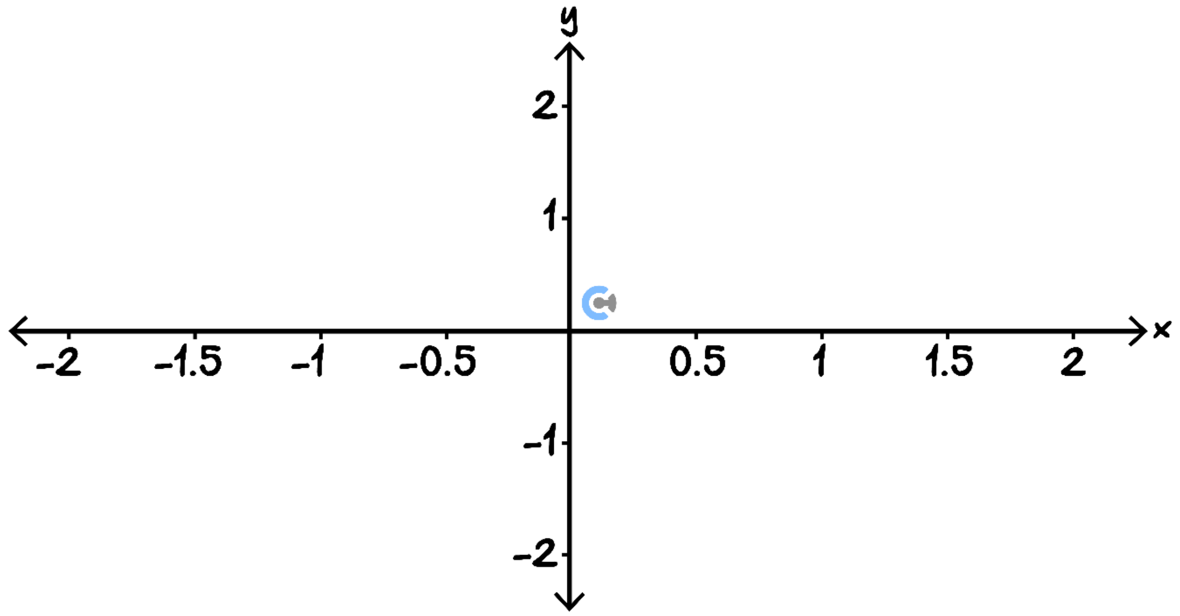
Question 62 (4 marks)

Consider the function $f : [-1, 1] \to R, f(x) = \arccos(x) - \frac{\pi}{2}$.

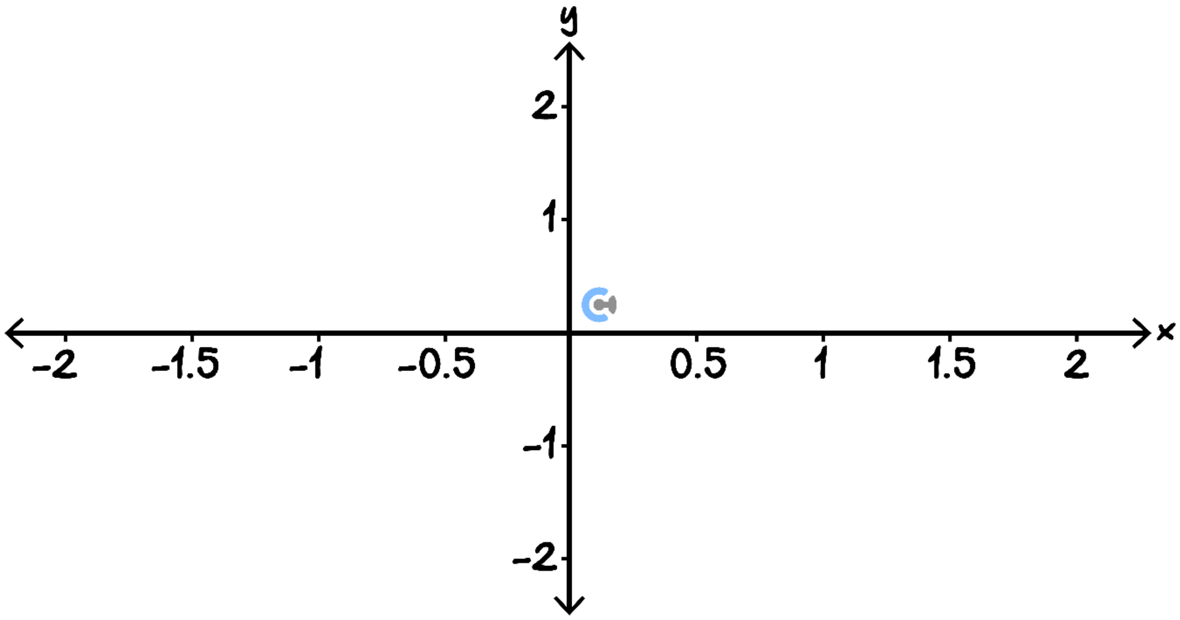
a. Sketch the graph of f on the axes below, labelling the endpoints with their coordinates. (2 marks)



b. Sketch the graph of $y = |f(x)|$ on the axes below. (1 mark)



c. Sketch the graph of $y = f(|x|)$ on the axes below. (1 mark)



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Question 63 (3 marks)

If $\sin(x) = 3\cos(x)$, find the value of $\sin(2x)$, where $x \in \left(0, \frac{\pi}{2}\right)$.

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Section G: [3.1-3.5] - Exam 2 Overall (Checkpoints) (36 Marks)

Question 64

A building is 72 m tall. From the top of the building, the angle of depression to a certain point on level ground is $30°$. How far is that point from the base of the building?

A. $36\sqrt{3}\,m$

B. $48\sqrt{3}\,m$

C. $72\sqrt{3}\,m$

D. $60\sqrt{3}\,m$

Question 65

If $\cot(\theta) = -\frac{12}{5}$ and $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, then $\sin(\theta)$ is equal to:

A. $\frac{12}{13}$

B. $-\frac{5}{13}$

C. $-\frac{12}{13}$

D. $\frac{5}{13}$

Question 66

The solutions of the equation: $2\sin\left(2x - \frac{\pi}{4}\right) + 1 = 0$ are:

A. $x = \pi n + \frac{17\pi}{24} \quad or \quad x = \pi n - \frac{25\pi}{24}, n \in \mathbb{Z}.$

B. $x = \pi n + \frac{17\pi}{24} \quad or \quad x = \pi n + \frac{25\pi}{24}, n \in \mathbb{Z}.$

C. $x = \pi n - \frac{17\pi}{24} \quad or \quad x = \pi n + \frac{25\pi}{24}, n \in \mathbb{Z}.$

D. $x = \pi n - \frac{17\pi}{24} \quad or \quad x = \pi n - \frac{25\pi}{24}, n \in \mathbb{Z}.$

Question 67

Let $\sin(\theta) = \frac{-7}{13}$ and $\cos^2(\alpha) = \frac{81}{169}$, where $\theta \in \left[\pi, \frac{3\pi}{2}\right]$ and $\alpha \in \left[\frac{3\pi}{2}, 2\pi\right]$.
The value of $\sin(\theta) + \cos(\alpha)$ is:

A. $\frac{2}{13}$

B. $-\frac{2}{13}$

C. $\frac{16}{13}$

D. $-\frac{16}{13}$

Question 68

Jack's line of sight, while looking at a bird on top of a tree, makes a $30°$ angle of elevation. He then walks 150 metres toward the tree to observe the bird more closely, causing his line of sight to make a $45°$ angle of elevation. How far was Jack from the tree initially?

A. $\frac{150\sqrt{3}}{\sqrt{3}+1}$

B. $\frac{150\sqrt{3}}{\sqrt{3}-1}$

C. $50\sqrt{3}$

D. $75\sqrt{3}$

Question 69 (1 mark)

The implied domain of $y = \arccos\left(\frac{x-a}{b}\right)$, where $b > 0$ is:

A. $[-1, 1]$

B. $[a - b, a + b]$

C. $[a - 1, a + 1]$

D. $[a, a + b\pi]$

E. $[-b, b]$

Question 70 (1 mark)

The implied domain of $f(x) = 2\cos^{-1}\left(\frac{1}{x}\right)$ is:

A. R

B. $[-1, 1]$

C. $(-\infty, -1] \cup [1, \infty)$

D. $R\backslash\{0\}$

E. $[-1, 1]\backslash\{0\}$

Question 71 (1 mark)

The solutions to $\cos(x) > \frac{1}{4}\text{cosec}\,(x)$ for $x = (0, 2\pi)\backslash\{\pi\}$ are given by:

A. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\frac{5\pi}{12}, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

B. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\frac{13\pi}{12}, \frac{17\pi}{12}\right)$

C. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\pi, \frac{13\pi}{12}\right) \cup \left(\frac{13\pi}{12}, 2\pi\right)$

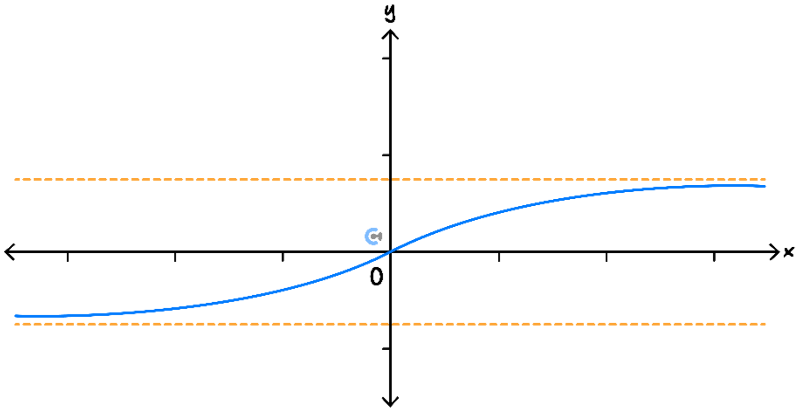
D. $x \in \left(\frac{\pi}{12}, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

E. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\pi, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

Space for Personal Notes

Question 72 (1 mark)

Part of the graph of $y = \frac{1}{2}\tan^{-1}(x)$ is shown below:



The equations of its asymptotes are:

A. $y = \pm\frac{1}{2}$

B. $y = \pm\frac{3}{4}$

C. $y = \pm 1$

D. $y = \pm\frac{\pi}{2}$

E. $y = \pm\frac{\pi}{4}$

Question 73 (1 mark)

Consider the function f with rule $f(x) = -\frac{1}{\sqrt{\sin^{-1}(cx+d)}}$, where $c, d \in R$ and $c > 0$. The domain of f is:

A. $x > -\frac{d}{c}$

B. $-\frac{d}{c} < x \leq \frac{1-d}{c}$

C. $\frac{-1-d}{c} \leq x \leq \frac{1-d}{c}$

D. $x \in R\backslash\left\{-\frac{d}{c}\right\}$

E. $x \in R$

Question 74 (1 mark)

If $\cos(x) = -a$ and $\cot(x) = b$, where $a, b > 0$, then cosec $(-x)$ is equal to:

A. $\frac{b}{a}$

B. $-\frac{b}{a}$

C. $-\frac{a}{b}$

D. $\frac{a}{b}$

E. $-ab$

Question 75 (1 mark)

The implied domain of the function with rule $f(x) = 1 - \sec\left(x + \frac{\pi}{4}\right)$ is:

A. R

B. $[0, 2]$

C. $R\backslash\left\{\frac{(4n-1)\pi}{4}\right\}, n \in Z$

D. $R\backslash\left\{\frac{(4n+1)\pi}{4}\right\}, n \in Z$

E. $R\backslash\left\{\frac{(2n-1)\pi}{2}\right\}, n \in Z$

Question 76 (1 mark)

A function f has the rule $f(x) = |b\cos^{-1}(x) - a|$, where $a > 0$, $b > 0$ and $a < \frac{b\pi}{2}$. The range of f is:

A. $[-a, b\pi - a]$

B. $[0, b\pi - a]$

C. $[a, b\pi - a]$

D. $[0, b\pi + a]$

E. $[a - b\pi, a]$

Question 77 (1 mark)

Let $f(x) = \frac{\sqrt{x-1}}{x}$ over its implied domain and $g(x) = \text{cosec}^2 x$ for $0 < x < \frac{\pi}{2}$.

The rule for $f(g(x))$ and the range, respectively, are given by:

A. $f\left(g(x)\right) = \text{cosec}^2\left(\frac{\sqrt{x-1}}{x}\right), [1, \infty)$

B. $f\left(g(x)\right) = \text{cosec}^2\left(\frac{\sqrt{x-1}}{x}\right), [2, \infty)$

C. $f\left(g(x)\right) = \sin(x)\cos(x), [-0.5, 0.5]\backslash\{0\}$

D. $f\left(g(x)\right) = \sin(x)\cos(x), \left(0, \frac{1}{2}\right)$

E. $f\left(g(x)\right) = \frac{1}{2}\sin(2x), \left(0, \frac{1}{2}\right]$

Question 78 (1 mark)

Let $f(x) = \frac{1}{\sec(3x)+\frac{3}{2}}$.

The number of asymptotes that the graph of f has in the interval $\left[-\frac{\pi}{6}, \pi\right]$ is:

A. 2

B. 3

C. 4

D. 5

E. 6

Space for Personal Notes

Question 79 (1 mark)

The implied domain of the function with rule $f(x) = \cos^{-1}(\log_e(bx))$, $b > 0$ is:

A. $(0, 1]$

B. $[1, e]$

C. $\left[\frac{1}{b}, \frac{e}{b}\right]$

D. $\left[\frac{1}{b}, \frac{e^{\pi}}{b}\right]$

E. $\left[\frac{1}{be}, \frac{e}{b}\right]$

Question 80 (1 mark)

The expression $1 - \frac{4\sin^2(x)}{\tan^2(x)+1}$ simplifies to:

A. $\sin(x)\cos(x)$

B. $1 - 2\cos^2(2x)$

C. $2\sin(2x)$

D. $2\sin^2(2x)$

E. $\cos^2(2x)$

Question 81 (1 mark)

In the interval $-\pi \leq x \leq \pi$, the graph of $y = a + \sec(x)$, where $a \in R$, has two x-intercepts when:

A. $0 \leq a \leq 1$

B. $-1 < a < 1$

C. $a \leq -1$ or $a > 1$

D. $-1 \leq a < 0$

E. $a < -1$ or $a \geq 1$

Question 82 (1 mark)

Given that $\sin(x) = a$, where $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, the $\cos\left(\frac{x}{2}\right)$ is equal to:

A. $-\frac{\sqrt{1+\sqrt{1-a^2}}}{\sqrt{2}}$

B. $\frac{\sqrt{1-\sqrt{a^2-1}}}{\sqrt{2}}$

C. $\frac{\sqrt{1+\sqrt{1-a^2}}}{\sqrt{2}}$

D. $-\frac{\sqrt{\sqrt{1-a^2}-1}}{\sqrt{2}}$

Question 83 (1 mark)

For the function $f: R \to R, f(x) = k \arctan(ax - b) + c$, where $k > 0$, $c > 0$ and $a, b = R$, $f(x) > 0$ if:

A. $c < \frac{k\pi}{2}$

B. $c \geq \frac{k\pi}{2}$

C. $x > \frac{b}{a}$

D. $c + k > \frac{\pi}{2}$

E. $c \geq \frac{\pi}{2}$

Space for Personal Notes

Question 84 (1 mark)

If $\sin(\theta + \emptyset) = a$ and $\sin(\theta - \emptyset) = b$, then $\sin(\theta)\cos(\emptyset)$ is equal to:

A. ab

B. $\sqrt{a^2 + b^2}$

C. $\sqrt{ab}$

D. $\sqrt{a^2 - b^2}$

E. $\frac{a+b}{2}$

Question 85 (1 mark)

Let $f(x) = \text{cosec}\,(x)$. The graph of f is transformed by:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 1 unit horizontally to the right, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the y-axis.

The rule of the transformed graph is:

A. $g(x) = 2\text{cosec}(3x + 1)$

B. $g(x) = 3\text{cosec}(2x - 1)$

C. $g(x) = 3\text{cosec}(2(x - 1))$

D. $g(x) = 2\text{cosec}\left(\frac{x}{3} - 1\right)$

E. $g(x) = 3\text{cosec}\left(\frac{x-1}{2}\right)$

Space for Personal Notes

Question 86 (1 mark)

Let $f(x) = \frac{\sqrt{x+1}}{x}$ and $g(x) = \tan^2(x)$, where $0 < x < \frac{\pi}{2}$. $f(g(x))$ is equal to:

A. $\sin(x)\sec^2(x)$

B. $\sec(x)\tan^2(x)$

C. $\cos(x)\cot^2(x)$

D. $\cos(x)\operatorname{cosec}^2(x)$

E. $\operatorname{cosec}(x)\cos^2(x)$

Question 87 (1 mark)

The implied domain of the function with rule $f(x) = \frac{3x}{\frac{\pi}{2} - \arccos(2-x)}$ is:

A. $[1, 3]$

B. $[-1, 1]$

C. $[0,1] \cup (1, 2]$

D. $[-1,0) \cup (0, 1]$

E. $[1, 2) \cup (2, 3]$

Question 88 (1 mark)

The maximal domain and range of the function $f(x) = a\cos^{-1}(bx) + c$, where a, b, and c are real constants with $a > 0$, $b < 0$ and $c > 0$, are respectively:

A. $[0, \pi]$ and $[-a, a]$

B. $[0, \pi]$ and $[-a + c, a + c]$

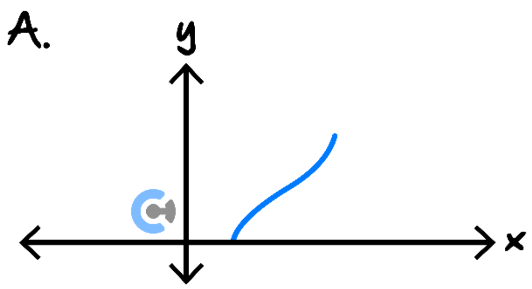
C. $\left[-\frac{1}{b}, \frac{1}{b}\right]$ and $[c, a\pi + c]$

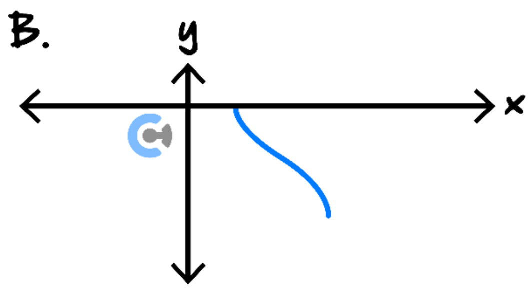
D. $\left[\frac{1}{b}, -\frac{1}{b}\right]$ and $[c, a\pi + c]$

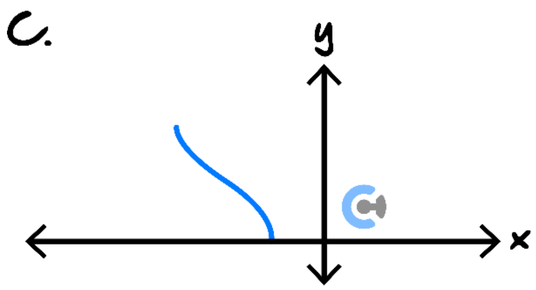
E. $\left[\frac{1}{b}, -\frac{1}{b}\right]$ and $[-a\pi + c, a\pi + c]$

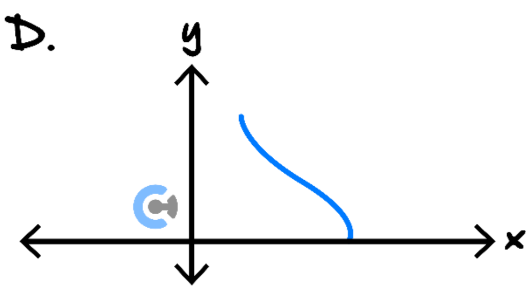
Question 89 (1 mark)

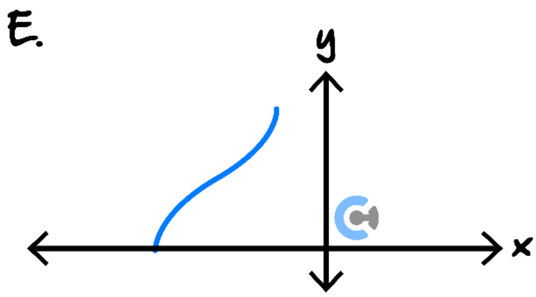
The graph of $y = \cos^{-1}(2 - bx)$, where b is a positive real constant, could be:











Question 90 (1 mark)

If the implied domain of $y = \sin(\cos^{-1}(ax - 1))$, where $a \in R \setminus \{0\}$, is the same as the range, then the value of a is:

A. -2

B. -1

C. 1

D. 2

E. 3

Question 91 (1 mark)

The implied domain and range of $f(x) = \sin(\cos^{-1}(1-2x))$ are respectively:

A. $[0, 1]$ and $[0, 1]$

B. $[-1, 0]$ and $[0, 1]$

C. R and $[-1, 1]$

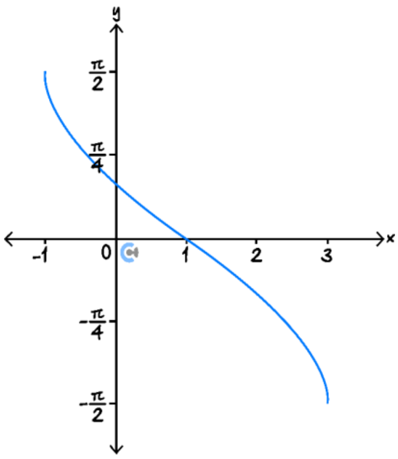
D. $[0, 1]$ and $[1, 1]$

E. R and $[0, 1]$

Question 92 (1 mark)

Let $f(x) = \arcsin(x)$ and $g(x) = ax + b$, where $a, b \in R$.

The graph of $y = f(g(x))$ is shown below.



The values of a and b are, respectively:

A. $\frac{1}{2}$ and $\frac{1}{2}$

B. $-\frac{1}{2}$ and $-\frac{1}{2}$

C. $-\frac{1}{2}$ and $\frac{1}{2}$

D. $\frac{1}{2}$ and 1

E. $-\frac{1}{2}$ and 1

Question 93 (1 mark)

The solutions of $\frac{1+5\sin(x)\cos(x)}{\cos^2(x)} - 7 = 0$ can be found by solving:

A. $(\tan(x) - 2)(\tan(x) + 3) = 0$

B. $(\tan(x) - 1)(\tan(x) + 7) = 0$

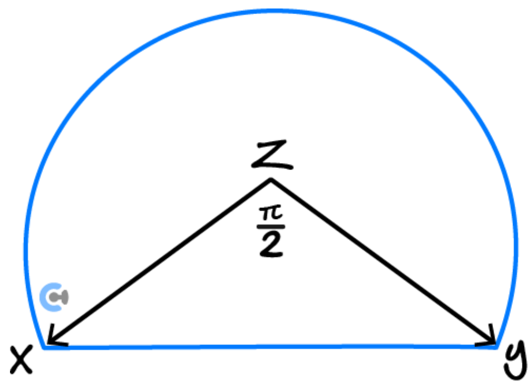
C. $(\tan(x) - 3)(\tan(x) - 2) = 0$

D. $(\tan(x) - 1)(\tan(x) + 6) = 0$

E. $(\tan(x) + 1)(\tan(x) + 6) = 0$

Question 94

The figure below shows the cross-section of a railway tunnel, modelled as the major segment of a circle, centre at Z and radius of 5 m. The angle $\angle XZY$ is $\frac{\pi}{2}$ radians.



a. Find the exact length of XY.

b. Determine the area of the triangle ACB.

c. Find the cross-sectional area of the tunnel.

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Question 95

The distance between the town of Algebraville (A) and the town of Baseville (B) is 80 km. Baseville is on a bearing of 80° from Algebraville.
The village of Contourville (C) is on a bearing of 110° from Algebraville and on a bearing of 190° from Baseville. The village of Desmosville (D) is on a bearing of 150° from Algebraville and on a bearing of 230° from Baseville.

a. Find, correct to one decimal place where appropriate, the distance between:

i. Baseville and Contourville.

ii. Baseville and Desmosville.

iii. Contourville and Desmosville.

b. Find the bearing of Desmosville from Contourville.

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Question 96

The population of birds in a particular location varies according to the rule:

$$b(t) = 1200 + 300\cos\left(\frac{\pi t}{6}\right),$$

where b is the number of birds and t is the number of months after 1 April 2020.

a. Find the period and amplitude of the function $b(t)$.

b. Find the maximum and minimum populations of birds in this location.

c. Find $b(4)$.

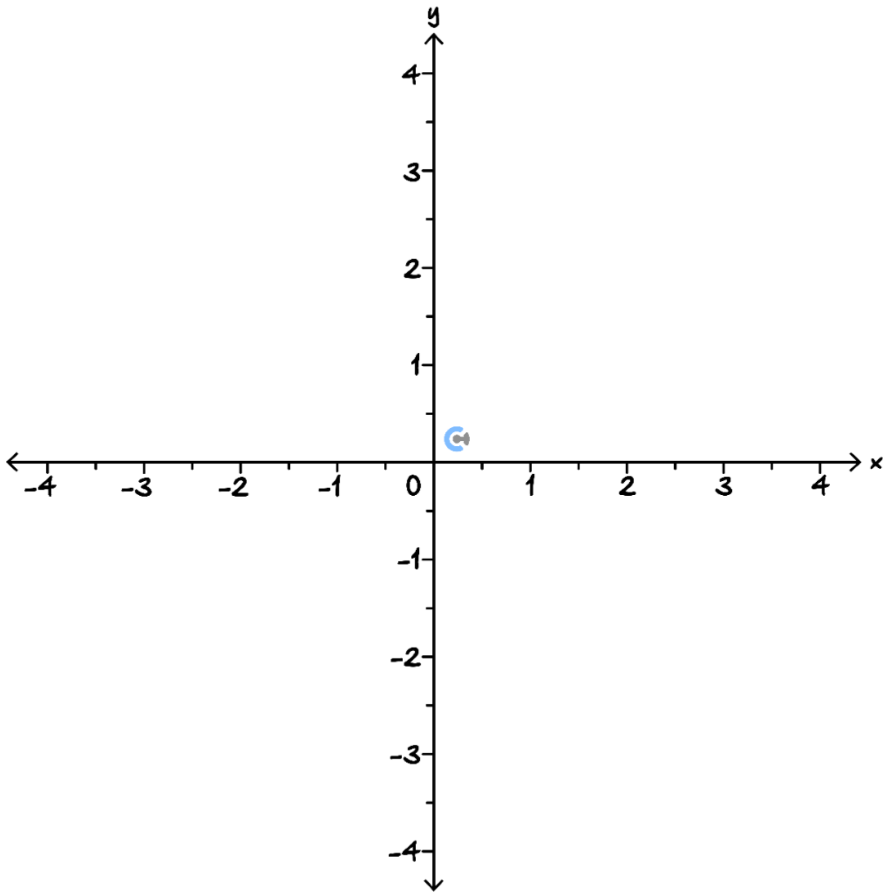
d. Over the 10 months from 1 April 2020, find the fraction of time when the population of birds in this location was less than $b(4)$.

Question 97 (5 marks)

Consider the function $f: D \to R$, where $f(x) = 2\arcsin(x^2 - 1)$.

a. Determine the maximal domain D and the range of f. (2 marks)

b. Sketch the graph of $y = f(x)$ on the axes below, labelling any endpoints and the y-intercept with their coordinates. (3 marks)



Question 98 (6 marks)

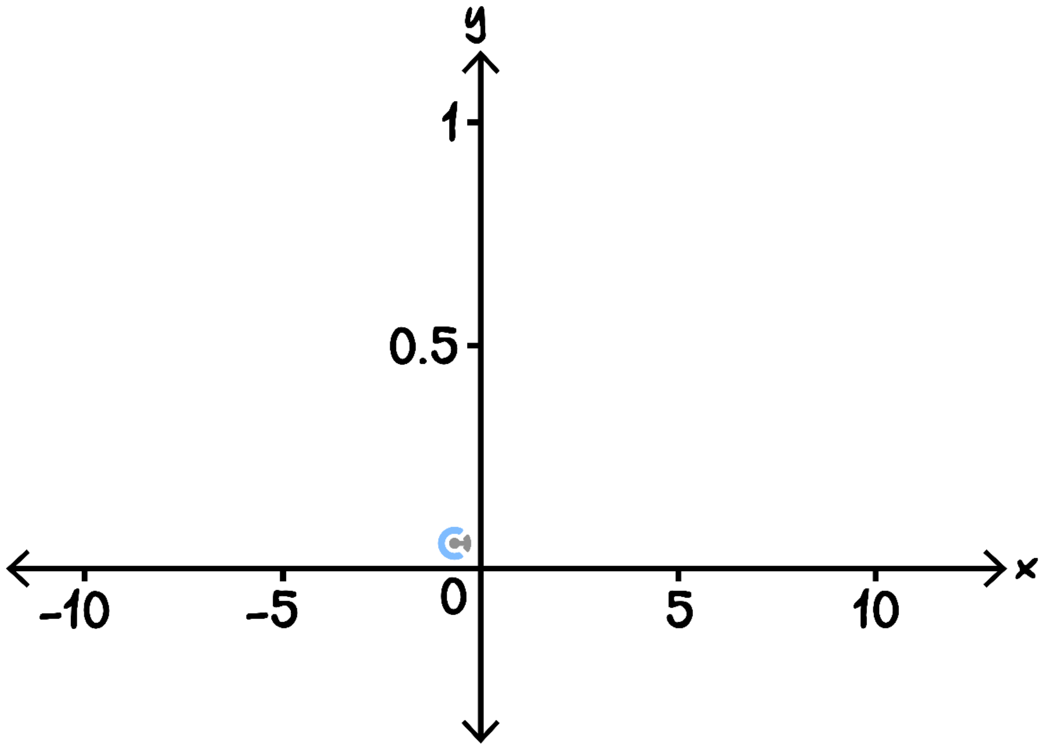
a.

i. Use an appropriate double-angle formula with $t = \tan\left(\frac{5\pi}{12}\right)$ to deduce a quadratic equation of the form $t^2 + bt + c = 0$, where b and c are real values. (2 marks)

ii. Hence, show that $\tan\left(\frac{5\pi}{12}\right) = 2 + \sqrt{3}$. (1 mark)

Consider $f: \left[\sqrt{3}, 6 + 3\sqrt{3}\right] \to R, f(x) = \arctan\left(\frac{x}{3}\right) - \frac{\pi}{6}$.

b. Sketch the graph of f on the axes below, labelling the endpoints with their coordinates. (3 marks)



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