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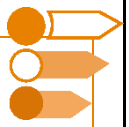
VCE Specialist Mathematics ½
Logic & Algorithms II [2.5]
Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 - Pg 12
Supplementary Questions	Pg 13- Pg 22

Section A: Compulsory Questions



Sub-Section [2.5.1]: Understand the Basics of Logic and Propositional Statements

Question 1

Translate the following to English:
$A =$ I study hard.
$B =$ I understand the material.
$C =$ I will pass the course.

$$A \wedge B \Rightarrow C$$

If I study hard and I understand the material, then I will pass the course.

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Question 2

Translate the following to English:
J = I keep my phone charged.
K = I am reachable.
L = There is an emergency.

$$\neg J \wedge L \Rightarrow \neg K$$

If I do not keep my phone charged and there is an emergency, then I am not reachable.

Question 3

Translate into propositional logic using correct syntax:

If the student studies diligently, then he will pass the exam and not need a retake.

Let S = The student studies diligently.
Let P = He passes the exam.
Let R = He needs a retake.
$S \Rightarrow (P \wedge \neg R)$

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Sub-Section [2.5.2]: Construct Truth Tables and Recognise Equivalent Logical Expressions

Question 4

Write the truth table for:

$$\sim(p \vee q)$$

p	q	pVq	~ (pVq)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Question 5

Write the truth table for:

$$(p \wedge q) \vee \sim q$$

p	q	p∧q	~q	(p∧q) ∨~q
T	T	T	F	T
T	F	F	T	T
F	T	F	F	F
F	F	F	T	T

Question 6

Construct a truth table for the statement $(p \vee q) \wedge \neg r$.

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

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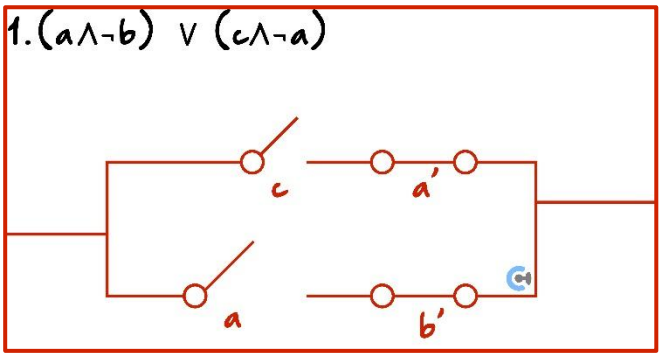
Sub-Section [2.5.3]: Represent Logical Expressions using Switching Circuits and Logic Gates

Question 7

Consider the expression:

$$(a \wedge \neg b) \vee (c \wedge \neg a)$$

Draw the switching circuit that is represented by this expression.



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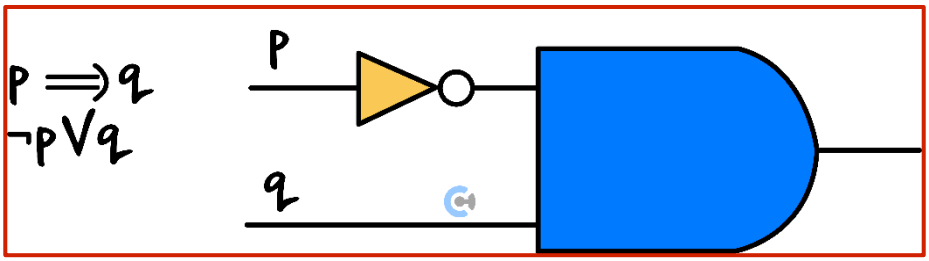
Question 8

Use logic gates to represent the following expression and draw the corresponding truth table:

$$p \Rightarrow q$$

Expression $p \Rightarrow q$:

p	q	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1



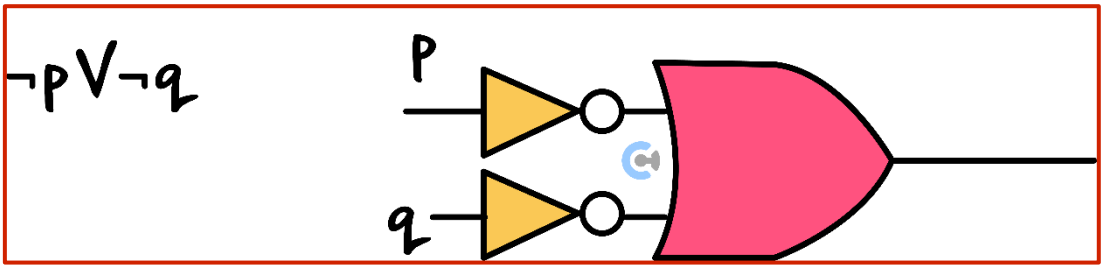
Question 9

Use logic gates to represent the following expression and draw the corresponding truth table:

$$\neg p \vee \neg q$$

Expression $\neg p \vee \neg q$:

p	q	$\neg p \vee \neg q$
1	1	0
1	0	1
0	1	1
0	0	1



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Sub-Section [2.5.4]: Simplify and Evaluate Boolean Algebra Expressions using Algebraic Identities and Karnaugh Maps

Question 10

Simplify each expression by algebraic manipulation.

a. $a + 0 =$

a

b. $a + \bar{a} =$

1

c. $a + ab =$

$a(1 + b) = a$

Question 11

Simplify each expression by algebraic manipulation.

a. $a(\bar{a} + b) =$

$a\bar{a} + ab = ab$

b. $ab + \bar{a}b =$

$b(a + \bar{a}) = b$

c. $a(a + b + c) =$

$aa + ab + ac + \cdots = a + ab + ac + .. = a$

Question 12

Simplify each expression by algebraic manipulation where $f(a, b, c) = a + b + c$.

a. $f(a, b, ab) =$

$a + b + ab = a + b$

b. $f(a, b, \bar{a} \cdot \bar{b}) =$

$a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$

c. $f[a, b, (\overline{ab})] =$

$a + b + (\overline{ab}) = a + b + \bar{a} + \bar{b} = 1$

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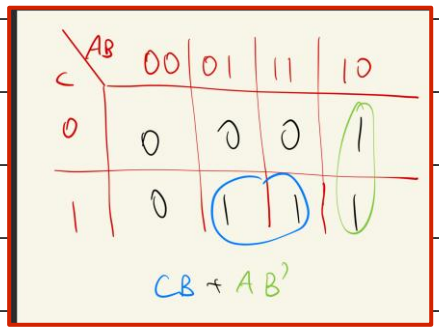
Question 13

Using a Karnaugh map, identify the Boolean expression corresponding to each of the following truth tables.

a.

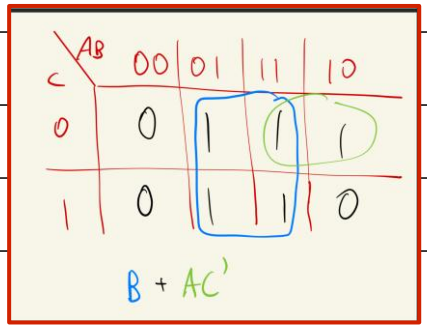
A	B	C	Result
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



$AB' + CB$

b.

A	B	C	Result
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



$B + AC'$

Sub-Section: The 'Final Boss'

Question 14

A traffic control system at an intersection is designed to activate a warning signal based on the status of three road sensors (A,B,C). Each sensor detects whether a vehicle is present (1) or absent (0). If at least two of the three sensors detect vehicles (i.e., receive a '1' signal), the warning light turns on (1) to alert drivers. Otherwise, the light remains off (0).

a. Construct a truth table with entries 0s and 1s that describes the operation of the traffic light control system.

We have three inputs:

- A, B, C (road sensors)
- Output F (warning light ON or OFF)

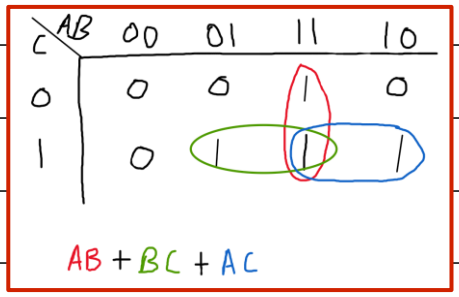
The warning light turns ON ($F = 1$) when at least two of the three sensors detect vehicles (A, B, C).

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

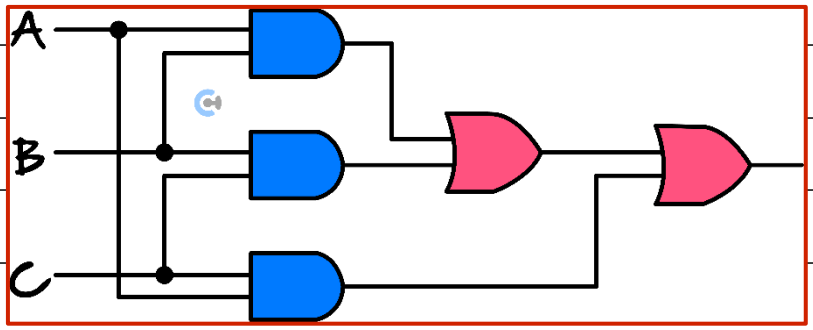
b. Derive a Boolean expression for the traffic light based on your truth table from **part a.**

$$\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

c. Use a Karnaugh map to simplify the Boolean expression obtained in **part b.**



d. Draw a logic circuit for the traffic light system using logic gates, based on your simplified Boolean expression from **part c.**



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Section B: Supplementary Questions

Sub-Section [2.5.1]: Understand the Basics of Logic and Propositional Statements

Question 15

Translate the following to English:
P = I eat healthy.
Q = I exercise regularly.
R = I will lose weight.

$$P \wedge Q \Rightarrow R$$

If I eat healthy and I exercise regularly, then I will lose weight.

Question 16

Translate the following to English:
A = I go jogging.
B = The weather is good.
C = I will feel energised.

$$\neg B \Rightarrow (\neg A \wedge \neg C)$$

If the weather is not good, then I will not go jogging and I will not feel energised.

Question 17

Translate into propositional logic using correct syntax:

If the team wins the match, then the fans will celebrate and the opposing team will be disappointed.

Let W = The team wins the match.
Let C = The fans celebrate.
Let D = opposing team is disappointed.
$W \Rightarrow (C \wedge D)$

Question 18

Translate into propositional logic using correct syntax:

If the baker uses old flour, then the bread will not rise and the customers will complain.

Let F = The baker uses old flour.
Let B = The bread rises.
Let C = The customers complain.
$F \Rightarrow (\neg B \wedge C)$

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Sub-Section [2.5.2]: Construct Truth Tables and Recognise Equivalent Logical Expressions

Question 19

Write the truth table for:

$$\sim p \vee q$$

p	q	~p	~pvq
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Question 20

Write the truth table for:

$$(p \wedge q) \vee (p \vee q)$$

p	q	p∧q	p∨q	(p∧q)∨(p∨q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Question 21

Construct a truth table for the statement $(p \oplus q) \Rightarrow r$, where $\oplus$ is the exclusive or.

p	q	r	$p \oplus q$	$(p \oplus q) \Rightarrow r$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

Question 22

Construct a truth table for the statement $\neg(p \wedge q) \oplus r$.

p	q	r	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \oplus r$
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	F	T	F
F	T	F	F	T	T
F	F	T	F	T	F
F	F	F	F	T	T

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Sub-Section [2.5.3]: Represent Logical Expressions using Switching Circuits and Logic Gates

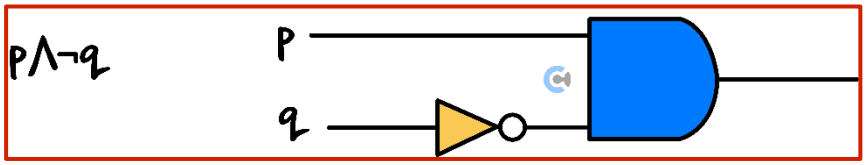
Question 23

Use logic gates to represent the following expression and draw the corresponding truth table:

$$p \wedge \neg q$$

Expression $p \wedge \neg q$:

p	q	$p \wedge \neg q$
1	1	0
1	0	1
0	1	0
0	0	0



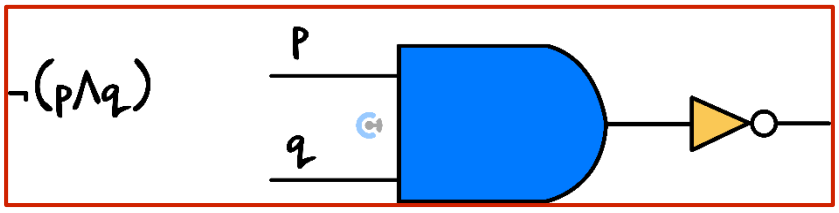
Question 24

Use logic gates to represent the following expression and draw the corresponding truth table:

$$\neg(p \wedge q)$$

Expression $\neg(p \wedge q)$:

p	q	$\neg(p \wedge q)$
1	1	0
1	0	1
0	1	1
0	0	1



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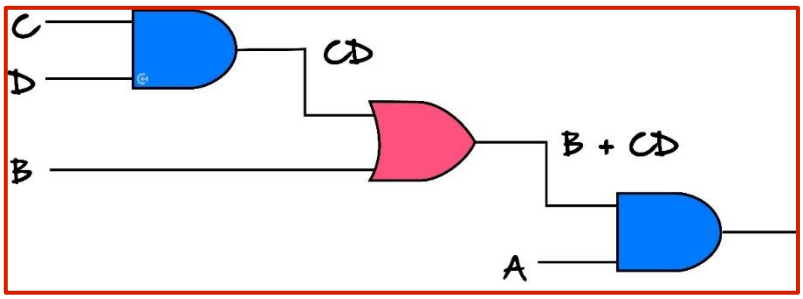
Question 25

Sketch a logic gate for the following expression:

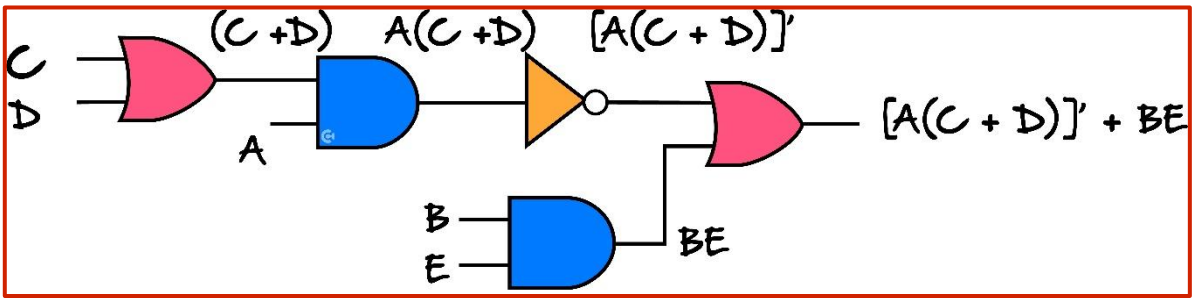
$$A(B + CD)$$



Question 26

Sketch a logic gate for the following expression:

$$[A(C + D)]' + BE$$



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Sub-Section [2.5.4]: Simplify and Evaluate Boolean Algebra Expressions using Algebraic Identities and Karnaugh Maps

Question 27

Simplify each expression by algebraic manipulation.

a. $\bar{a} \cdot 0 =$

0

b. $a + a =$

a

c. $a + \bar{a}b =$

$(a + \bar{a})(a + b) = a + b$

Question 28

Simplify each expression by algebraic manipulation.

a. $y + y\bar{y} =$

y

b. $xy + x\bar{y} =$

$x(y + \bar{y}) = x$

c. $\bar{x} + y\bar{x} =$

$\bar{x}(1 + y) = \bar{x}$

Question 29

Simplify each expression by algebraic manipulation.

a. $(w + \bar{x} + y + \bar{z})y =$

y

b. $(x + \bar{y})(x + y) =$

x

c. $w + (w\bar{x}yz) =$

$w(1 + \bar{x}yz) = w$

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Question 30

Simplify the following expression by algebraic manipulation:

$$(x + z)(\bar{x} + y)(z + y) =$$

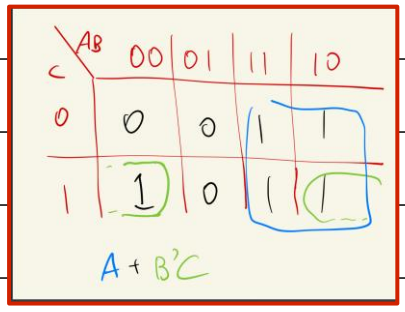
$xy + z\,\bar{x}$

Question 31

Using a Karnaugh map, identify the Boolean expression corresponding to each of the following truth tables:

a.

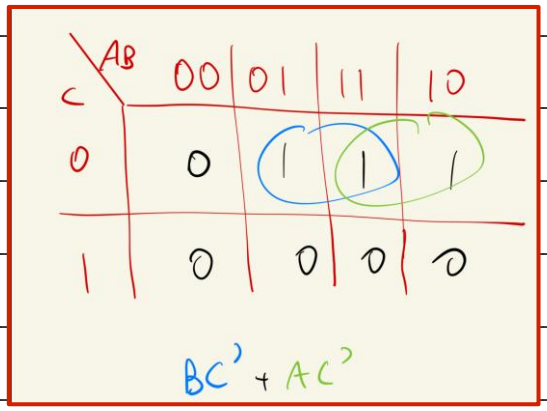
A	B	C	Result
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



$A + B'C$

b.

A	B	C	Result
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	1	0	1
1	1	1	0



$(A + B)C' = AC' + BC'$

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