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VCE Specialist Mathematics ½
Logic & Algorithms I [2.4]
Workbook

Outline:

Learning Objectives:

□ **SM12 [2.4.1]** - Write and understand basic algorithms

□ **SM12 [2.4.2]** - Understand and evaluate algorithms that have conditional statements and represent hybrid functions as algorithms

□ **SM12 [2.4.3]** - Understand and evaluate algorithms with loops

□ **SM12 [2.4.4]** - Write and evaluate functions using pseudocode

Section A: Algorithms

Sub-Section: Introduction to Algorithm

What is an algorithm?

Algorithm

- An algorithm is a clearly specified ____.

Question 1

Write down the steps necessary (i.e., create an algorithm) to do proof by contrapositive.

Sub-Section: Assigning Variables

Assigning Variables

- To construct algorithms for more mathematical/complex problems, ______________________ variables will be useful.
  - E.g., $A \leftarrow 3$ assigns the **value 3** to the **variable A**.
- We can also ____________________ our variables using the arrow.
  - E.g., $A \leftarrow A + 3$ assigns the value $\boldsymbol{A + 3}$ to the **variable A**.
  - Since the value of A was already 3, its new value will be 6.

Question 2 Walkthrough.

For the following algorithm, evaluate the final output of the algorithm.

```
Step 1. $A \leftarrow 3$
Step 2. $A \leftarrow A + 2$
Step 3. $A \leftarrow 2A - 1$
Step 4: Print $A$.
```

Question 3

Construct an algorithm that doubles any input given.

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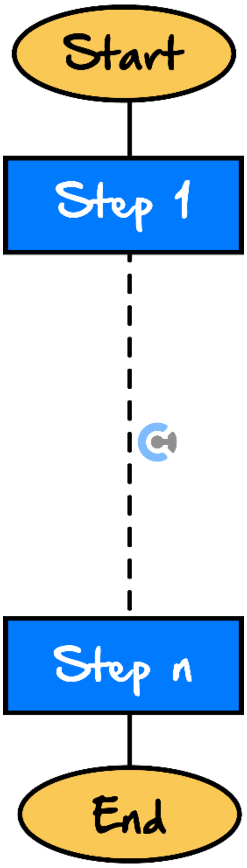
Sub-Section: Flowchart

How do we visualise the algorithm?

Flowcharts

- The visual way of representing the algorithm.



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Question 4 Walkthrough.

Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} x - 2, & x > 0 \\ 2x - 3, & x \leq 0 \end{cases}$$

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Question 5

Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} 5x - 1, & x > 2 \\ \frac{1}{3}x - 1, & x \leq 2 \end{cases}$$

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Sub-Section: Table of Values

How do we track the value of the variables?

Table of Values

- Used to check if the algorithm works by following the steps one by one.

	Variable (s)
Step 1	Value 1
. . .	. . .
Step n	Value n

Question 6 Walkthrough.

Consider the following algorithm.

$$\text{Step 1: } A \leftarrow 2 \text{ and } B \leftarrow 3$$

$$\text{Step 2: } A \leftarrow 2A + 4 \text{ and } B \leftarrow 2B - 1$$

$$\text{Step 3: } A \leftarrow B$$

Perform a desk check (construct the table of values) for the values of A and B.

Question 7

The following algorithm was intended to swap the values of two variables, A and B.

```
Step 1: $Input\ A, B$
Step 2: $A \leftarrow B$
Step 3: $B \leftarrow A$
Step 4: $Print\ A, B$
```

a. Perform a desk check (construct the table of values) for the values of A and B as $2, 4$ for the above algorithm.

b. Did the algorithm perform what it was designed for?

c. Construct a new algorithm that swaps the value of A and B, and performs a desk check.

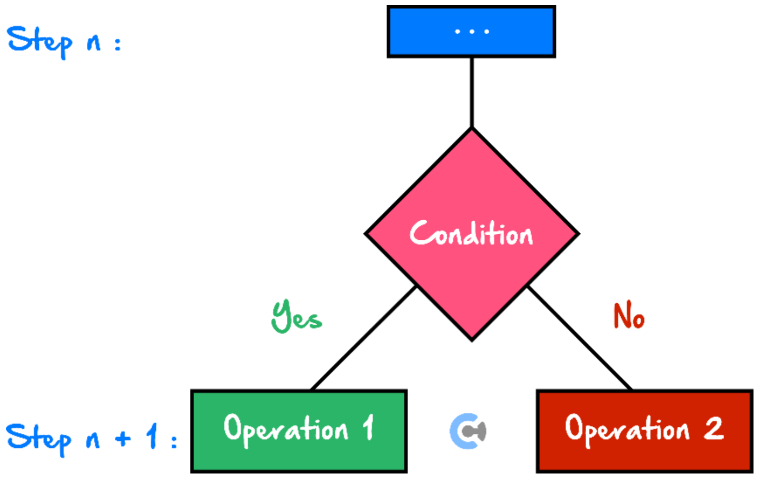
Section B: Selections and Loops

Sub-Section: Selections

Discussion: What could we do if we want to selectively perform an operation?

Selections

- Allows us to perform different operations at a given step depending on a certain condition.



- In other words, we are ____.
- Keywords for selection:

If, then

Otherwise/Else/Else If

- 'If . . . then . . .' instructions are used in algorithms to enable ______________________________ to be made within the algorithm.
- Selection is similar to how _______________________________ work in both Mathematical Methods and Specialist Maths.
- Doing different operations (equations) depending on the condition (domain).

Question 8 Walkthrough.

Construct a table of values for the following algorithm.

```
Step 1: $n \leftarrow 2$

Step 2: $\boldsymbol{If}\ n\ is\ even, \boldsymbol{then}\ T \leftarrow 3n + 5$
        $\boldsymbol{Otherwise}\ T \leftarrow n + 5$

Step 3: $n \leftarrow n + 3$

Step 4: $Print\ n, T.$
```

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Question 9 Tech-Active.

Following is an algorithm for calculating the Australian tax.

```
Step 1: $Input\ income$
Step 2a: $\boldsymbol{If\ income \leq 18200, then\ tax \leftarrow 0}$
Step 2b: $\boldsymbol{Else\ If\ income \leq 37000, then\ tax \leftarrow 0.19 \times income - 3458}$
Step 2c: $\boldsymbol{Else\ If\ income \leq 90000, then\ tax \leftarrow 0.325 \times income - 8453}$
Step 2d: $\boldsymbol{Else\ If\ income \leq 180000, then\ tax \leftarrow 0.37 \times income - 12503}$
Step 2e: $\boldsymbol{Else\ If\ tax \leftarrow 0.45 \times income - 26903}$
Step 3: $Print\ tax$
```

Calculate the tax for $75000.

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Question 10

Turn the following hybrid function into an algorithm.

$$f(n) = \begin{cases} 1 - 2n, & if\ n\ is\ even \\ 4, & if\ n = 5 \\ 2n + 1, & otherwise \end{cases}$$

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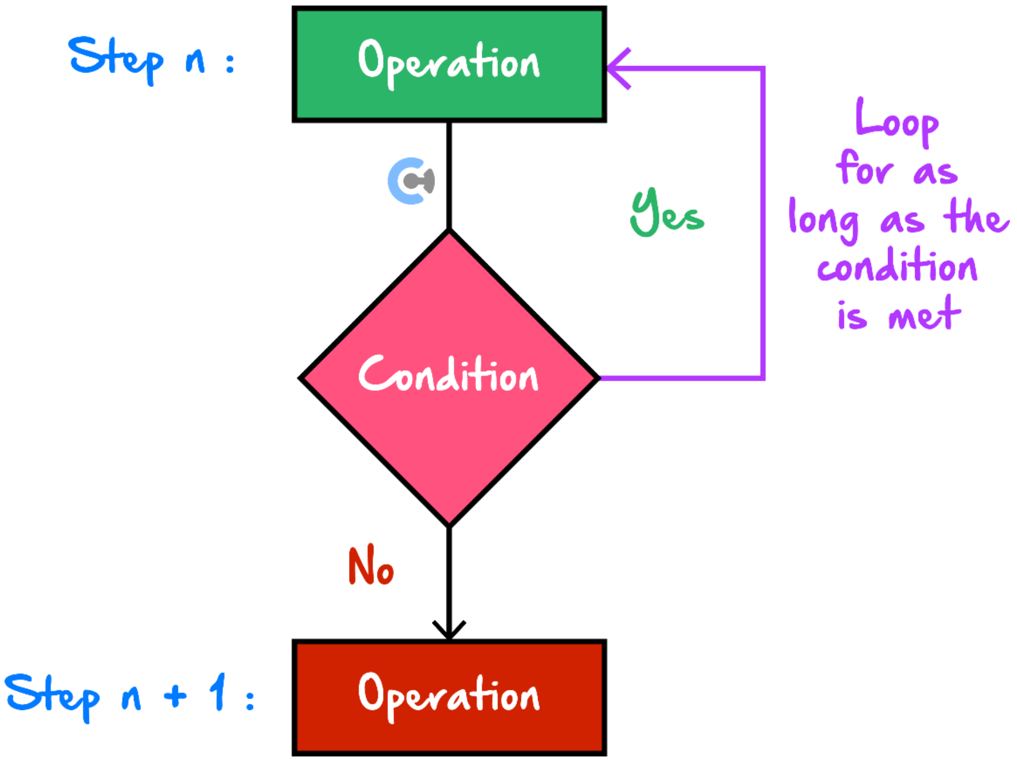
Sub-Section: Loops

Discussion: What do we do if we want to do something repeatedly?

Loops (Iterations)

- Iteration (looping) allows us to repeat steps in a ____.
- It is controlled by the ____________________. We only repeat when a ____________________ is met.



- Keywords for iterations:

Repeat ... For...

Repeat ... while...

Discussion: What are the differences between for and while loops?

Analogy: Your parents taking care of you.

- Consider the two parents below.
  - Parent 1:

$$\textit{I'll take care of you for the next } 10 \textit{ years!}$$

  - Parent 2:

$$\textit{I'll take care of you while you are not married!}$$

- For which parent do we exactly know how many years they will take care of us?
- Hence, which parent is a for loop and which parent is a while loop?

For v/s While Loop

- **For Loop**
  - We know how many iterations will happen.
- **While Loop**
  - We don't know how many loops will happen.

Question 11 Walkthrough.

Create a table of values for the following algorithm.

```
Step 1. $A \leftarrow 10$ and $n \leftarrow 0$
Step 2. $A \leftarrow 2A$ and $n \leftarrow n + 2$
Step 3. **Repeat** from step 2 **while** $n < 5$
```

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Question 12

Construct an algorithm that outputs the largest power of 3 that is less than 1000.

NOTE: We use a while loop here as we do not know straight away how many loops this will take.

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Question 13

Consider the following algorithm:

```
Step 1. $A \leftarrow 87$
Step 2. $A \leftarrow 2A - 1$
Step 3. **Repeat** from step 2 **while** $A > 50$
```

What is the problem with this algorithm?

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Section C: Pseudocode

Sub-Section: Introduction to Pseudocodes

Pseudocode

```
if condition then
    operation 1
else
    operation 2
end if
```

- "Pseudo" = fake, so pseudocode = fake code.
- Concise way of representing algorithms.

Pseudocodes: Indentation (Spacing)

- For pseudocodes, the placement of codes is important.
- For every rabbit hole of codes, we fall into, we write out codes more towards the **right direction.**
    - For every rabbit hole of codes, we come out of, we write out codes back towards the **left direction.**
    - A rabbit hole of codes can be loops, selections, etc.

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Sub-Section: Pseudocode for Selection

Pseudocode: Selections (If, Else, Else if, Then)

- "If-then" allows us to perform operations when a certain condition is met.

```
if  condition  then
    operation
end if
```

- "Else" provides an opportunity to perform a different operation when a condition is **NOT** met.

```
if  condition  then
    operation 1
else
    operation 2
end if
```

- "Else if" provides an opportunity to add multiple pathways each with different conditions.

```
if  condition 1  then
    operation 1
else if  condition 2  then
    operation 2
else
    operation 3
end if
```

Question 14 Walkthrough.

Evaluate the final output from each of the following:

```
$x \leftarrow 7$
if $x < 15$
    $x \leftarrow x + 10$
else
    $x \leftarrow x - 10$
print $x$
```

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Question 15

Evaluate the final output from each of the following:

```
$a \leftarrow 1$
$b \leftarrow 2$
if $a + b < 5$
    $b \leftarrow b + 5$
    $a \leftarrow a - 1$
end if
print $a, b$
```

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Sub-Section: Pseudocode for Loops

Pseudocode: Iterations (For Loops, While Loops, and Nested Loops)

- **For Loops:** It is a loop that increases the variable by 1 each time it loops.
  - Variable moved from ______________________________ to ______________________________ by 1.

```
for   variable   from   lower bound   to   upper bound
                          condition

    operation

end for
```

- **While Loops:** It is a loop that does **NOT** change the value of a variable by default.
  - To change the value of the variable, it needs to be described in the ___________________ section.
  - Without this, we can create an ________________________________.

```
while   condition

    operation

end while
```

- **Nested Loop:** We can have a loop happening within another loop.
  - The first pass of the outer loop starts the inner loop, which executes to completion. Then the second pass of the outer loop starts the inner loop again. This repeats until the outer loop finishes.

```
for   var 1   from   lower 1   to   upper 1

    for   var 2   from   lower 2   to   upper 2

        operation 2

    end for

    operation 1

end for
```

- **Example of a Nested Loop:** Minutes and seconds.

```
Declare integer seconds, minutes

For minutes = 0 to 59

    For seconds = 0 to 59

        Output “”, seconds

    Next For

Next For
```

Question 16 Walkthrough.

Evaluate the final output from each of the following:

```
$total \leftarrow 0$
for $i$ from 1 to 3
    $total \leftarrow total + i$
end for
print $total$
```

Question 17

Evaluate the final output from each of the following:

a.

```
$a \leftarrow 5$
$\mathrm{b} \leftarrow 7$
while $a + b < 15$
    $b \leftarrow b - 1$
    $a \leftarrow a + 2$
end while
print $a, b$
```

b.

```
$c \leftarrow 0$
for $a$ from $1$ to $2$
    for $b$ from $1$ to $2$
        $c \leftarrow c + ab$
    end for
end for
```

Sub-Section: Functions

Function

- Think of a function as a bag of algorithms.

Instead of saying "an algorithm for picking the largest prime number smaller than *n*", we can save this algorithm as $f(n)$.

- Using functions allows us to easily change the input of the algorithm.
- By incorporating functions within another algorithm, it allows us to have an ______________________ ____.

  - By simply mentioning the function within the pseudocode, we can incorporate another algorithm.

```
define $function(input)$:
    $follow\ these\ instructions$
    return $output$
```

Question 18 Walkthrough.

Construct a pseudocode for a function named "*dist*" which calculates the distance between two points.

NOTE: Within the function, the two points can be defined as any arbitrary value.

Question 19

Using pseudocode, define a function for finding each of the following:

a. The gradient of the line through two points (a, b) and (c, d).

b. Number of solutions for a quadratic equation $ax^2 + bx + c = 0$.

Sub-Section: Lists

Lists

- Lists are merely a collection of values. More formally, it is defined as a finite sequence of values.

$A \leftarrow [1, 3, 5, 7, 9]$: Defines A as a list of odd numbers up to 10.

- The notation $A[n]$ spits out the "n^{th}" value in the list. E.g., $A[3] = 5$.

- To add more values to the list we can ______________________.

Append 11 to A (From above)

Result: $A = [1, 3, 5, 7, 9, 11]$.

- Index:
  - The position of an entry in a list is called its index. In this book, we use 1 as the index of the first entry. However, most programming languages use 0 as the index of the first entry.

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Question 20 Walkthrough.

Consider the algorithm below and evaluate the output.

```
$A \leftarrow [\,]$
for $i$ from 1 to 10
    if $i < 8$ then
        append $i$ to $A$
    end if
end for
return $A$
```

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Question 21

Consider the algorithm below and evaluate the output.

```
$A \leftarrow [\,]$
for $i$ from 1 to 8
    if $i < 4$ then
        append $2i$ to $A$
    else
        append $\mathrm{i} - 2$ to $A$
    end if
end for
return $A$
```

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Contour Checklist

□ Learning Objective: [2.4.1] - Write and Understand Basic Algorithms

Key Takeaways

Algorithm

□ An algorithm is a clearly specified ______________________________________.

Assigning Variables

□ To construct algorithms for more mathematical/complex problems, ______________________ variables will be useful.

- ○ E.g., $A \leftarrow 3$ assigns the **value** $\mathbf{3}$ to the **variable** $\boldsymbol{A}$.

□ We can also ____________________ our variables using the arrow.

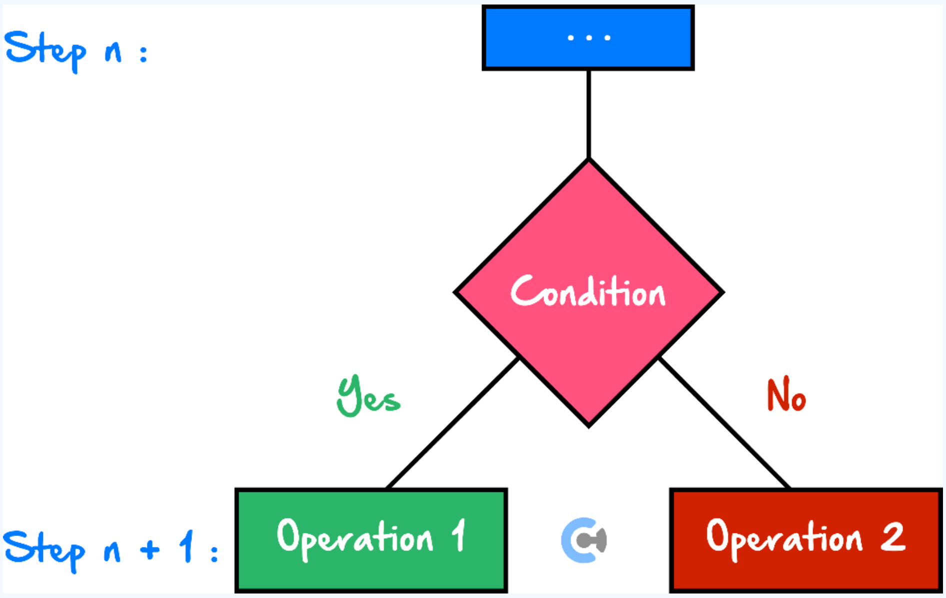
- ○ E.g., $A \leftarrow A + 3$ assigns the value $\boldsymbol{A} + \mathbf{3}$ to the **variable** $\boldsymbol{A}$.

□ **Learning Objective: [2.4.2] - Understanding and Evaluate Algorithms that have Conditional Statements and Represent Hybrid Functions as Algorithms**

Key Takeaways

Selections

□ Allows us to perform different operations at a given step depending on a certain condition.



□ In other words, we are ____.

□ Keywords for selection:

If, then

Otherwise/Else/Else If

- ○ 'If . . . then . . .' instructions are used in algorithms to enable __________________________ to be made within the algorithm.

□ Selection is similar to how ____________________________ work in both Mathematical Methods and Specialist Maths.

□ Doing different operations (equations) depending on the condition (domain).

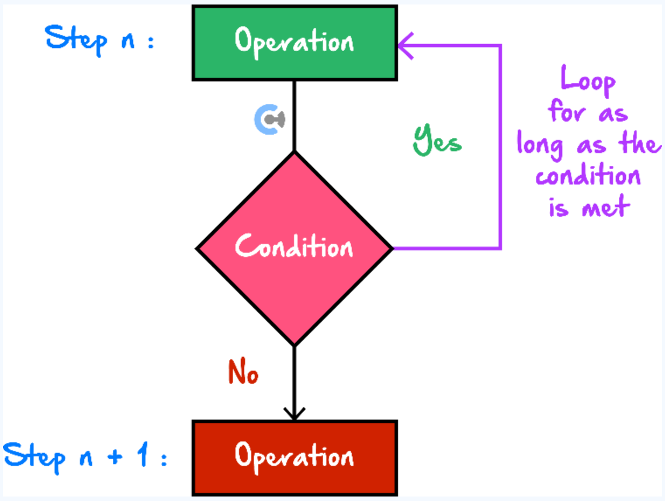
□ Learning Objective: [2.4.3] – Understand and Evaluate Algorithms with Loops

Key Takeaways

Loops (Iterations)

□ Iteration (looping) allows us to repeat steps in a ______________________________________.

□ It is controlled by the _________________. We only repeat when a __________________ is met.



□ Keywords for iterations:

Repeat ... For...

Repeat ... while...

□ Learning Objective: [2.4.4] - Write and Evaluate Functions Using Pseudocode

Key Takeaways

Pseudocode

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□ "Pseudo" = fake, so pseudocode = fake code.

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Function

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```

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