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VCE Specialist Mathematics ½

Logic & Algorithms I [2.4]

Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2- Pg 14
Supplementary Questions	Pg 17- Pg 26

Section A: Compulsory Questions

Sub-Section [2.4.1]: Write and Understand Basic Algorithms

Question 1

Construct an algorithm that triples any input given.

Step *1*. Input A,

Step *2*. $A \leftarrow 3A$

Step *3*: Print *A*.

Question 2

Construct an algorithm that halves any input given and adds 1.

Step *1*. Input A,

Step *2*. $A \leftarrow 1/2A+1$

Step *3*: Print *A*.

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Question 3

Construct an algorithm that subtracts 5 from an input and then multiplies by 3.

Step 1. Input A

Step 2. $A \leftarrow 3(A-5)$

Step 3. Print A

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Sub-Section [2.4.2]: Understanding and Evaluating Algorithms that have Conditional Statements and Represent Hybrid Functions as Algorithms

Question 4

a. Turn the following function into an algorithm.

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Step 1: Input x

Step 2: **If** x>=0, **then** $y \leftarrow x$

else $y \leftarrow -x$

Step 3: Print y.

b. Evaluate the final output:

$a \leftarrow 1$
$b \leftarrow 1$
if $a + b < 7$
  $b \leftarrow b - 2$
  $a \leftarrow a + 3$.
end if
print a, b.

$a = 4$ and $b = -1$.

Question 5 Tech-Active.

Following is an algorithm for calculating the Australian tax.

Step 1: Input income.

Step 2a: If income ≤ 18200, then tax $\leftarrow 0$.

Step 2b: Else If income ≤ 37000, then tax $\leftarrow 0.19 \times$ income $- 3458$.

Step 2c: Else If income ≤ 90000, then tax $\leftarrow 0.325 \times$ income $- 8453$.

Step 2d: Else If income ≤ 180000, then tax $\leftarrow 0.37 \times$ income $- 12503$.

Step 2e: Else If tax $\leftarrow 0.45 \times$ income $- 26903$.

Step 3: Print tax.

a. Calculate tax for 200000.

$$200000 \times 0.45 - 26903 = 63097$$

b. Calculate tax for 45888.

$$45888 \times 0.325 - 8453 = 6460.6$$

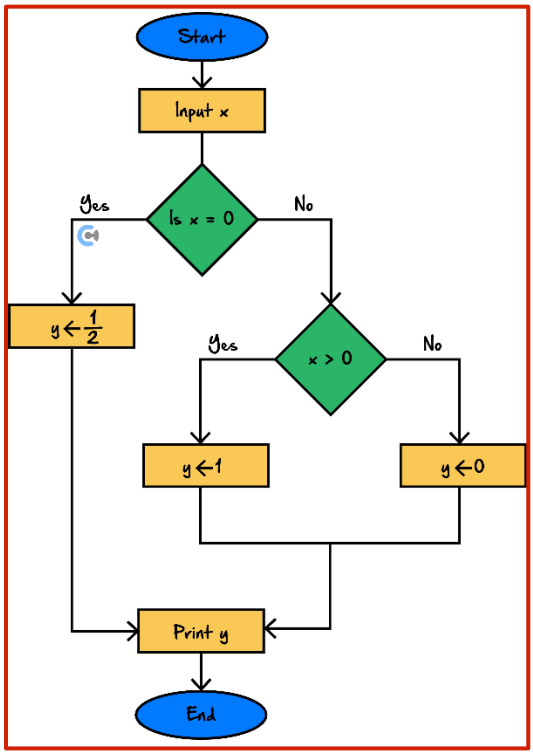
c. Calculate tax for 90001.

$$90001 \times 0.37 - 12503 = 20797.37$$

Question 6

a. Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \\ \frac{1}{2} & x = 0 \end{cases}$$



b. Write an algorithm for the following hybrid function.

$$f(x) = \begin{cases} 1 & x > 3 \\ 0 & x = 3 \\ -1 & x < 3 \end{cases}$$

Step 1: Input x

Step 2: **If $x > 3$, $y \leftarrow 1$**

else if $x = 3$. then $y \leftarrow 0$

else $y \leftarrow -1$

Step 3: print y

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Sub-Section [2.4.3]: Understand and Evaluate Algorithms with Loops

Question 7

Check whether the following algorithm has any problems. If there is a problem, state the problem; if there is no problem, give the final output of the algorithm.

Step 1: $A \leftarrow 60$

Step 2: $A \leftarrow 2A - 50$

Step 3: Repeat 2 while $A > 60$.

It goes on infinitely. The condition of the loop is ALWAYS met.

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Question 8

Evaluate the final output.

$a \leftarrow 1$
$b \leftarrow 1$
while $a + b < 7$
    $b \leftarrow b - 2$
    $a \leftarrow a + 3$.
end while
print a, b.

$a = 13$ and $b = -7$.

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Question 9

a. Evaluate the final output:

$c \leftarrow 0$
for a from 1 to 2
  for b from 1 to 2
    $c \leftarrow c - ab$
  end for
end for
print c.

$a = 13$ and $b = -7$.

a	b	c
1	1	-1
	2	-3
2	1	-5
	2	-9

Ans : $c = -9$.

b. Construct an algorithm that outputs the largest multiple of 5 that is less than or equal to 100.

Step 1: $x \leftarrow 0$
Step 2: $x \leftarrow x + 5$
Step 3: Repeat 2 while $x \leq 100$.
Step 4: Print x.

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Sub-Section [2.4.4]: Write and Evaluate Functions using Pseudocode

Question 10

Using pseudocode, define a function for finding each of the following:

a. Finding the area of the triangle given base and height.

Define area for (b, h)
$A \leftarrow \frac{1}{2}bh$
return A.

b. Finding the surface area of a sphere given radius.

Define area sphere (r)
$A \leftarrow 4\pi r$
return A.

c. Define a function for modulus.

Define modulus (X)
if $X > 0$
return X
else
return $-X$.

Question 11

Evaluate the following algorithm:

a. $A \leftarrow [\ ]$
for n from 1 to 6
  if $n =$ even then
    append 1 to A.
  else
    append 0 to A.

$A = [0, 1, 0, 1, 0, 1]$

b. $A \leftarrow [\ ]$
for n from 1 to 5
  $B \leftarrow 2n + 1$
  append B to A.

$A = [3, 5, 7, 9, 11]$

c. $A \leftarrow [1, 1]$
for n from 1 to 5
    $x \leftarrow A[n] + A[n+1]$
    append x to A.

$A = [1, 1, 2, 3, 5, 8, 13]$

Question 12

Using pseudocode, construct an algorithm for the following:

a. Find the biggest possible product from any 2 numbers from the list $= (23, 5, 2, 56, 34)$.

NOTE: Cannot multiply numbers by themselves.

```
Maxproduct= 1
for a from 1 to 5
   for b from 1 to 5
       if a = b
          break
       else
          product = A[a] × A[b]
            if product ≥ max product
               max product ← product
    end for.
end for.
print "maxproduct"
```

b. Find the smallest possible sum of 2 numbers from the same list as **part a**.

```
A ← [23, 5, 2, 56, 34]   minsum = 1000
    for a from 1 to 5
        for b from 1 to 5
            if a = b
                return
            else
                sum = A[a] + A[b]
                if sum ≤ minsum
                    minsum ← sum
        end for.
    end for.
    print "minsum"
```

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Sub-Section: Final Boss

Question 13

Consider the sequence $1, 4, 7, 10, 13, 16, 19 \ldots 3(n-1)+1$.

Using pseudocode, write an algorithm for:

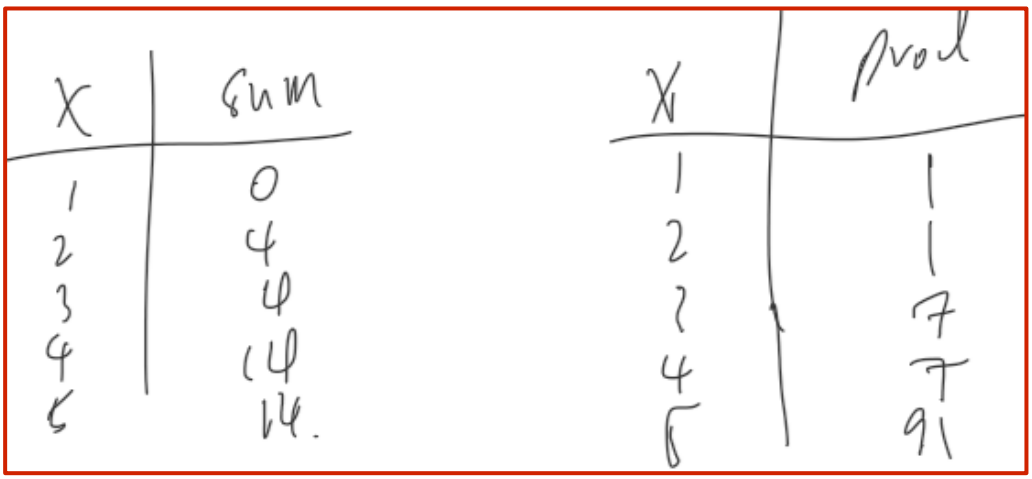
a. Calculate the sum of the even terms in the sequence up to the n^{th} term.

```
Step 1: input int (n), sum ← 0
Step 2: for x from 1 to n
            if x =even
              term ← 3(x − 1) + 1
              sum ← sum +term
          end for.
Step 3: print sum.
```

b. Calculate the product of the odd terms in the sequence up to the n^{th} term.

```
Step 1: input int (n), prod ← 1
Step 2: for x from 1 to n
            if x =odd
              term ← 3(x − 1) + 1
             prod ← prod × term
          end for.
Step 3: print prod.
```

c. Construct a table of values to demonstrate each algorithm when $n = 5$.



x	sum
1	0
2	4
3	9
4	14
5	14.

x	prod
1	1
2	1
3	7
4	7
5	91

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Section B: Supplementary Questions

Sub-Section [2.4.1]: Write and Understand Basic Algorithms

Question 14

Construct an algorithm that multiplies any input given by 10.

Step *1*. Input A,

Step *2*. $A \leftarrow 10A$

Step *3*: Print *A*.

Question 15

Construct an algorithm that adds any input given by 5.

Step *1*. Input A,

Step *2*. $A \leftarrow A+5$

Step *3*: Print *A*.

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Question 16

Construct an algorithm that subtracts any input given by 5 and multiplies by 2.

Step *1*. Input A,

Step *2*. $A \leftarrow 1/2A-5$

Step *3*: Print *A*.

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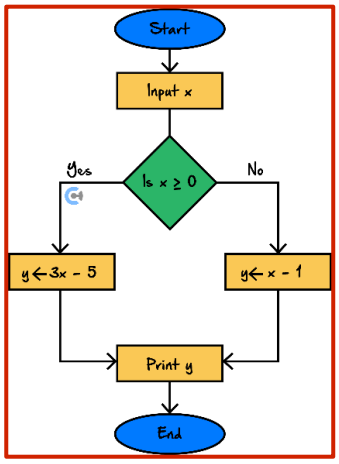
Sub-Section [2.4.2]: Understanding and Evaluating Algorithms that have Conditional Statements and Represent Hybrid Functions as Algorithms

Question 17

Using a flowchart, describe an algorithm of the following hybrid function.

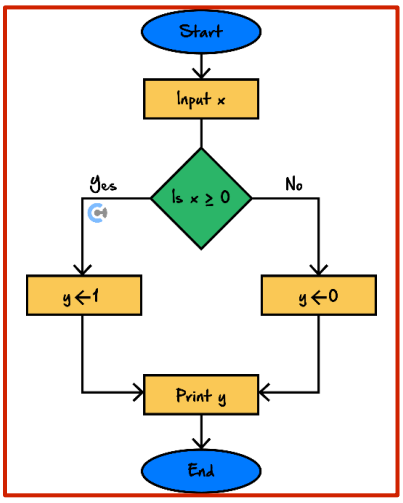
$$f(x) = \begin{cases} 3x - 5 & x \geq 0 \\ x - 1 & x < 0 \end{cases}$$



Question 18

Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



Question 19

Turn the following function into an algorithm.

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ -2x + 1 & x < 1 \end{cases}$$

Step 1: Input x

Step 2: **If** x>=1, **then** $y \leftarrow x\text{^}2$

else $y \leftarrow -2x+1$

Step 3: Print y.

Question 20

Turn the following function into an algorithm.

$$f(x) = max\{n \in \mathbb{R} | n \leq x\}$$

Step 1: input x
Step 2: $y \leftarrow max\{n \in \mathbb{R} | n \leq x\}$
Step 3: print y.

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Sub-Section [2.4.3]: Understand and Evaluate Algorithms with Loops

Question 21

Check whether the following algorithm has any problems. If there is a problem, state the problem; if there is no problem, give the final output of the algorithm.

Step 1: $A \leftarrow 30$
Step 2: $A \leftarrow 3A - 20$
Step 3: Repeat 2 while $A > 65$.

It goes on infinitely. The condition of the loop is ALWAYS met.

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Question 22

Evaluate the following algorithm:

for a from 1 to 10
  if $a =$ even, then
    print "yes"
else
    print "no"
end for.

no yes no yes no yes no yes no yes

Question 23

Check whether the following algorithm has any problems. If there is a problem, state the problem; if there is no problem, give the final output of the algorithm.

Step 1: $A \leftarrow 60$
Step 2: $A \leftarrow 2A - 50$
Step 3: Repeat 2 while $A \leq 130$.

$A = 210$

Question 24

Evaluate the following output:

$a \leftarrow 5$
$b \leftarrow 10$
if $a - b < 5$
  $a \leftarrow a - 5$
  $b \leftarrow b - 10$
end if
print a, b.

$a = 0, b = 0.$

Space for Personal Notes

Sub-Section [2.4.4]: Write and Evaluate Functions using Pseudocode

Question 25

$A \leftarrow [\ ]$
for n from 1 to 5
  append n to A.
  if $n = 1$, then
    return
else
  $A = \sqrt{n^2 + A[n-1]}$
  if $A =$ integer
    print "$A[n-1], n, A$ is a perfect triangle."
end for.

3, 4, 5 is a perfect triangle.

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Question 26

a. Roger decides to invest \$1000 at an interest rate of 10% compounded monthly. Construct an algorithm that computes the number of years needed for Roger's investment to double.

Step 1: $\tau \leftarrow 1000$, rate $\leftarrow 10$, $T \leftarrow 0$
Step 2: $\tau \leftarrow \tau \times \left(1 + \frac{0.1}{12}\right)^{12}, T \leftarrow T + 1$
Step 3: repeat 2 while $\tau \leq 2000$.
Step 4: print T.

b. Jacob decides to invest \$500 at an interest rate of 15% compounded annually. Construct an algorithm that computes the number of years needed for Jacob's investment to increase by 50%.

Step 1: $\tau \leftarrow 500$, rate $\leftarrow 0.15$, $T \leftarrow 0$
Step 2: $\tau \leftarrow \tau \times (1 + \text{rate})^1, T \leftarrow T + 1$
Step 3: repeat 2 while $\tau \leq 750$.
Step 4: print T.

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Question 27

Using pseudocode, write an algorithm to find all the primes less or equal to 100.

```
Prime 1st ← [1]        plistless= 1
    For number from 2 to 100
        check ← 0
        For index from 1 to plistless
            if number / prime list [index] = int,
                check ← check +1
            else
                return
        end for.

        if check = 0
            append number to prime list.
            plistless ← plistless +1
end for.
print "primelist"
```

Question 28

Using pseudocode, construct an algorithm for the following:

Find the shortest distance between any 2 ***different*** coordinates from the list of coordinates.

$$Y \text{ coord} = [1, 35, 5, 41, 5]$$
$$X \text{ coord} = [123, 2, 74, 213, 2]$$

```
Mindist = 300 [or any high enough initial number]
for a from 1 to 5
    for b from 1 to 5
        if a = b,
            break
        else
            distance = √((x[a] − x[b])² + (y[a] − y[b])²)
            if distance ≤ mindist
                mindist ← distance
    end for.
end for.
print min dist.
```

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