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VCE Specialist Mathematics ½
Further Proof Techniques [2.2]
Test Solutions

20 Marks. 22 Minutes Writing.

Results:

Test Questions	_____ / 20

Section A: Test Questions (20 Marks)

Question 1 (4 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. De Morgan Law says that $\neg(A \vee B) = \neg A \vee \neg B$.		✓
b. Contrapositive statement of "if you are a Contour student then you will enjoy learning maths" is given by "if you enjoy learning maths then you are a Contour student".		✓
It should be: If you do not enjoy learning maths then you are Not a contour student.		
c. Proof by contradiction requires assuming that the contradicting statement is true.	✓	
d. Equivalent statement is when a statement and its converse are both true.	✓	
e. Universal statements can be proven by simply giving an example within the set.		✓
f. To disprove an existence statement, you prove the universal statement with the opposite conclusion.	✓	
g. In proof by induction, you can assume that $\mathrm{P}(1), \mathrm{P}(2), \mathrm{P}(3), \ldots \mathrm{P}(k)$ is true and from there, prove that $\mathrm{P}(k) \rightarrow \mathrm{P}(k+1)$.		✓
Only assume $\mathrm{P}(k)$ is true for arbitrary k, then prove $\mathrm{P}(k+1)$.		
h. Induction proof can be done for when k is all real numbers.		✓

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Question 2 (4 marks)

Let $n \in N$. If $5^n - 1$ is prime, then n is odd.

a. Write down the contrapositive of the statement. (1 mark)

If n is even, $5^n - 1$ is not prime (has other factors than itself and 1).

b. Prove that the contrapositive is true. (3 marks)

$n = 2k$, $k \in \mathbb{N}$

Then $5^n - 1 = 5^{2n} - 1$

$= (5^k)^2 - 1^2 = (5^k - 1)(5^k + 1)$

$\therefore$ $5^n - 1$ has a non 5, 1 factor

$5^n - 1$ is not prime

$\therefore$ $5^n - 1$ is prime $\Rightarrow$ n odd

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Question 3 (3 marks)

Prove that if x is irrational then, $\sqrt{x+1}$ is irrational. Use the contradiction method in your proof.

Suppose $x \in \mathbb{Q}'$ AND $\sqrt{x+1} \in \mathbb{Q}$

$\sqrt{x+1} = p/q$, $p, q \in \mathbb{Z} \setminus \{0\}$

$x+1 = \frac{p^2}{q^2}$

$x = \frac{p^2 - q^2}{q^2}$

$p^2 - q^2 \in \mathbb{Z}$

$q^2 \in \mathbb{Z}$

So $x \in \mathbb{Q}$ (contradiction)

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Question 4 (4 marks)

Prove that the following is true for all positive integers n: n is odd if and only if $3n^2 + 8$ is odd.

n odd $\Rightarrow$ $3n^2+8$ odd

$n = 2k+1$, $k \in \mathbb{Z}$

Then $3n^2+8 = 3(2k+1)^2+8$

$= 3(4k^2+4k+1)+8$

$= 12k^2+12k+11$

$= 2(6k^2+6k+5)+1$

But $6k^2+6k+5 \in \mathbb{Z}$, so result is odd

$3n^2+8$ odd $\Rightarrow$ n odd

(equivalent to n even $\Rightarrow$ $3n^2+8$ even

Direct proof:

If n is even, then $n = 2k$ for some int. k.

Therefore $3n^2+8 = 3(2k)^2+8 = 12k^2+8$

$= 2(6k^2+4)$,

which is even.

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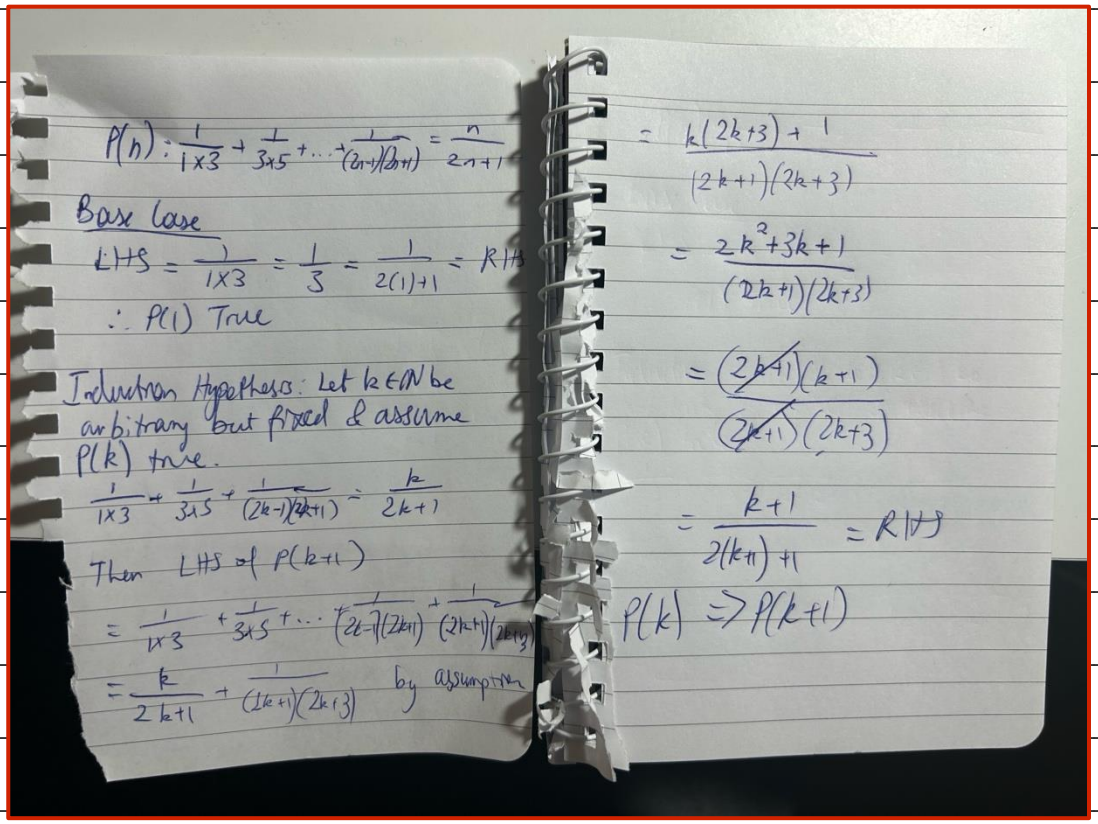
Question 5 (5 marks)

Prove that $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$ for all $n \in N$.



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