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VCE Specialist Mathematics ½
Proofs I [2.1]
Workbook

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Outline:

Sets and Notations Pg 2-14
- Rationalising the Denominator

Statements Pg 15-21
- Operation of Statements
- De Morgans's Law

Proving Number Sets Pg 22-37
- Conditional Statements
- Even and Odd Numbers
- Divisibility
- Rational Numbers

Learning Objectives:

- ☐ SM12 [2.1.1] – Number sets
- ☐ SM12 [2.1.2] – Statements and operations
- ☐ SM12 [2.1.3] – Proving number sets

Section A: Sets and Notations

Context

- We will learn about proofs involving number sets later in this workbook.
- We need to first revise/preview the different number sets encountered in Specialist Mathematics.

What is a set?

Set

$\{2, 4, 6, 8 \ldots\}$

$$\{value_1, \quad value_2, \quad \ldots\}$$

- A set is simply a collection of multiple values.

Let's look at some questions together!

Question 1 Walkthrough.

Interpret the following set and simplify:

$$\{x | x^2 = 4\}$$

$x^2 = 4$

$x = \pm 2$

$\{-2, 2\}$

Your Turn!

Question 2

Interpret the following set and simplify:

$(x-1)(x-3)$

$$\{x: x^2 - 4x + 3 = 0\}$$

$\{1, 3\}$

Question 3 Additional Question.

Interpret the following set and simplify:

$|\,|$ ← modulus
absolute value

$$\{x: |x| = 8\}$$

$\{-8, 8\}$

Space for Personal Notes

Definition

Interval Notation

- Between a and b.

$$(a, b)$$

- Between a and b inclusive.

$$[a, b]$$

- The interval of real numbers between a and b inclusive, but excluding c.

$$[a, b]\backslash\{c\}$$

Let's look at some questions together!

Question 4 Walkthrough.

Express each of the following subsets of R in interval notation.

a. $\left\{x: -\frac{\pi}{2} < x \leq 3\pi\right\}$

b. $\{x: x \neq -2\}$

c. $\{x: x < 3\} \cap \{x: x \neq -2\}$

Your Turn!

Question 5

Express each of the following subsets of R in interval notation.

a. $\{x: x > 5\}$

$x \in (5, \infty)$

b. $\{x: -1 \leq x < 6\} \cap \{x \neq 5\}$

$x \in [-1, 6) \setminus \{5\}$

c. $\{x: x < 4\} \cap \{x: x \neq 0\}$

$x \in (-\infty, 4) \setminus \{0\}$

Question 6 Additional Question.

Express each of the following subsets of R in interval notation.

a. $\{x: x < 3\} \cap \{x: x \geq 2\}$

b. $\{x: x^2 \leq 4\} \cap \{x: x \neq 2\}$

How could we combine two sets?

Definition

Operation of Sets

- Negation: Everything **but**

$$\neg A \text{ or } A'$$

- Intersection

$$A \cap B$$

and

- Union

$$A \cup B$$

or

- Difference

$$A \backslash B$$

Remove

- Product

$$A \times B = \{(a, b), a \in A, b \in B\}$$

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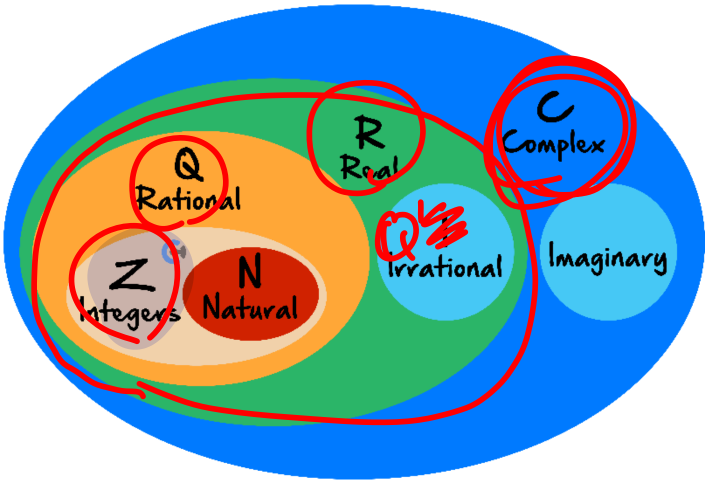
A quick recap of number sets!

Number Sets



$$N \subseteq Z \subseteq Q \subseteq R \subseteq C$$

- Naturals (N)

$N =$ 1, 2, 3 . . . (+ve) integers

- Integers (Z)

$Z =$ +ve or −ve whole number

- Rationals (Q)

$Q = \frac{a}{b}$ $a \in Z$, $b \in Z$

- Real (R)

$R =$ we can draw on a number

- Complex (C)

$C =$ Real or imaginary

i $\sqrt{-1}$

Let's look at some questions together!

Question 7 Walkthrough.

For the following numbers, state all the number sets they are an element of.

a. $\sqrt{3}$

$\mathbb{R}$ $(\mathbb{Q}')$ $\mathbb{C}$

b. $\frac{1}{3}$

$\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

c. 3

$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

d. $3 + i$

i $\sqrt{-1}$

$\mathbb{C}$

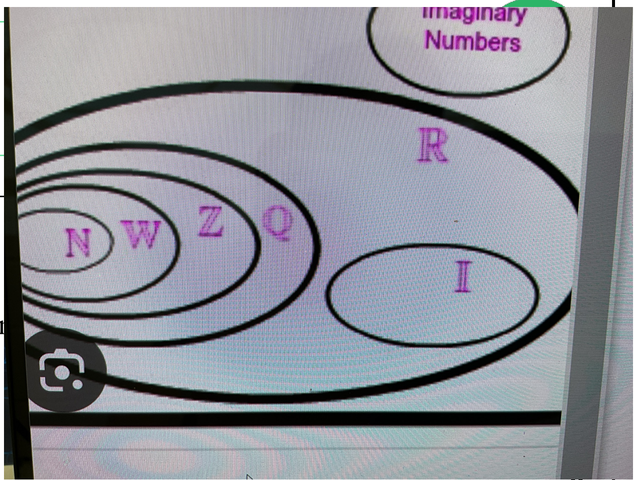
Space for Personal Notes

Recall!

Active Recall: Determining Number Sets

- N is the set of natural number (1, 2, 3...).
- Z is the set of integer (whole number).
- Q is the set of rational ($\frac{a}{b}$ ∈ Z).
- R is the set of real number (number line).
- C is the set of complex (real or imaginary).

Your Turn!



Question 8

For the following numbers, state all the number sets they are an ele

a. -4

$\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

b. $\frac{5}{12}$

$\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

c. $-4+i$

$\mathbb{C}$

Question 9 Additional Question.

For the following numbers, state all the number sets they are an element of.

a. e

b. 10

c. i

Space for Personal Notes

Sub-Section: Rationalising the Denominator

Context

- Take a look at the fraction below:

$$\frac{\sqrt{2}+3}{\sqrt{2}-1}$$

- How can we express this in the form of $\frac{a+\sqrt{b}}{c}$?
- VCAA loves these types of questions!
- EG: 2022 VCAA Specialist Exam 1.

The graph of $y = f(x)$ for $x \in \left[-\frac{\pi}{24}, \frac{\pi}{48}\right]$ is rotated about the x-axis to form a solid of revolution.

Find the volume of this solid. Give your answer in the form $\frac{\left(a-\sqrt{b}\right)\pi}{c}$, where $a, b, c \in Z$. 3 marks

Rationalising the Denominator

- **Aim**: To remove surds in the denominator.

$$\frac{1}{a-\sqrt{b}} = \frac{1}{a-\sqrt{b}} \times \frac{a+\sqrt{b}}{a+\sqrt{b}}$$

- Multiply the top and bottom by the **conjugate of the denominator**.

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Discussion: How does this work? What does $(a-\sqrt{b})\times(a+\sqrt{b})$ equal to?

Difference of perfect squares

Let's look at some questions together!

Question 10 Walkthrough.

Rationalise the denominator for the following:

$$\frac{1}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}}$$

$$= \frac{2-\sqrt{7}}{(2+\sqrt{7})(2-\sqrt{7})}$$

$$= \frac{2-\sqrt{7}}{4-7} = \frac{2-\sqrt{7}}{-3}$$

Recall!

Active Recall

- To rationalise the denominator, we multiply the numerator and denominator by conjugate denominator and expand.

Your Turn!

Question 11

Rationalise the denominator for the following.

a. $\frac{1+\sqrt{2}}{5-\sqrt{2}}$ $= \frac{1+\sqrt{2}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{(1+\sqrt{2})(5+\sqrt{2})}{(5-\sqrt{2})(5+\sqrt{2})} = \frac{5+6\sqrt{2}+2}{25-2}$

$= \frac{7+6\sqrt{2}}{23}$

b. $\frac{5-\sqrt{6}}{7+\sqrt{6}}$ $= \frac{5-\sqrt{6}}{7+\sqrt{6}} \times \frac{7-\sqrt{6}}{7-\sqrt{6}} = \frac{35-12\sqrt{6}+6}{49-6}$

$= \frac{41-12\sqrt{6}}{43}$

Question 12 Additional Question.

Rationalise the denominator for the following. Write your answer in the form $\frac{a+b\sqrt{c}}{d}$.

$$\frac{3+\sqrt{2}}{6-\sqrt{2}}.$$

TIP: We will need this skill again when we look at complex numbers later!

Key Takeaways

- A set is a collection of multiple values.
- When using interval notation, square brackets are used to include the endpoints and round brackets are used to exclude the endpoints.
- Common number sets that are encountered in Specialist Mathematics are N (positive whole numbers), Z (integers), Q (rational numbers), R (real numbers) and C (complex numbers).
- To rationalise a fraction with a surd in the denominator, we can first multiply the numerator and denominator by the conjugate of the denominator and simplify by expanding.

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Section B: Statements

Sub-Section: Operation of Statements

What is a statement?

Context

- Before we start looking at proof questions, we need to learn about statements.

Statements and its Operations

- Statements can be very general.

$$A = \text{Doing SM12}$$

B=Doing MM12

- Negation: Everything **but**

$$\neg A$$

Not doing SM12

- Intersection

$$A \wedge B$$

doing SM12 & MM12

- Union

$$A \vee B$$

doing SM12 or MM12

NOTE: We can use ∪ notation instead of ∨.

Space for Personal Notes

Let's look at some questions together!

Question 13 Walkthrough.

Consider the following statements:

$$A = \text{Doing SM12}$$

$$B = \text{Liking Maths}$$

Write down the statement for:

a. $\neg A$

b. $A \wedge B$

c. $A \vee B$

Recall!

Active Recall

- For two statements A and B, $\neg A$ is the statement Not A, $A \wedge B$ is the statement A and B and $A \vee B$ is the statement A or B.

negation intersection union

Your Turn!

Question 14

Consider the following statements:

$$A = \text{Travelling to Argentina}$$

$$B = \text{Learning Spanish}$$

Write down the statement for:

a. $\neg A$

Not trav. to A.

b. $A \wedge B$

Trav to A & learn S.

c. $A \vee \neg B$

Trav to A OR not learning S

Question 15 Additional Question.

Consider the following statements:

$$A = \text{Taking Accounting } \frac{1}{2}$$

$$B = \text{Taking Accounting } \frac{3}{4}$$

Write down the statement for:

a. $\neg A$

b. $A \wedge \neg B$

c. $A \vee B$

Space for Personal Notes

Sub-Section: De Morgans's Law

Context

- We can do expanding and factorising involving the symbols $\neg$, $\land$ and $\lor$.

Exploration: De Morgan's Law

- Consider the statements:

$$A = \text{I Like Chocolate}$$

$$B = \text{I Like Strawberry}$$

- Consider $\neg(A \land B)$. Write it in words below!

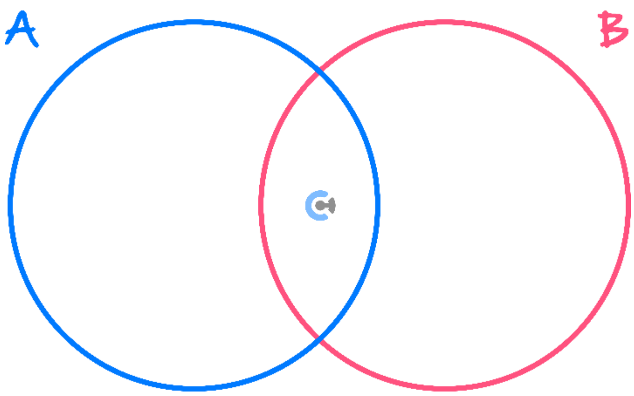
- Consider $\neg A \lor \neg B$. Write it in words below!

- Hence, we can say,

$$\neg(A \land B) = \neg A \lor \neg B$$

- We can also say,

$$\neg(A \lor B) = \neg A \land \neg B$$



NOTE: If you expand or factorise negation, the union and intersection flips!

De Morgan's Law

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

Let's look at some questions together!

Question 16 Walkthrough.

Using De Morgan's Law, write down the statement below in a different way.

Negation of "Sam doesn't like SM or MM."

A: Sam doesn't like SM

B: Sam doesn't like MM

$\neg(A \vee B) = \neg A \wedge \neg B$

Sam likes SM and Sam likes MM

Recall!

Active Recall

- $\neg(A \wedge B) =$ $\neg A \vee \neg B$ and $\neg(A \vee B) =$ $\neg A \wedge \neg B$.

Your Turn!

Question 17

Using De Morgan's Law, write down the statement below in a different way.

Negation of "Bobby has an iPhone but doesn't have a Mac."

$\neg \, (A \wedge B)$

$\neg A \vee \neg B$

Bobby doesn't have an iphone

OR

bobby has a mac

Question 18 Additional Question.

Using De Morgan's Law, write down the statement below in a different way.

Negation of "Bobby can speak Portuguese and English."

Key Takeaways

- We can do the following operations involving statements: $\neg A$ meaning "not A," $A \wedge B$ meaning "A or B," and $A \vee B$, meaning "A and B."
- De Morgan's Laws state that $\neg(A \wedge B) = \neg A \vee \neg B$ and $\neg(A \vee B) = \neg A \wedge \neg B$.
- We notice that in De Morgan's Laws, the intersections and unions flip.

Section C: Proving Number Sets

Sub-Section: Conditional Statements

Context

- Conditional statements will be the most important for us when doing a question involving proof.
- Check out this question from the sample Specialist Mathematics 3/4 exam. The "If ..., then ..." statement is an example of a conditional statement.

A ⇒ B

Question 1

Consider the following statement.

'For all integers n, if n^2 is even, then n is even.'

Which one of the following is the contrapositive of this statement?

A. For all integers n, if n^2 is odd, then n is odd.

B. There exists an integer n such that n^2 is even and n is odd.

C. There exists an integer n such that n is even and n^2 is odd.

D. For all integers n, if n is odd, then n^2 is odd.

E. For all integers n, if n is even, then n^2 is even.

Discussion: If you are living, what must you be doing?

→ breathing
→ heart beating
→ moving
→ eating
→ smelling

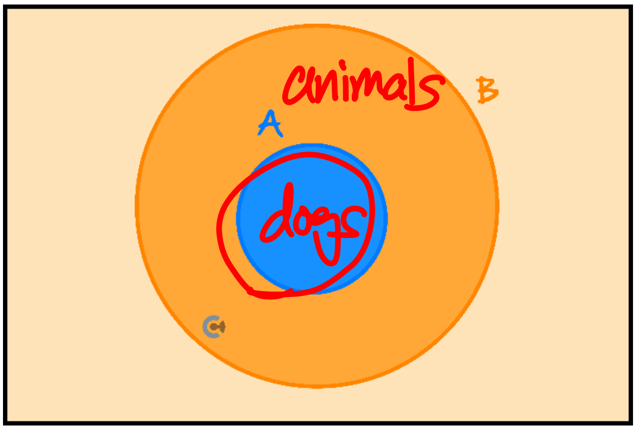
⇒

moving ⇐ living

If living then moving

NOTE: Visually, imply is the same as "subset".

Conditional Statements



$$A \implies B$$

- Notation for "implies": $\implies$

$$\text{"}Hypothesis \implies Conclusion\text{"}$$

A is a ________ of B.

Let's look at some questions together!

Question 19 Walkthrough.

Write a conditional statement from each of the following:

All students at Contour Education are hard-working and diligent.

NOTE: Order Matters!

Recall!

<u>Active Recall:</u> What is the relationship between the hypothesis and the conclusion?

$H \Rightarrow C$

Your Turn!

Question 20

Write a conditional statement from each of the following:

Students doing SM12 are all brave.

SM12 students $\Rightarrow$ brave

Question 21 Additional Question.

Write a conditional statement from each of the following:

You must do well enough in English in order to study Law at University.

Sub-Section: Even and Odd Numbers

Context

- Exam 1 of Specialist Mathematics 3/4 usually has a proof question. Here is a question from the sample exam. After this workbook, you will be proving the statement!

  Question 4 (3 marks)
  Use proof by contradiction to prove that if n is odd, where $n \in N$, then $n^3 + 1$ is even.

Exploration: Even and Odd Numbers

- Write out a few **even** numbers:

  2, 4, 6

- We can rewrite them in the following way:

  2×1, 2×2, 2×3

- Do you notice a pattern?

  $Even = 2k, \; k \in \mathbb{Z}$

- Now write out a few **odd** numbers:

  −1, 3, 5, 7

- We can rewrite them in the following way:

- There is a similar pattern for odd numbers!

  $Odd = 2k + 1, \; k \in \mathbb{Z}$

Proofs involving Even and Odd numbers

- Simply show:

$$Even = 2k, k \in Z$$

$$Odd = 2k + 1, k \in Z$$

Let's look at some questions together!

Question 22 Walkthrough.

Prove that if n is even, then n^2 is also even.

H C

Since n is even

Let $n = 2k$, $k \in \mathbb{Z}$

$n^2 = (2k)^2 = 4k^2 = 2 \times (2k^2)$

$2k^2$ is an integer

Hence $2 \times (2k^2)$ is even

$\therefore$ n^2 is even

NOTE: Start with the hypothesis by saying "let..."

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Recall!

Active Recall: Odd and Even Numbers

- An even number can be written in the form $n = 2k,\ k \in \mathbb{Z}$.
- An odd number can be written in the form $n = 2k+1,\ k \in \mathbb{Z}$.
- It is important to include $k \in Z$.

Your Turn!

Question 23

Prove that if n is odd, then n^2 is also odd.

n is an odd number

Let $n = 2k+1,\ k \in \mathbb{Z}$

$n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$2k^2 + 2k \in \mathbb{Z}$

Hence n^2 is odd

Space for Personal Notes

Question 24 Additional Question.

Prove that if two numbers a and b are odd, then their sum is even.

$a = 2k+1, \quad k \in \mathbb{Z}$

$b = 2m+1, \quad m \in \mathbb{Z}$

$a+b = 2k+1+2m+1$

$= 2k+2m+2$

$= 2(k+m+1)$

$k+m+1 \in \mathbb{Z}$

$\therefore a+b$ is even

Space for Personal Notes

Sub-Section: Divisibility

Context

- Even numbers are divisible by 2. How about divisibility by other numbers? Check out this divisibility question from the sample exam.

  Question 3 (4 marks)
  Prove by mathematical induction that the number $9^n - 5^n$ is divisible by 4 for all $n \in N$.

Exploration:

- Write out a few numbers that are divisible by 3.

  9, 27, 30

- Notice that these numbers can be written in the form:

  3×3, 3×9, 3×10

- Therefore, any number that is divisible by 3 can be written in the form:

$$n = 3k, \ k \in Z$$

Space for Personal Notes

Let's look at some questions together!

Question 25 Walkthrough.

Prove that if n is a multiple of 3, n^2 must be divisible by 9

look for $9k$

$n = 3k,\ k \in \mathbb{Z}$

$n^2 = (3k)^2 = 9k^2$

$k^2 \in \mathbb{Z}$

Hence n^2 is divisible by 9

NOTE: For a number n to be divisible by a, n MUST be an integer multiple of a, that is you MUST be able to write $n = a \times \text{INTEGER}$

Recall!

Active Recall

- If n is divisible by a, then n can be written as $a \cdot k,\ k \in \mathbb{Z}$.

Space for Personal Notes

Your Turn!

Question 26

Prove the following statement: If n is a multiple of 4, n^2 must be divisible by 8.

8 ×

$n = 4k, k \in \mathbb{Z}$

$n^2 = (4k)^2 = 16k^2$

$= 8 \times (2k^2)$

$2k^2 \in \mathbb{Z}$

$\therefore n^2$ is divisible by 8

Question 27 Additional Question.

Prove the following statements:

a. If n is a multiple of 3, then $n^2 + 10n + 9$ is divisible by 3

b. If n is a multiple of 3, then $n^2 - 1$ is not divisible by 3.

c. If n is divisible by 3 and m is divisible by 10, then $5n + 7m$ is divisible by 5.

Definition

Divisibility Proof - Splitting by Cases

- For divisibility of 2, we split the cases into ___________.

$$\textit{Case } 1: 2k, \quad 2k+1$$

- For divisibility of n we split the cases into ___________.

$$\textit{Case } 1: nk, \quad nk+1, \quad nk+2 \quad \cdots \quad nk+(n-1)$$

Let's look at some questions together!

Question 28 Walkthrough.

Prove the following statement:

For any integer n, $(n+1)(n+2)$ is an even number (divisible by 2).

NOTE: Split the case into two because we are checking the divisibility of 2.

Recall!

Active Recall: Splitting by cases

- For a proof question where we want to show something is divisible by 3, we can split it into three cases where $n = 3k$, $n = 3k+1$, $n = 3k+2$

Your Turn!

Question 29

Prove the following statement: For any integer n, $(n+1)(n+2)n$ is divisible by 3.

$\to 3\times$

CASE 1: $n = 3k$, $k \in \mathbb{Z}$

$(3k+1)(3k+2)\times(3k)$

$3\times(3k+1)(3k+2)(k)$, where $(3k+1)(3k+2)(k) \in \mathbb{Z}$

$\therefore$ div. by 3

CASE 2: $n = 3k+1$, $k \in \mathbb{Z}$

$(3k+1+1)(3k+1+2)(3k+1)$

$= (3k+2)(3k+3)(3k+1)$

$= 3\times(3k+2)(k+1)(3k+1)$, where $(3k+2)(k+1)(3k+1) \in \mathbb{Z}$

$\therefore$ div. by 3

CASE 3: $n = 3k+2$, $k \in \mathbb{Z}$

$(3k+2+1)(3k+2+2)(3k+2)$

$(3k+3)(3k+4)(3k+2)$

$\to 3\times(k+1)(3k+4)(3k+2)$, where $(k+1)(3k+4)(3k+2) \in \mathbb{Z}$

$\therefore$ div by 3

$\Rightarrow$ Always divisible by 3

Space for Personal Notes

Question 30 Additional Question.

Prove the following statement below:

Prove that if $n = m^2$ for some integer m, then $n = 4k$ or $n = 4k + 1$ for some integer k.

Space for Personal Notes

Sub-Section: Rational Numbers

Proving Rationals ($\mathbb{Q}$)

- Write out a few rational numbers:

$$\frac{7}{3}$$

- Write out a few numbers that are not rational numbers!

$$\frac{1}{\pi} \qquad \frac{\sqrt{3}}{2}$$

- Not all fractions are rational numbers! What is the general form for a rational number?

$$n = \frac{a}{b}, \quad a \in \mathbb{Z}, \; b \in \mathbb{Z} \setminus \{0\}$$

Space for Personal Notes

Let's look at some questions together!

Question 31 Walkthrough.

Prove the following conditional statement:

If x is a rational number, $\frac{x+2}{x+3}$ is also a rational number.

$x = \frac{a}{b}$, $a \in \mathbb{Z}$, $b \in \mathbb{Z} \setminus \{0\}$

$\frac{x+2}{x+3} = \frac{\frac{a}{b}+2}{\frac{a}{b}+3} \begin{matrix} \times b \\ \times b \end{matrix} = \frac{a+2b}{a+3b}$

WANT: $\in \mathbb{Z}$, $\in \mathbb{Z} \setminus \{0\}$

$a+2b \in \mathbb{Z}$

$a+3b \in \mathbb{Z} \setminus \{0\}$

Hence, $\frac{x+2}{x+3}$ is rational

Recall!

Active Recall: Rational numbers

- A rational number can be written in the form:

$\frac{a}{b}$ $a \in \mathbb{Z}$, $b \in \mathbb{Z} \setminus \{0\}$

Question 32

Prove the following conditional statement:

If x and y are rational numbers, $\frac{xy}{x+y}$ is also a rational number.

Let $x = \frac{a}{b}$, $y = \frac{c}{d}$

$a, c \in \mathbb{Z}$ $\quad b, d \in \mathbb{Z} \setminus \{0\}$

$\frac{\frac{a}{b} \times \frac{c}{d} \times bd}{\left(\frac{a}{b} + \frac{c}{d}\right) \times bd} = \frac{ac}{ad + bc}$

$ac \in \mathbb{Z}$

$ad + bc \in \mathbb{Z} \setminus \{0\}$

$\therefore$ Rational

Question 33 Additional Question.

Prove the following statement below:

If x and y are rational numbers and y is not equal to zero, then x/y is also a rational number.

Key Takeaways

- Adding, subtracting and multiplying integers result in integers.
- An even number can be expressed as $2k$ where $k \in Z$ and an odd number can be expressed as $2k + 1$ where $k \in Z$.
- If n is divisible by a, then $n = a \times k$ where $k \in Z$, meaning $n = a \times \text{INTEGER}$.
- Where helpful, we can approach a proof involving divisibility by considering different cases.

Contour Check

Learning Objective: [2.1.1] – Number sets

Key Takeaways

- A set is a collection of multiple values.
- When using interval notation, square brackets are used to include the endpoints and round brackets are used to exclude the endpoints.
- Common number sets that are encountered in Specialist Mathematics are N (positive whole numbers), Z (integers), Q (rational numbers), R (real numbers) and C (complex numbers).
- To rationalise a fraction with a surd in the denominator, we can first multiply the numerator and denominator by the conjugate of the denominator and simplify by expanding.

Learning Objective: [2.1.2] – Statements and operations

Key Takeaways

- We can do the following operations involving statements: $\neg A$ meaning "not A", $A \wedge B$ meaning "A or B", and $A \vee B$, meaning "A and B".
- De Morgan's Laws state that $\neg(A \wedge B) = \neg A \vee \neg B$ and $\neg(A \vee B) = \neg A \wedge \neg B$.
- We notice that in De Morgan's Laws, the intersection/union flips.

Learning Objective: [2.1.3] – Proving number sets

Key Takeaways

- Adding, subtracting and multiplying integers result in integers.
- An even number can be expressed as $2k$ where $k \in Z$ and an odd number can be expressed as $2k+1$ where $k \in Z$.
- If n is divisible by a, then $n = a \times k$ where $k \in Z$, meaning $n = a \times \text{INTEGER}$.
- Where helpful, we can approach a proof involving divisibility by considering different cases.

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