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VCE Specialist Mathematics ½

Proofs I [2.1]

Test Solutions

20 Marks. 1 Minute Reading. 16 Minutes Writing.

Results:

Test Questions	_____ / 20

Section A: Test Questions (20 Marks)

Question 1 (5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. Set given by $\{x\|x^2 \neq 16\}$ can be simplified to $R\backslash\{-4,4\}$.	✓	
b. All integers are natural numbers hence $Z \subseteq N$.		✓
c. 1.91 is a rational number.	✓	
d. To simplify $\frac{1}{2+\sqrt{3}}$, you multiply $2+\sqrt{3}$ on both top and bottom.		✓
e. Opposite of liking maths and science is not liking maths or not liking science.	✓	
f. $2k+1$ is an odd number regardless of what k is.		✓
g. $\frac{m}{n}$ is a rational number only if m and n are integers. 5/2.5 is also rational. Order matters!		✓
h. If m and n are non-zero integers, then $\frac{m}{n}$ is a rational number.	✓	
i. To prove that a number is divisible by 5, we simply show that the number is 5 times by an integer.	✓	
j. Product of 5 consecutive numbers is always divisible by 5.	✓	

Space for Personal Notes

Question 2 (2 marks)

Express $\frac{2+\sqrt{5}}{-1-\sqrt{3}}$ in the form $\frac{a}{b}$ where $a \in R$ and $b \in N$.

$$\frac{2-2\sqrt{3}+\sqrt{5}-\sqrt{15}}{2}$$

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Question 3 (2 marks)

James claims the following.

All living humans breathe and eat.

Pranit comes along and opposes the idea.

a. What did Pranit say? (1 mark)

There is at least one human that does not breathe or eat

b. Who is correct? (1 mark)

James

Space for Personal Notes

Question 4 (5 marks)

Prove the following conditional statements.

a. If n is an even number, then $n^3 + n^2$ is also an even number. (2 marks)

let $n = 2k, k \in \mathbb{Z}$

$n^3 + n^2 = n^2(n+1)$

$= (2k)^2(2k+1)$

$= 4k^2(2k+1)$

$= 2[2k^2(2k+1)]$

$= 2m$ where $m \in \mathbb{Z}$

b. If n is a natural number, then $n^3 - n$ is divisible by 3. (3 marks)

(C1) let $n = 3k, k \in \mathbb{Z}$

$n^3 - n$

$= n(n^2-1) = n(n+1)(n-1)$

$= 3k(3k+1)(3k-1)$

$= 3m,$

$m = k(3k+1)(3k-1)$

$\in \mathbb{Z}.$

(C2) let $n = 3k+1, k \in \mathbb{Z}$

$n^3 - n$

$= n(n^2-1) = n(n+1)(n-1)$

$= (3k+1)(3k+2)(3k+1-1)$

$= 3(3k+1)(3k+2)k$

$= 3m,$

$m = k(3k+1)(3k+2)$

$\in \mathbb{Z}$

(C3) let $n = 3k+2, k \in \mathbb{Z}$

$n^3 - n$

$= n(n^2-1) = n(n+1)(n-1)$

$= (3k+2)(3k+3)(3k+1)$

$= 3(3k+2)(k+1)(3k+1)$

$= 3m.$

$m = (3k+2)(k+1)(3k+1)$

$\in \mathbb{Z}$

Question 5 (6 marks)

Prove the following statements:

a. For any integer n, if n is divisible by 3, then n^2 is divisible by 3. (2 marks)

Solution: We have $n = 3k$ where $k \in \mathbb{Z}$. Then

$$\begin{aligned} n^2 &= (3k)^2 \\ &= 9k^2 \\ &= 3(3k^2) \\ &= 3m \end{aligned}$$

where $m = 3k^2 \in \mathbb{Z}$, and so n^2 is divisible by 3.

b. For any integer m and n, if m is divisible by 2 and n is divisible by 5 then $7m + 4n$ is even. (2 marks)

Solution: Let $m = 2k$ and $n = 5p$ where $k, p \in \mathbb{Z}$. Then

$$\begin{aligned} 7m + 4n &= 14k + 20p \\ &= 2(7k + 10p) \\ &= 2r \end{aligned}$$

where $r = 7k + 10p \in \mathbb{Z}$, and so $7m + 4n$ is even.

c. For any integer n, $(2n-1)^2 + (2n+2)^2$ is odd. (2 marks)

Solution: We have that

$$\begin{aligned} (2n-1)^2 + (2n+2)^2 &= 4n^2 - 4n + 1 + 4n^2 + 8n + 4 \\ &= 8n^2 + 4n + 4 + 1 \\ &= 2(4n^2 + 2n + 2) + 1 \\ &= 2m + 1 \end{aligned}$$

where $m = 4n^2 + 2n + 2 \in \mathbb{Z}$, and so $(2n-1)^2 + (2n+2)^2$ is always odd.

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