CONTOUREDUCATION

VCE Specialist Mathematics ½
Proofs I [2.1]
Homework

Homework Outline:

Compulsory Questions	Pg 2-Pg 15
Supplementary Questions	Pg 16-Pg 31

Section A: Compulsory Questions

Sub-Section [2.1.1]: Number Sets
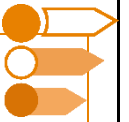
Question 1

State all the number sets that the following are an element of:

a. $\sqrt{5}$

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b. 5

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c. $\pi + i$

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d. $-\frac{3}{7}$

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Question 2

Express each of the following subsets of $\mathbb{R}$ in interval notation:

a. $\{x: x > -4\}$

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b. $\{x: x \neq 1\} \cap \{x: x \geq -3\}$

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c. $\{x: x \neq 0\} \cup \{x: x \leq 2\}$

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Question 3

Rationalise the denominator, then simplify the following expressions:

a. $\dfrac{5}{\sqrt{3}}$

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b. $\dfrac{2}{1+\sqrt{2}}$

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c. $\dfrac{\sqrt{8}+3}{2+\sqrt{5}}$

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Sub-Section [2.1.2]: Operations on Statements

Question 4

Consider the following statements:

$$A = \text{It is raining.}$$
$$B = \text{I go out running.}$$

Write down the following:

a. $A \wedge B$

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b. $\neg A$

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c. $\neg A \vee B$

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Question 5

Use De Morgan's Law to write down the negation of the following statements:

a. The cake is delicious and the coffee is hot.

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b. It is raining or the sun is shining.

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c. The computer is cheap and slow.

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Question 6

Write the following as conditional statements:

a. Customers that spend over $500 get a voucher.

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b. All students who study hard pass their exams.

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c. People who go to the gym grow their muscles.

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Sub-Section [2.1.3]: Proofs Involving Even and Odd Numbers

Question 7

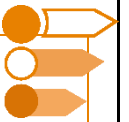For an integer n, show that if n is odd then n^2 is odd.

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Question 8

Show that $(2n + 5)^2 - (2n - 1)$ is always even for any $n \in \mathbb{Z}$.

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Question 9

Show that $n^2 + 5n + 6$ is even for all $n \in \mathbb{N}$.

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Sub-Section [2.1.4]: Proofs Involving Divisibility

Question 10

Show that if n is divisible by 5, then n^2 is divisible by 5 for any $n \in \mathbb{N}$.

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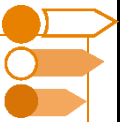_____

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Question 11

Show that if n is divisible by 2 and m is divisible by 3, then $5n + 10m$ is even for all $n, m \in \mathbb{N}$.

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Question 12

Show by cases that for $n \in \mathbb{Z}$, if n is not divisible by 3, then n^2 is not divisible by 9.

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Sub-Section [2.1.5]: Proofs Involving Rational Numbers

Question 13

Show that if $\sqrt{x}$ is rational, then x is rational for any $x \in \mathbb{R}$.

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Question 14

Show that if both x and y are rational, then $x^2 - y^3$ is rational.

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Question 15

Find possible values for $x, y \in \mathbb{R}$ such that x and y are irrational, but x^y is rational.

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Sub-Section: The 'Final Boss'

Question 16

Consider the statements:

$$A : n \text{ is an even integer.}$$
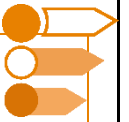$$B : n < 0$$

a. Write the statement $A \wedge \neg B$ in words.

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b. Prove that if $A \wedge \neg B$ is true, then $n^2 + 3n + 1$ is odd.

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c. Prove that the product of any two odd integers is odd.

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Section B: Supplementary Questions

Sub-Section [2.1.1]: Number Sets

Question 17

State all the number sets that the following are an element of:

a. $\sqrt{5}$

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b. 5

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c. $\pi + i$

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d. $-\frac{3}{7}$

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Question 18

Express each of the following subsets of $\mathbb{R}$ in interval notation.

a. $\{x: x > 3\}$

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b. $\{x: -8 < x < 1\} \cap \{x: x \geq -3\}$

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c. $\{x: x \neq 1\} \cup \{x: x \leq 5\}$

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Question 19

Rationalise the denominator and then simplify the following expressions:

a. $\dfrac{2}{\sqrt{3}}$

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b. $\dfrac{5}{3+\sqrt{2}}$

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c. $\dfrac{\sqrt{30}+7}{4+\sqrt{7}}$

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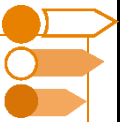_____

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Question 20

Rationalise the denominator of the following expression and simplify:

$$\frac{x+\sqrt{y}}{\sqrt{a}+\sqrt{b}},$$

where $x, y, a, b > 0$ and $a \neq b$.

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Question 21

Consider the following statements:

$$A = \text{It is hot outside.}$$
$$B = \text{I go to the beach.}$$

Write down the following:

a. $A \land B$

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b. $\neg B$

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c. $\neg A \lor \neg B$

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Question 22

Use De Morgan's Law to write down the negation of the following statements:

a. The movie is entertaining and the popcorn is tasty.

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b. The traffic is light or the weather is clear.

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c. The phone is affordable and has a good camera.

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Question 23

Write the following as conditional statements:

a. People who recycle help the environment.

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b. Employees who work overtime earn extra pay.

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c. Athletes who practice regularly improve their performance.

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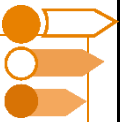_____

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Question 24

Simplify the following logical expression using De Morgan's Laws:

$$\neg((P \wedge Q) \vee (\neg R \wedge S)).$$

Give your answer in the form

$$(A \vee B) \wedge (C \vee D)$$

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Sub-Section [2.1.3]: Proofs Involving Even and Odd Numbers

Question 25

For an integer n, show that if n is even then n^3 is even.

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Question 26

Show that, $(4n + 2)^2 - (2n - 1)$ is always odd for any $n \in \mathbb{Z}$.

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Question 27

Show that $n^2 + 7n + 10$ is even for all $n \in \mathbb{N}$.

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Question 28

Prove that the product of any two odd integers minus the sum of the same two integers is always even.

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Sub-Section [2.1.4]: Proofs Involving Divisibility

Question 29

Show that if n is divisible by 7, then n^2 is divisible by 7 for any $n \in \mathbb{N}$.

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Question 30

Show that if n is divisible by 2 and m is divisible by 3, then $3n + 4m$ is divisible by 3 for all $n, m \in \mathbb{N}$.

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Question 31

Prove that if m and n are even integers, then $m^2 + n^2$ and $m^2 - n^2$ are both divisible by 4.

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Question 32

Prove that the sum of any two consecutive odd numbers is divisible by 4.

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Sub-Section [2.1.5]: Proofs Involving Rational Numbers

Question 33

Show that if $\sqrt[3]{x}$ is rational, then x is rational for any $x \in \mathbb{R}$.

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Question 34

Show that if both x and y are rational, then $x^2 + y^2$ is rational.

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Question 35

Prove that if x is rational and $x \neq 0$, then $\frac{1}{x}$ is also rational.

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Question 36

Prove that if x and y are rational and $x, y \neq 0$ then

$$\frac{(x - 2y)^5 + x^2 + 3y}{x^2 + 2y^2}$$

is rational.

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