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VCE Specialist Mathematics ½
AOS 2 Revision [2.0]
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Section A: [2.1] - Proofs I (Checkpoints)

Sub-Section [2.1.1]: Number Sets

Question 1

State all the number sets that the following are an element of:

a. $\sqrt{5}$

---

b. 5

---

c. $\pi + i$

---

d. $-\frac{3}{7}$

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Question 2

Express each of the following subsets of $\mathbb{R}$ in interval notation.

a. $\{x: x > 3\}$

b. $\{x: -8 < x < 1\} \cap \{x: x \geq -3\}$

c. $\{x: x \neq 1\} \cup \{x: x \leq 5\}$

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Question 3

Rationalise the denominator and then simplify the following expressions.

a. $\frac{2}{\sqrt{3}}$

b. $\frac{5}{3+\sqrt{2}}$

c. $\frac{\sqrt{30}+7}{4+\sqrt{7}}$

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Question 4

Rationalise the denominator of the following expression and simplify

$$\frac{x+\sqrt{y}}{\sqrt{a}+\sqrt{b}},$$

Where $x, y, a, b > 0$ and $a \neq b$.

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Sub-Section [2.1.2]: Operations on Statements

Question 5

Consider the following statements:

$$A = \text{It is hot outside.}$$
$$B = \text{I go to the beach.}$$

Write down the following:

a. $A \wedge B$.

b. $\neg B$.

c. $\neg A \vee \neg B$.

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Question 6

Use De Morgan's Law to write down the negation of the following statements:

a. The movie is entertaining and the popcorn is tasty.

b. The traffic is light or the weather is clear.

c. The phone is affordable and has a good camera.

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Question 7

Write the following as conditional statements.

a. People who recycle help the environment.

b. Employees who work overtime earn extra pay.

c. Athletes who practice regularly improve their performance.

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Question 8

Simplify the following logical expression using De Morgan's Laws:

$$\neg((P \wedge Q) \vee (\neg R \wedge S)).$$

Give your answer in the form:

$$(A \vee B) \wedge (C \vee D).$$

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Sub-Section [2.1.3]: Proofs Involving Even and Odd Numbers

Question 9

For an integer n, show that if n is even then n^3 is even.

Question 10

Show that $(4n + 2)^2 - (2n - 1)$ is always odd for any $n \in \mathbb{Z}$.

Question 11

Show that $n^2 + 7n + 10$ is even for all $n \in \mathbb{N}$.

Question 12

Prove that the product of any two odd integers minus the sum of the same two integers is always even.

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Sub-Section [2.1.4]: Proofs Involving Divisibility

Question 13

Show that if n is divisible by 7, then n^2 is also divisible by 7 for any $n \in \mathbb{N}$.

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Question 14

Show that if n is divisible by 2 and m is divisible by 3, then $3n + 4m$ is divisible by 3 for all $n, m \in \mathbb{N}$.

Question 15

Prove that if m and n are even integers, then $m^2 + n^2$ and $m^2 - n^2$ are both divisible by 4.

Question 16

Prove that the sum of any two consecutive odd numbers is divisible by 4.

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Sub-Section [2.1.5]: Proofs Involving Rational Numbers

Question 17

Show that if $\sqrt[3]{x}$ is rational, then x is rational for any $x \in \mathbb{R}$.

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Question 18

Show that if both x and y are rational, then $x^2 + y^2$ is rational.

Question 19

Prove that if x is rational and $x \neq 0$, then $\frac{1}{x}$ is also rational.

Question 20

Prove that if x and y are rational and $x, y \neq 0$ then,

$$\frac{(x-2y)^5 + x^2 + 3y}{x^2 + 2y^2}$$

is rational.

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Section B: [2.2] - Proofs II (Checkpoints)

Sub-Section [2.2.1]: Direct and Indirect Proofs

Question 21

Prove that all numbers of the form $n^3 - n$, where $n \in \mathbb{Z}$, are multiples of 6.

Question 22

Prove the following statement using a proof by contrapositive: If n^5 is odd, then n is odd.

Question 23

Prove the following statement using a proof by contradiction: $\sqrt{5}+\sqrt{7} < 5$.

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Question 24

Prove that for $a, b > 0$, we have $a + b \geq \left(\frac{1}{a} + \frac{1}{b}\right)^{-1}$.

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Sub-Section [2.2.2]: Proofs involving Converse and Equivalent Statements

Question 25

Write the converse of the following statements.

a. If a person exercises regularly, they stay healthy.

b. If a car is fuel-efficient, it saves money on gas.

c. If a student studies, they pass their exams.

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Question 26

Suppose $n \in \mathbb{Z}$. Prove that n is odd, if and only if $3n + 1$ is even.

Question 27

Prove the following statement: $\frac{n(n+1)}{2}$ is a natural number, if and only if n is a natural number.

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Question 28

Prove the following statement: For any integer n, n is divisible by 3, if and only if the sum of its digits is divisible by 3.

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Sub-Section [2.2.3]: Proofs involving the Universal and Existence Quantifiers

Question 29

Write the following statements in terms of the universal ($\forall$) and existential ($\exists$) quantifiers.

a. All positive integers are greater than zero.

b. There exists an integer that is a perfect square.

c. For all real numbers x, if $x > 0$, then $\frac{1}{x} > 0$.

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Question 30

Negate the following statements involving universal and existential quantifiers.

a. $\forall n \in \mathbb{Z}, n + 0 = n$

b. $\exists x \in \mathbb{R}, x^3 = 8$

c. $\forall x \in \mathbb{R}, x^2 \geq 0$

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Question 31

Disprove the following statements by providing a counterexample.

a. Disprove that for all integers $n, n^3 - n$ is always odd.

b. Disprove that there exists an integer n such that, $2n + 1 = 0$.

c. Disprove that for all real numbers $x, x^2 + x$ is greater than 1.

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Question 32

Prove that:

$$\forall a, b \in \mathbb{R}^+ \cup \{0\}, \frac{a+b}{2} \geq \sqrt{ab}$$

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Sub-Section [2.2.4]: Telescoping Series and Proofs by Induction

Question 33

Simplify the following telescoping series using partial fraction decomposition and simplification.

$$\sum_{k=1}^{n+1} \frac{1}{(k+1)(k+2)} = \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)}$$

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Question 34

Prove the following statement by induction:

$$2 + 4 + 6 + \cdots + 2n = n(n + 1) \text{ for all integers } n \geq 1.$$

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Question 35

Prove the following statement by induction:

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(r^n-1)}{r-1} \text{ for all integers } n \geq 1.$$

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Question 36

Prove the following statement by induction:

$$a + (a + d) + (a + 2d) + \cdots + (a + (n - 1)d) = \frac{n}{2}(2a + (n - 1)d), \text{ for all integers } n \geq 1.$$

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Section C: [2.3] - Proofs Exam Skills (Checkpoints)

Sub-Section [2.3.1]: Solve Problems Using AM-GM Inequalities

Question 37

Show using the AM-GM inequality that for $x > 0$ we have:

$$5x + \frac{5}{x} \geq 10$$

Question 38

Minimise $2x + \frac{2}{x}$ over $x > 0$ by applying the AM-GM inequality, and hence maximise $6 - 2x - \frac{2}{x}$.

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Question 39

Find an expression for the area of a rectangle that has a perimeter of 4 units and a width of x units, and hence use the AM-GM inequality to maximise the area of such a rectangle.

Question 40

Let $x, y > 0$. Furthermore, suppose that $xy = 4$. Find the minimum value of $xy^3 + x^3y$.

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Sub-Section [2.3.2]: Solve Arithmetic and Geometric Proofs

Question 41

Prove using induction that $2 + 7 + 12 + \cdots + (5n - 3) = \frac{n(5n-1)}{2}$.

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Question 42

Prove using induction that $1 \cdot 7 + 2 \cdot 8 + \cdots + n(n+6) = \frac{n(n+1)(2n+19)}{6}$.

Question 43

Prove using induction that $2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \cdots + 2 \cdot 3^n = 3^{n+1} - 3$.

Question 44

a. Prove using induction that for all $n \in \mathbb{N}$, $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$.

b. Hence, write a rule for $2^3 + 4^3 + \cdots + (2n)^3$.

Hint: $2^3 + 4^3 + \cdots + (2n)^3$ is related to $1^3 + 2^3 + \cdots + n^3$ in a reasonably simple way.

c. Now, deduce a rule for $1^3 + 3^3 + \cdots + (2n-1)^3$ using the rule you obtained above.

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Sub-Section [2.3.3]: Prove Divisibility With Induction

Question 45

Prove using induction that if $n \in \mathbb{N}$, then $8^n - 1$ is divisible by 7.

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Question 46

Prove using induction that if $n \in \mathbb{N}$, then $n^3 + 3n^2 + 2n$ is divisible by 3.

Note: If you want to make this question a bit harder, you can instead show that $n^3 + 3n^2 + 2n$ is also divisible by 6. You might need to use the fact that the product of two consecutive integers is always even.

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Question 47

Prove using induction that if $n \in \mathbb{N}$, then $10^{n+1} + 10^n + 1$ is divisible by 3.

Note: The statement says that 111, 1101, 11001, etc., are all divisible by 3.

Question 48

Recall that $n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$. For example, $3! = 1 \cdot 2 \cdot 3$. Prove using induction that if $n \in \mathbb{N}$, then $(2n)!$ is divisible by 2^n.

Section D: [2.4] - Logic & Algorithms I (Checkpoints)

Sub-Section [2.4.1]: Write and Understand Basic Algorithms

Question 49

Construct an algorithm that multiplies any input given by 10.

Question 50

Construct an algorithm that adds any input given by 5.

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Question 51

Construct an algorithm that subtracts any input given by 5 and multiplies by 2.

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Sub-Section [2.4.2]: Understanding and Evaluating Algorithms That Have Conditional Statements and Represent Hybrid Functions as Algorithms

Question 52

Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} 3x - 5 & x \geq 0 \\ x - 1 & x < 0 \end{cases}$$

Question 53

Using a flowchart, describe an algorithm of the following hybrid function.

$$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Question 54

Turn the following function into an algorithm.

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ -2x + 1 & x < 1 \end{cases}$$

Question 55

Turn the following function into an algorithm.

$$f(x) = max\{n \in \mathbb{R} | n \leq x\}$$

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Sub-Section [2.4.3]: Understand and Evaluate Algorithms with Loops

Question 56

Check whether the following algorithm has any problems. If there is a problem, state the problem; if there is no problem, give the final output of the algorithm.

Step 1: $A \leftarrow 30$
Step 2: $A \leftarrow 3A - 20$
Step 3: Repeat 2 while $A > 65$.

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Question 57

Evaluate the following algorithm:

For a from 1 to 10
  if $a =$ even, then
    print "yes"
else
    print "no"
end for.

Question 58

Check whether the following algorithm has any problems. If there is a problem, state the problem; if there is no problem, give the final output of the algorithm.

Step 1: $A \leftarrow 60$
Step 2: $A \leftarrow 2A - 50$
Step 3: Repeat 2 while $A \leq 130$.

Question 59

Evaluate the following output:

$a \leftarrow 5$
$b \leftarrow 10$
if $a - b < 5$
  $a \leftarrow a - 5$
  $b \leftarrow b - 10$
end if
print a, b.

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Sub-Section [2.4.4]: Write and Evaluate Functions using Pseudocode

Question 60

$A \leftarrow [\ ]$
for n from 1 to 5
  append n to A.
  if $n = 1$, then
   return
else
  $A = \sqrt{n^2 + A[n-1]}$
  if $A =$ integer
   print "$A[n-1], n, A$ is a perfect triangle."
end for.

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Question 61

a. Roger decided to invest \$1000 at an interest rate of 10% compounded monthly. Construct an algorithm that computes the number of years needed for Roger's investment to double.

b. Jacob decided to invest \$500 at an interest rate of 15% compounded annually. Construct an algorithm that computes the number of years needed for Jacob's investment to increase by 50%.

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Question 62

Using pseudocode, write an algorithm to find all the primes less or equal to 100.

Question 63

Using pseudocode, construct an algorithm for the following:

Find the shortest distance between any 2 ***different*** coordinates from the list of coordinates.

$$Y \text{ coordinate} = [1, 35, 5, 41, 5]$$
$$X \text{ coordinate} = [123, 2, 74, 213, 2]$$

Section E: [2.5] - Logic & Algorithms II (Checkpoints)

Sub-Section [2.5.1]: Understand the Basics of Logic and Propositional Statements

Question 64

Translate the following to English:
P = I eat healthy.
Q = I exercise regularly.
R = I will lose weight.

$$P \wedge Q \Rightarrow R$$

Question 65

Translate the following to English:
A = I go jogging.
B = The weather is good.
C = I will feel energised.

$$\neg B \Rightarrow (\neg A \wedge \neg C)$$

Question 66

Translate into propositional logic using the correct syntax:

If the team wins the match, then the fans will celebrate and the opposing team will be disappointed.

Question 67

Translate into propositional logic using the correct syntax:

If the baker uses old flour, then the bread will not rise and the customers will complain.

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Sub-Section [2.5.2]: Construct Truth Tables and Recognise Equivalent Logical Expressions

Question 68

Write the truth table for:

$$\sim p \vee q$$

Question 69

Write the truth table for:

$$(p \wedge q) \vee (p \vee q)$$

Question 70

Construct a truth table for the statement $(p \oplus q) \Rightarrow r$, where $\oplus$ is the exclusive or.

Question 71

Construct a truth table for the statement $\neg(p \wedge q) \oplus r$.

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Sub-Section [2.5.3]: Represent Logical Expressions using Switching Circuits and Logic Gates

Question 72

Use logic gates to represent the following expression and draw the corresponding truth table:

$$p \wedge \neg q$$

Question 73

Use logic gates to represent the following expression and draw the corresponding truth table:

$$\neg(p \wedge q)$$

Expression $\neg(p \wedge q)$:

p	q	$\neg(p \wedge q)$
1	1	0
1	0	1
0	1	1
0	0	1

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Question 74

Sketch a logic gate for the following expression:

$$A(B + CD)$$

Question 75

Sketch a logic gate for the following expression:

$$[A(C + D)]' + BE$$

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Sub-Section [2.5.4]: Simplify and Evaluate Boolean Algebra Expressions using Algebraic Identities and Karnaugh Maps

Question 76

Simplify each expression by algebraic manipulation.

a. $\bar{a} \cdot 0 =$

b. $a + a =$

c. $a + \bar{a}b =$

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Question 77

Simplify each expression by algebraic manipulation.

a. $y + y\bar{y} =$

b. $xy + x\bar{y} =$

c. $\bar{x} + y\bar{x} =$

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Question 78

Simplify each expression by algebraic manipulation.

a. $(w + \bar{x} + y + \bar{z})y =$

b. $(x + \bar{y})(x + y) =$

c. $w + (w\bar{x}yz) =$

Question 79

Simplify the following expression by algebraic manipulation:

$$(x + z)(\bar{x} + y)(z + y) =$$

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Question 80

Using a Karnaugh map, identify the Boolean expression corresponding to each of the following truth tables:

a.

A	B	C	Result
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b.

A	B	C	Result
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

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Section F: [2.1-2.5] - Exam 1 Overall (Checkpoints) (27 Marks)

Question 81 (3 marks)

Inspired from VCAA Specialist Mathematics Exam 1 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024specmaths1-w.pdf#page=3

Prove that if x is an odd integer then $2x^2 - 3x - 7$ is even, using direct proof.

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Question 82 (4 marks)

Inspired from VCAA Specialist Mathematics Exam 1 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024SM1-nht-w.pdf#page=7

Prove by mathematical induction that $1 \times 7 + 2 \times 15 + 3 \times 23 + \cdots + n(8n - 1) = \frac{1}{6}n(n+1)(16n+5)$ for all $n \in N$.

Question 83 (4 marks)

Prove using induction that for all $n \in \mathbb{N}$, $n < 2^n$.

Question 84 (3 marks)

Consider the statement below:

There cannot exist two integers m and n such that $5m + 10n = 3$.

a. Write down a statement to begin a proof by contradiction for the statement above. (1 mark)

b. Hence, obtain a contradiction and prove the original statement. (2 marks)

Question 85 (4 marks)

Prove using induction that $6^n + 4$ is divisible by 5 for all $n \in \mathbb{N}$.

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Question 86 (4 marks)

Prove using induction that for all $n \in \mathbb{N}$, it holds that $\left(1+\frac{1}{1}\right)\left(2+\frac{1}{2}\right)\cdots\left(1+\frac{1}{n}\right)=n+1$.

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Question 87 (5 marks)

Prove the following biconditional statement for $x, y \in \mathbb{Z}$:

$$x + y \text{ is even, if and only if, } x^2 + y^2 \text{ is even.}$$

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Section G: [2.1-2.5] - Exam 2 Overall (Checkpoints) (7 Marks)

Question 88 (1 mark)

Inspired from VCAA Specialist Mathematics Exam 2 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024specmaths2-w.pdf#page=2

Consider the statement:

'For any integers m and n, if $m + n \geq 9$ then $m \geq 5$ or $n \geq 5$'.

The contrapositive of this statement is:

A. If $m < 5$ or $n < 5$, then $m + n < 9$.

B. If $m \geq 5$ or $n \geq 5$, then $m + n \geq 9$.

C. If $m < 5$ and $n < 5$, then $m + n < 9$.

D. If $m \leq 5$ and $n \leq 5$, then $m + n \leq 9$.

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Question 89 (1 mark)

Inspired from VCAA Specialist Mathematics Exam 2 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024SM2-nht-w.pdf#page=2

Consider the following proof:

$$\text{Prove that } \sqrt{15}+\sqrt{7}>\sqrt{19}.$$
$$\text{Assume } \sqrt{15}+\sqrt{7}\leq\sqrt{19}.$$
$$\text{Then } \left(\sqrt{15}+\sqrt{7}\right)^2\leq 19$$
$$15+2\sqrt{105}+7\leq 19$$
$$2\sqrt{105}\leq -3$$
$$\text{Hence, } \sqrt{15}+\sqrt{7}>\sqrt{19}.$$

This proof can be best described as a:

A. Direct proof.

B. Proof by contrapositive.

C. Proof by contradiction.

D. Proof by counter-example.

E. Proof by mathematical induction.

Question 90 (1 mark)

The contrapositive to the statement, "If n is even, then n^2 is even." is:

A. If n^2 is odd, then n is odd.

B. If n^2 is even, then n is even.

C. If n is odd, then n^2 is even.

D. If n is odd, then n^2 is odd.

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Question 91 (1 mark)

Consider the following:

$$\text{For all } k > K,\ 1.5^k < (k-1)!$$

What is the smallest value of $K \in \mathbb{N}$ such that the above holds?

A. 2

B. 3

C. 4

D. 5

Question 92 (1 mark)

The negation of the statement, "All the cars in the carpark are black." is:

A. All the vans in the carpark are black.

B. There exists a car in the carpark that is not black.

C. There exists a bus in the carpark without a mirror.

D. All the cars in the carpark are yellow.

Question 93 (1 mark)

Find the minimum value of $6x^2 + \frac{6}{x^2}$.

A. 6

B. 25

C. 15

D. 12

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Question 94 (1 mark)

Consider the following statement:

If a car in the carpark is black, then it costs a lot of money.

Which of the following is the converse of the above?

A. If a bus in the carpark costs a lot of money, then it is not black.

B. If a car costs a lot of money, then it is in the carpark.

C. If a car in the carpark is not black, then it costs a lot of money.

D. If a car in the carpark costs a lot of money, then it is black in colour.

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