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VCE Specialist Mathematics ½
Sequences & Series [1.3]
Homework Solutions

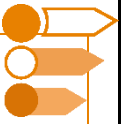
Homework Outline:

Compulsory Knowledge	Pg 2 – Pg 15
Extension Work	Pg 16 – Pg 25



Section A: Compulsory Questions

Sub-Section [1.3.1]: Finding Sequence from Recurrence Relations



Question 1



Construct the first five terms for the sequence given by, $t_n = 3 + t_{n-1}$, where $t_1 = 3$.

$$t_1 = 3, t_2 = 6, t_3 = 9, t_4 = 12, t_5 = 15.$$

Question 2



Given that $t_n = 5 \cdot t_{n-1}$ and $t_1 = 2$, find the value of n for which t_n is equal to 250.

$$n = 4$$

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Question 3


Given that $t_n = 4 \cdot t_n^2$ and $t_1 = 3$, find the value of t_3 .

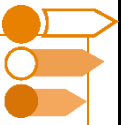
$$t_3 = 5184$$

Question 4 Tech-Active.


Given $t_{n+1} = t_n + \frac{1}{t_n}$ and $t_1 = 7$, find the value of n for which t_n is equal to $\frac{2549}{350}$.

$$n = 3$$

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Sub-Section [1.3.2]: Finding Arithmetic Sequence, Mean, and Series

Question 5



Consider the arithmetic sequence $t_n = 6n + 3$.

a. Find t_{10} .

$$t_{10} = 63$$

b. Find the arithmetic mean of t_5 and t_{15} .

$$\frac{t_5 + t_{15}}{2} = \frac{33 + 93}{2} = 63$$

c. Evaluate S_5 .

$$S_5 = \frac{5}{2}(t_1 + t_5) = 105$$

Question 6



It is known that, $t_2 = 8$ and $t_4 = 18$.

a. Find the first term and the common difference of the sequence.

$$d = \frac{t_4 - t_2}{4 - 2} = \frac{18 - 8}{2} = 5$$

$$a = t_1 = t_2 - d = 8 - 5 = 3$$

b. Find the general term t_n .

$$t_n = 3 + 5(n - 1) \text{ (or anything equivalent)}$$

c. Evaluate S_4 .

$$\text{Method 1 : } S_4 = \frac{n}{2}(2a + d(n-1)) = \frac{4}{2}(2 \cdot 3 + 5 \cdot (4-1)) = 42$$

$$\text{Method 2 : } S_4 = \frac{4}{2}(t_1 + t_4) = \frac{4}{2}(3 + 18) = 42$$

Question 7



Find the sum of all the multiples of 4 between 0 and 100.

$$4 + 8 + \dots + 96 + 100$$

$$4 + 8 + \dots + 100 = \frac{25}{2}(4 + 100) = 1300$$

NOTE: How do we know there are 25 terms? Because $t_n = 4 + 4(n-1)$ so you can solve for $t_n = 100$ for n .

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Question 8 Tech-Active.

Given that $S_4 = 64$ and $S_{10} = 280$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 10, d = 4$$

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Sub-Section [1.3.3]: Finding Geometric Sequence, Mean, and Series

Question 9



Given $t_n = 7\left(\frac{1}{2}\right)^n$.

a. Find t_6 .

$$t_6 = \frac{7}{64}$$

b. Find the geometric mean of t_5 and t_7 .

$$\sqrt{t_5 \cdot t_7} = \frac{7}{64}$$

c. Evaluate S_5 .

$$S_5 = \frac{\frac{7}{2} \left(\left(\frac{1}{2} \right)^5 - 1 \right)}{\frac{1}{2} - 1} = \frac{217}{32}$$

Question 10



It is known that, $t_2 = \frac{8}{9}$ and $t_4 = \frac{8}{81}$.

a. Find the common ratio (given that it is positive) and first term.

$$a = \frac{8}{3}, r = \frac{1}{3}$$

b. Find the general term t_n .

$$t_n = \frac{8}{3} \left(\frac{1}{3} \right)^{n-1}$$

c. Evaluate S_4 .

$$S_4 = \frac{\frac{8}{3} \left(\left(\frac{1}{3} \right)^4 - 1 \right)}{\frac{1}{3} - 1} = \frac{320}{81}$$

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Question 11


Consider $t_n = \frac{1}{2} \cdot t_{n-1}$. Find t_1 if, $S_{10} = \frac{3069}{256}$.

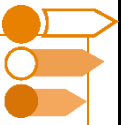
$$t_1 = 6$$

Question 12 Tech-Active.


Given that $S_5 = 155$ and $S_8 = 1275$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 5, r = 2, t_n = 5 \cdot 2^{n-1}$$

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Sub-Section [1.3.4]: Infinite Geometric Series

Question 13



Identify first term, common ratio and hence, find the value of series.

$$\frac{9}{5} + \frac{9}{25} + \frac{9}{125} + \frac{9}{625} + \dots$$

$$S_{\infty} = \frac{9}{4}$$

Question 14



Identify first term, common ratio and hence, find the value of series.

$$2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \dots$$

$$S_{\infty} = \frac{3}{2}$$

Question 15


Find the value of r , given that:

$$3 + 3r + 3r^2 + 3r^3 + \dots = 9$$

$$r = \frac{2}{3}$$

Question 16 Tech-Active.


Find the value of a , given that:

$$a - \frac{a}{2} + \frac{a}{4} - \frac{a}{8} + \frac{a}{16} + \dots = 18$$

$$a = 36$$

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Sub-Section: The 'Final Boss'



Question 17



Consider a geometric sequence, $t_n = 6 \cdot r^{n-1}$ where, $-1 < r < 1$. Suppose that, $S_2 = \frac{48}{7}$.

a. Show that, $r = \frac{1}{7}$.

$$6 + 6r + 6r^2 = \frac{342}{49} \implies r = \frac{1}{7} \text{ or } r = -\frac{8}{7} \text{ (reject because } r < -1)$$

b. Write the rule for S_n .

$$S_n = \frac{6 \left(\left(\frac{1}{7} \right)^n - 1 \right)}{\frac{1}{7} - 1} = 7 \left(1 - \left(\frac{1}{7} \right)^n \right)$$

c. Find the value of S_∞ .

Method 1: $S_n = \frac{6 \left(\left(\frac{1}{7} \right)^n - 1 \right)}{\frac{1}{7} - 1} = 7 \left(1 - \left(\frac{1}{7} \right)^n \right) \rightarrow 7 \text{ as } n \rightarrow \infty.$

Method 2: $S_\infty = \frac{6}{1 - \frac{1}{7}} = 7$

d. Hypothetically, you would need to add an infinite number of terms to obtain S_∞ . What is the least number of terms you need to add so that, the sum S_n is “sufficiently close” to S_∞ ? For the purpose of this question, this means to find the smallest value of n so that, $S_n > 0.99S_\infty$.

The answer is $n = 3$. Note that $0.99 \cdot S_\infty = 6.93$.

Method 1 (Brute force):

$$S_1 = 6, S_2 \approx 6.857, S_3 \approx 6.980$$

Method 2 (Logarithms!): $S_n = 7 \left(1 - \frac{1}{7^n} \right) = 6.93 \implies n = 2.37$ Round up to get $n = 3$

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Section B: Supplementary Questions

Sub-Section [1.3.1]: Finding Sequence from Recurrence Relations

Question 18



Given $t_n = 6 + 4 \cdot t_{n-1}$ and $t_1 = 3$, find the value of t_3 . Is the sequence an arithmetic sequence, geometric sequence, or neither?

$$t_3 = 78$$

Question 19



Given $t_n = t_{n-1}^{t_{n-1}}$ and $t_1 = 2$, find the value of n so that, $t_n = 256$.

$$n = 3$$

Question 20


Given $t_n = t_{n-1}^2$ and $t_1 = 3$, find the smallest n so that, $t_n > 100$.

$$n = 4$$

Question 21


Given $t_n = -t_{n-1}$ and $t_1 = 2$. Write down the first few terms in the sequence and hence, write down a formula for the general term t_n .

$$t_n = 2(-1)^{n-1}$$

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Sub-Section[1.3.2]: Finding Arithmetic Sequence, Mean, and Series

Question 22



Consider the arithmetic sequence, $t_n = t_{n-1} + 5$ and $t_1 = 2$.

a. Find t_{10} .

$$t_n = 2 + 5(n - 1) \implies t_{10} = 47$$

b. Find the arithmetic mean of t_3 and t_{10} .

$$\frac{t_3 + t_{10}}{2} = \frac{59}{2}$$

c. Evaluate S_4 .

$$S_4 = 38$$

Question 23



Find the value of x so that, the arithmetic mean of 8 and $2x + 6$ is 17.

$$x = 10$$

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Question 24


Let $t_n = 5 + dn$. Find the value of d if, $S_4 = 50$.

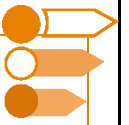
$$d = 3$$

Question 25


Given that $t_4 = 16$ and $S_8 = 136$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 10, d = 2, t_n = 10 + 2(n - 1)$$

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Sub-Section [1.3.3]: Finding Geometric Sequence, Mean, and Series

Question 26



Given $t_n = 4t_{n-1}$ and $t_1 = 3$.

a. Find t_3 .

$$t_3 = 48$$

b. Find the geometric mean of t_2 and t_5 .

$$\sqrt{t_2 \cdot t_5} = 96$$

c. Evaluate S_5 .

$$S_5 = \frac{3 \cdot (4^5 - 1)}{4 - 1} = 1023$$

Question 27



Suppose that t_n is a geometric series such that, $t_5 = 40.5$ and $t_9 = 3280.5$. Find the common ratio of the geometric series.

$$r = 3$$

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Question 28


Let $t_n = 4 \cdot r^n$. Find the value(s) of r given that, the geometric mean between t_4 and t_8 is 256.

$$r = \pm 2$$

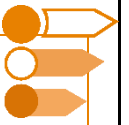
Question 29


Given $t_n = 6 \cdot t_{n-1}$ and $t_1 = 7$. Find the smallest value of n so that, S_n first exceeds 1000.

$$n = 4$$

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Sub-Section [1.3.4]: Infinite Geometric Series



Question 30



Find the value of the infinite series:

$$\frac{7}{2} - \frac{7}{4} + \frac{7}{8} - \frac{7}{16} + \dots$$

$$S_{\infty} = \frac{7}{3}$$

Question 31



Find the value of the infinite series:

$$2 + \frac{2}{7} + \frac{2}{49} + \frac{2}{343} + \dots$$

$$S_{\infty} = \frac{7}{3} \text{ (again!)}$$

Question 32


Find the value of r , given that:

$$5 + 5r + 5r^2 + 5r^3 + \dots = \frac{45}{8}$$

$$r = \frac{1}{9}$$

Question 33


Find the value of a , given that:

$$a - \frac{a}{6} + \frac{a}{36} - \frac{a}{216} + \dots = \frac{54}{5}$$

$$a = 9$$

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