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VCE Specialist Mathematics ½
Sequences & Series [1.3]
Homework Solutions

Homework Outline:

Compulsory Knowledge	Pg 2 – Pg 15
Extension Work	Pg 16 – Pg 25

Section A: Compulsory Questions

Sub-Section [1.3.1]: Finding Sequence from Recurrence Relations

Question 1

Construct the first five terms for the sequence given by, $t_n = 3 + t_{n-1}$, where $t_1 = 3$.

$t_1 = 3, t_2 = 6, t_3 = 9, t_4 = 12, t_5 = 15.$

Question 2

Given that $t_n = 5 \cdot t_{n-1}$ and $t_1 = 2$, find the value of n for which t_n is equal to 250.

$n = 4$

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Question 3

Given that $t_n = 4 \cdot t_n^2$ and $t_1 = 3$, find the value of t_3.

$t_3 = 5184$

Question 4 Tech-Active.

Given $t_{n+1} = t_n + \frac{1}{t_n}$ and $t_1 = 7$, find the value of n for which t_n is equal to $\frac{2549}{350}$.

$n = 3$

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Sub-Section [1.3.2]: Finding Arithmetic Sequence, Mean, and Series

Question 5

Consider the arithmetic sequence $t_n = 6n + 3$.

a. Find t_{10}.

$$t_{10} = 63$$

b. Find the arithmetic mean of t_5 and t_{15}.

$$\frac{t_5 + t_{15}}{2} = \frac{33 + 93}{2} = 63$$

c. Evaluate S_5.

$$S_5 = \frac{5}{2}(t_1 + t_5) = 105$$

Question 6

It is known that, $t_2 = 8$ and $t_4 = 18$.

a. Find the first term and the common difference of the sequence.

$$d = \frac{t_4 - t_2}{4 - 2} = \frac{18 - 8}{2} = 5$$

$$a = t_1 = t_2 - d = 8 - 5 = 3$$

b. Find the general term t_n.

$$t_n = 3 + 5(n - 1) \text{ (or anything equivalent)}$$

c. Evaluate S_4.

Method 1 : $S_4 = \frac{n}{2}(2a + d(n-1)) = \frac{4}{2}(2 \cdot 3 + 5 \cdot (4-1)) = 42$

Method 2 : $S_4 = \frac{4}{2}(t_1 + t_4) = \frac{4}{2}(3 + 18) = 42$

Question 7

Find the sum of all the multiples of 4 between 0 and 100.

$$4 + 8 + \ldots + 96 + 100$$

$4 + 8 + \ldots + 100 = \frac{25}{2}(4 + 100) = 1300$

NOTE: How do we know there are 25 terms? Because $t_n = 4 + 4(n-1)$ so you can solve for $t_n = 100$ for n.

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Question 8 Tech-Active.

Given that $S_4 = 64$ and $S_{10} = 280$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 10,\ d = 4$$

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Sub-Section [1.3.3]: Finding Geometric Sequence, Mean, and Series

Question 9

Given $t_n = 7\left(\frac{1}{2}\right)^n$.

a. Find t_6.

$$t_6 = \frac{7}{64}$$

b. Find the geometric mean of t_5 and t_7.

$$\sqrt{t_5 \cdot t_7} = \frac{7}{64}$$

c. Evaluate S_5.

$$S_5 = \frac{\frac{7}{2}\left(\left(\frac{1}{2}\right)^5 - 1\right)}{\frac{1}{2} - 1} = \frac{217}{32}$$

Question 10

It is known that, $t_2 = \frac{8}{9}$ and $t_4 = \frac{8}{81}$.

a. Find the common ratio (given that it is positive) and first term.

$$a = \frac{8}{3},\ r = \frac{1}{3}$$

b. Find the general term t_n.

$$t_n = \frac{8}{3}\left(\frac{1}{3}\right)^{n-1}$$

c. Evaluate S_4.

$$S_4 = \frac{\frac{8}{3}\left(\left(\frac{1}{3}\right)^4 - 1\right)}{\frac{1}{3} - 1} = \frac{320}{81}$$

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Question 11

Consider $t_n = \frac{1}{2} \cdot t_{n-1}$. Find t_1 if, $S_{10} = \frac{3069}{256}$.

$$t_1 = 6$$

Question 12 Tech-Active.

Given that $S_5 = 155$ and $S_8 = 1275$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 5,\ r = 2,\ t_n = 5 \cdot 2^{n-1}$$

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Sub-Section [1.3.4]: Infinite Geometric Series

Question 13

Identity first term, common ratio and hence, find the value of series.

$$\frac{9}{5}+\frac{9}{25}+\frac{9}{125}+\frac{9}{625}+\ldots$$

$$S_\infty = \frac{9}{4}$$

Question 14

Identity first term, common ratio and hence, find the value of series.

$$2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+\cdots$$

$$S_\infty = \frac{3}{2}$$

Question 15

Find the value of r, given that:

$$3 + 3r + 3r^2 + 3r^3 + \cdots = 9$$

$$r = \frac{2}{3}$$

Question 16 Tech-Active.

Find the value of a, given that:

$$a - \frac{a}{2} + \frac{a}{4} - \frac{a}{8} + \frac{a}{16} + \cdots = 18$$

$$a = 36$$

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Sub-Section: The 'Final Boss'

Question 17

Consider a geometric sequence, $t_n = 6 \cdot r^{n-1}$ where, $-1 < r < 1$. Suppose that, $S_2 = \frac{48}{7}$.

a. Show that, $r = \frac{1}{7}$.

$$6 + 6r + 6r^2 = \frac{342}{49} \implies r = \frac{1}{7} \text{ or } r = -\frac{8}{7} \text{ (reject because } r < -1)$$

b. Write the rule for S_n.

$$S_n = \frac{6\left(\left(\frac{1}{7}\right)^n - 1\right)}{\frac{1}{7} - 1} = 7\left(1 - \left(\frac{1}{7}\right)^n\right)$$

c. Find the value of S_∞.

Method 1: $S_n = \dfrac{6\left(\left(\frac{1}{7}\right)^n - 1\right)}{\frac{1}{7} - 1} = 7\left(1 - \left(\dfrac{1}{7}\right)^n\right) \to 7$ as $n \to \infty$.

Method 2: $S_\infty = \dfrac{6}{1 - \frac{1}{7}} = 7$

d. Hypothetically, you would need to add an infinite number of terms to obtain S_∞. What is the least number of terms you need to add so that, the sum S_n is "sufficiently close" to S_∞? For the purpose of this question, this means to find the smallest value of n so that, $S_n > 0.99S_\infty$.

The answer is $n = 3$. Note that $0.99 \cdot S_\infty = 6.93$.

Method 1 (Brute force): $S_1 = 6, S_2 \approx 6.857, S_3 \approx 6.980$

Method 2 (Logarithms!): $S_n = 7\left(1 - \dfrac{1}{7^n}\right) = 6.93 \implies n = 2.37$ Round up to get $n = 3$

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Section B: Supplementary Questions

Sub-Section [1.3.1]: Finding Sequence from Recurrence Relations

Question 18

Given $t_n = 6 + 4 \cdot t_{n-1}$ and $t_1 = 3$, find the value of t_3. Is the sequence an arithmetic sequence, geometric sequence, or neither?

$$t_3 = 78$$

Question 19

Given $t_n = t_{n-1}^{t_{n-1}}$ and $t_1 = 2$, find the value of n so that, $t_n = 256$.

$$n = 3$$

Question 20

Given $t_n = t_{n-1}^2$ and $t_1 = 3$, find the smallest n so that, $t_n > 100$.

$$n = 4$$

Question 21

Given $t_n = -t_{n-1}$ and $t_1 = 2$. Write down the first few terms in the sequence and hence, write down a formula for the general term t_n.

$$t_n = 2(-1)^{n-1}$$

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Sub-Section[1.3.2]: Finding Arithmetic Sequence, Mean, and Series

Question 22

Consider the arithmetic sequence, $t_n = t_{n-1} + 5$ and $t_1 = 2$.

a. Find t_{10}.

$$t_n = 2 + 5(n-1) \implies t_{10} = 47$$

b. Find the arithmetic mean of t_3 and t_{10}.

$$\frac{t_3 + t_{10}}{2} = \frac{59}{2}$$

c. Evaluate S_4.

$$S_4 = 38$$

Question 23

Find the value of x so that, the arithmetic mean of 8 and $2x + 6$ is 17.

$$x = 10$$

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Question 24

Let $t_n = 5 + dn$. Find the value of d if, $S_4 = 50$.

$$d = 3$$

Question 25

Given that $t_4 = 16$ and $S_8 = 136$, find the values of a (the first term) and d (the common difference) and hence, write down the general term t_n of the sequence.

$$a = 10,\ d = 2,\ t_n = 10 + 2(n-1)$$

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Sub-Section [1.3.3]: Finding Geometric Sequence, Mean, and Series

Question 26

Given $t_n = 4t_{n-1}$ and $t_1 = 3$.

a. Find t_3.

$$t_3 = 48$$

b. Find the geometric mean of t_2 and t_5.

$$\sqrt{t_2 \cdot t_5} = 96$$

c. Evaluate S_5.

$$S_5 = \frac{3 \cdot (4^5 - 1)}{4 - 1} = 1023$$

Question 27

Suppose that t_n is a geometric series such that, $t_5 = 40.5$ and $t_9 = 3280.5$. Find the common ratio of the geometric series.

$$r = 3$$

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Question 28

Let $t_n = 4 \cdot r^n$. Find the value(s) of r given that, the geometric mean between t_4 and t_8 is 256.

$$r = \pm 2$$

Question 29

Given $t_n = 6 \cdot t_{n-1}$ and $t_1 = 7$. Find the smallest value of n so that, S_n first exceeds 1000.

$$n = 4$$

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Sub-Section [1.3.4]: Infinite Geometric Series

Question 30

Find the value of the infinite series:

$$\frac{7}{2}-\frac{7}{4}+\frac{7}{8}-\frac{7}{16}+\cdots$$

$$S_\infty = \frac{7}{3}$$

Question 31

Find the value of the infinite series:

$$2+\frac{2}{7}+\frac{2}{49}+\frac{2}{343}+\cdots$$

$$S_\infty = \frac{7}{3} \text{ (again!)}$$

Question 32

Find the value of r, given that:

$$5 + 5r + 5r^2 + 5r^3 + \cdots = \frac{45}{8}$$

$$r = \frac{1}{9}$$

Question 33

Find the value of a, given that:

$$a - \frac{a}{6} + \frac{a}{36} - \frac{a}{216} + \cdots = \frac{54}{5}$$

$$a = 9$$

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