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VCE Specialist Mathematics ½

Modulus & Partial Fractions Exam Skills [1.2]

Workbook

Outline:

Section A: Recap of [1.1] – Modulus and Partial Fractions

Modulus Functions

- **Definition:**

$$f(x) = |x| = \begin{cases} x \text{ if } x\_\_\_\_\_ \\ -x \text{ if } x\_\_\_\_\_ \end{cases}$$

- Is a **hybrid function.**
- **Purpose:** Always return a non-negative number.
- **Range:** _____.

Alternative Definition of Modulus Functions

$$\sqrt{\_\_\_\_\_} = |x|$$

NOTE: Important not to forget the modulus in the exams!

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Sub-Section: Solving Modulus Equations

Solving Equations Involving Modulus Functions

$$|f(x)| = b$$

$$f(x) = \_\_\_\_\_$$

- **Interpretation:**
  - The _____ $f(x)$ equals to b.
  - $f(x)$ can be either _____.

TIP: Check your solutions by substituting them back into the equation!

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Sub-Section: Solving Modulus Inequalities

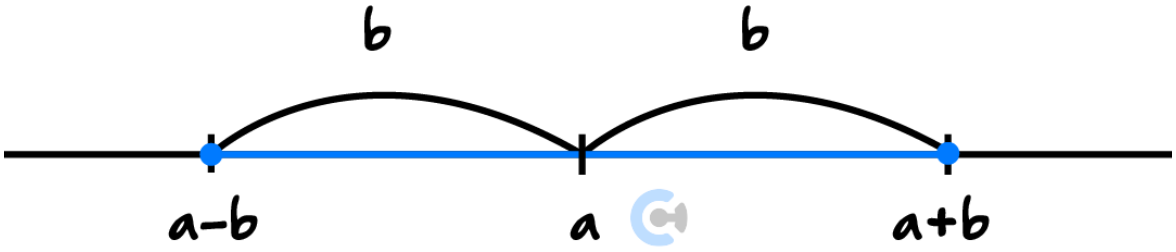
Solving Modulus Inequalities

$$|x - a| \leq b$$

- Interpretation:

x has a distance from 'a' that is less than or equal to 'b'

- Visualise:



$$\_\_\_\_\_ \leq x \leq \_\_\_\_\_$$

TIP: Always sketch a number line!

Question 1 Walkthrough.

Solve the following inequality.

$$|x + 2| < 5$$

Question 2

Solve the following inequality for x:

a. $|2x + 7| - 2 \geq 3$

b. $|2x - 4| - 3 \geq 5$

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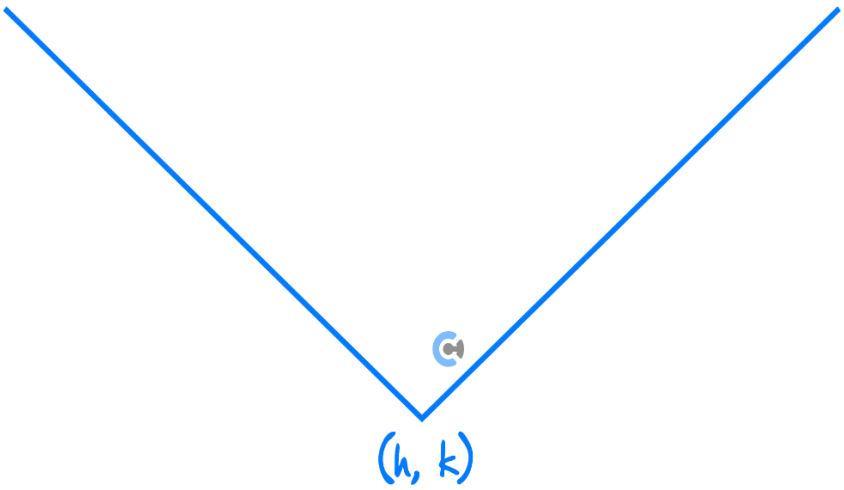
Sub-Section: Sketching Modulus Functions

Let's now consider the graph of modulus functions!

Graph of The Modulus Function



- **General form:**

$$y = a|x - h| + k$$

  - **Vertex** is at _____.

- **Hybrid form:**

$$y = \begin{cases} a(x-h)+k, & x \geq h \\ -a(x-h)+k, & x < h \end{cases}$$

TIP: Think of modulus functions as a "**straightened quadratic**".

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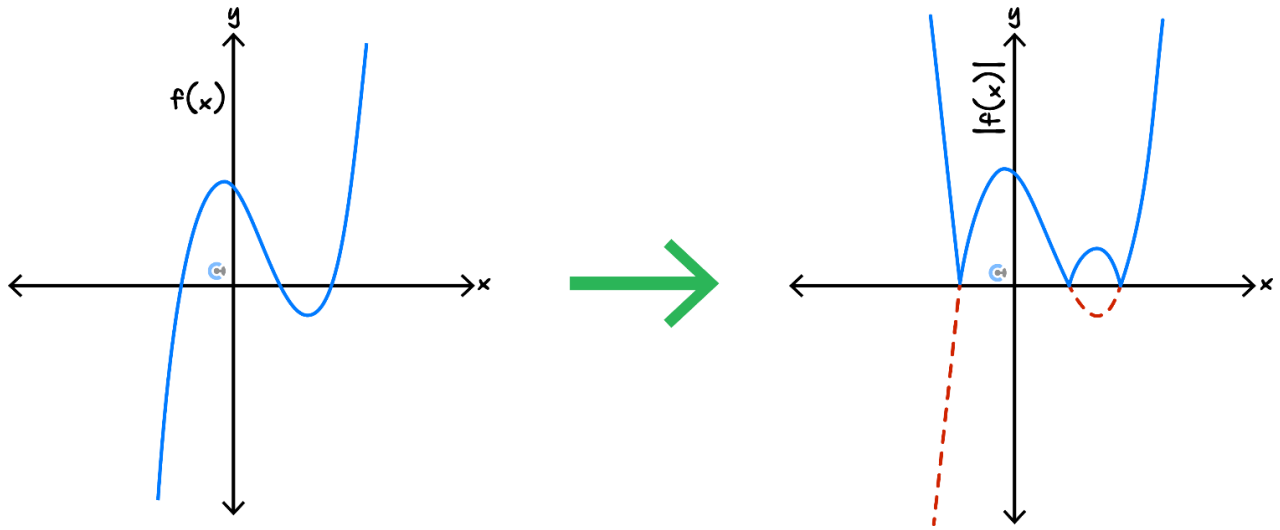
Sub-Section: Graphing Composite of Modulus Functions

Graphs of Composite Modulus Functions

- Modulus is the outer function.

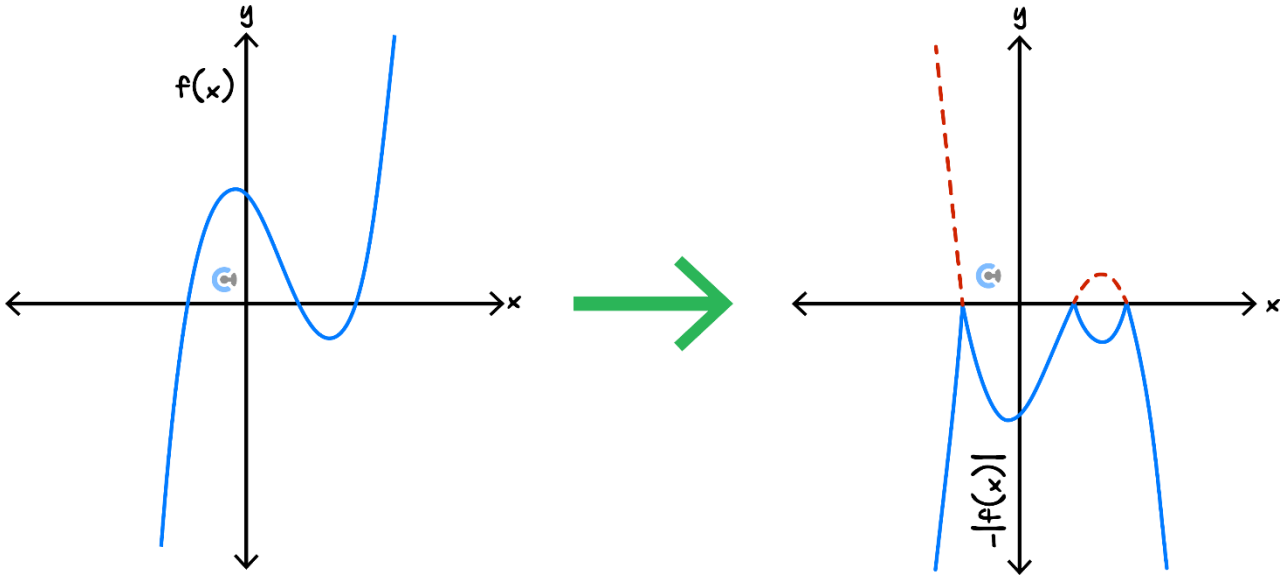
$$y = |f(x)|$$

- All negative y-values are flipped to be _____.



$$y = -|f(x)|$$

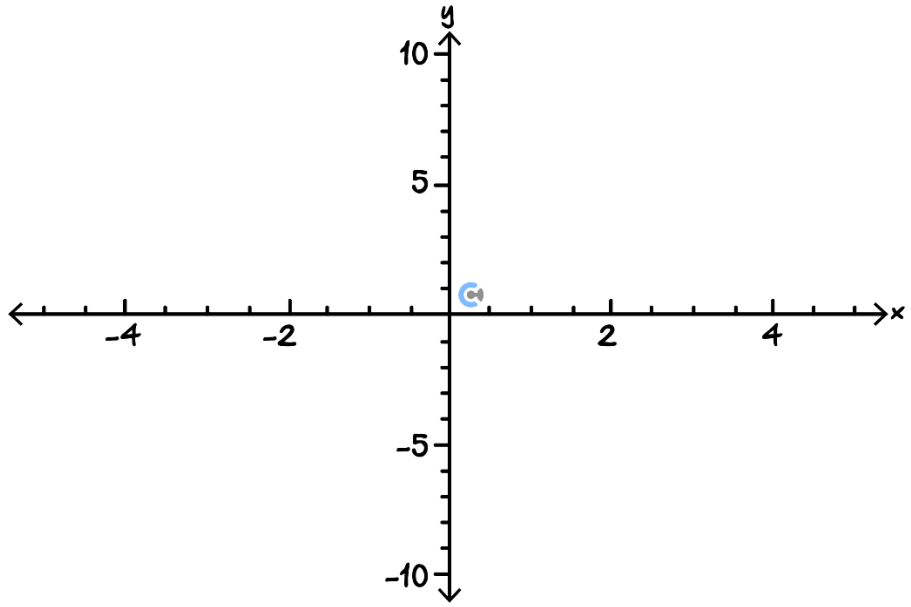
- The graph of $y = |f(x)|$ has undergone a reflection in the _____.



Question 3

Sketch the following graph over the specified domain. Label all key intercepts.

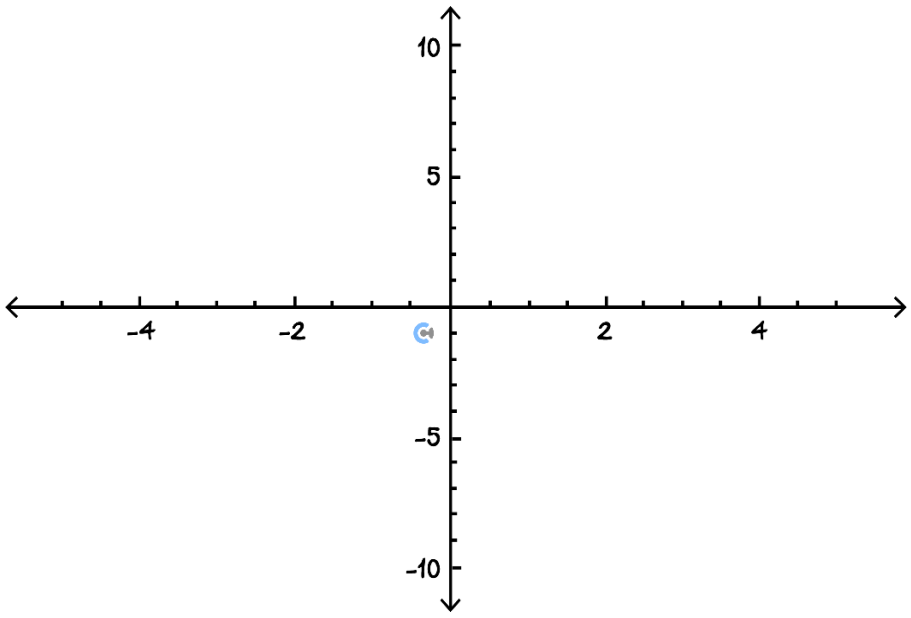
$$y = 2|x - 2| + 1$$



Question 4 Additional Question.

Sketch the following graph over the specified domain. Label all key intercepts.

$$y = -|(x + 1)(x - 3)|$$



Discussion: What would happen if $f(x)$ turned into $f(|x|)$?

- f will always take a _____ value, even if the x-value is negative.

$\rightarrow$ $\rightarrow$

- At:
  - $x = -2$: _____
  - $x = 2$: _____

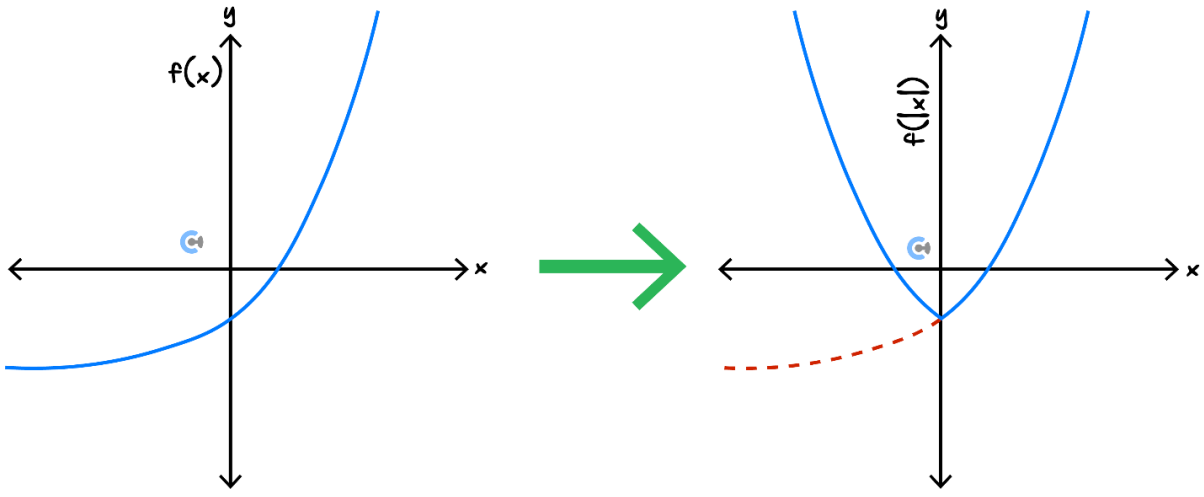
Discussion: Since $f(|-2|) = f(|2|)$, where is $f(|x|)$ symmetrical about?

Graphs of Composite Modulus Functions

- Modulus is the inside function.

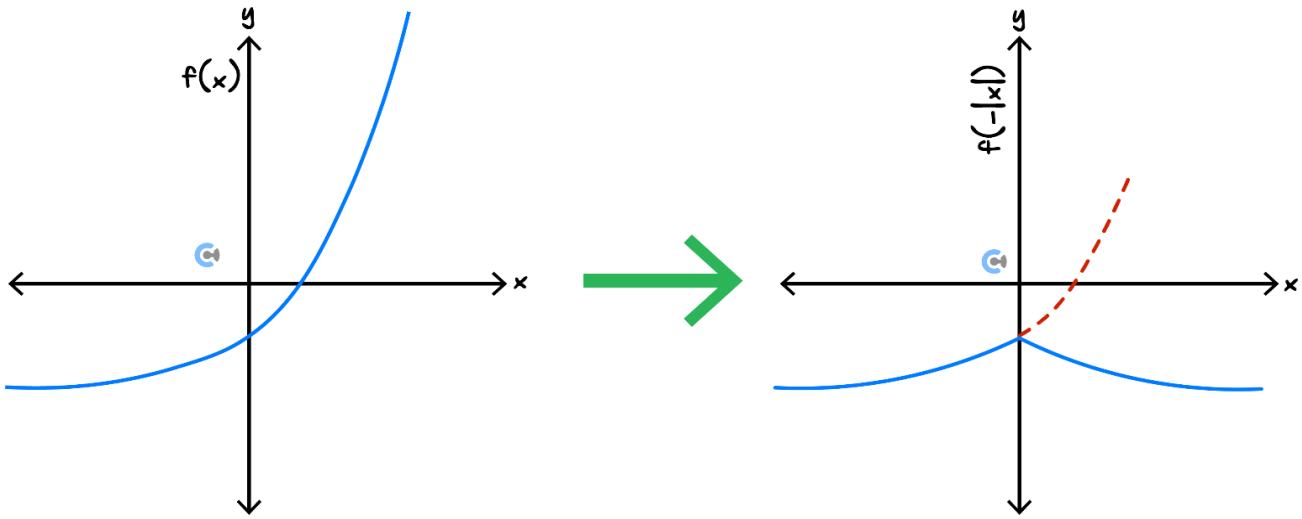
$$y = f(|x|)$$

- Take the positive side and flip it to the other side.



- Take the negative side and flip it to the other side.

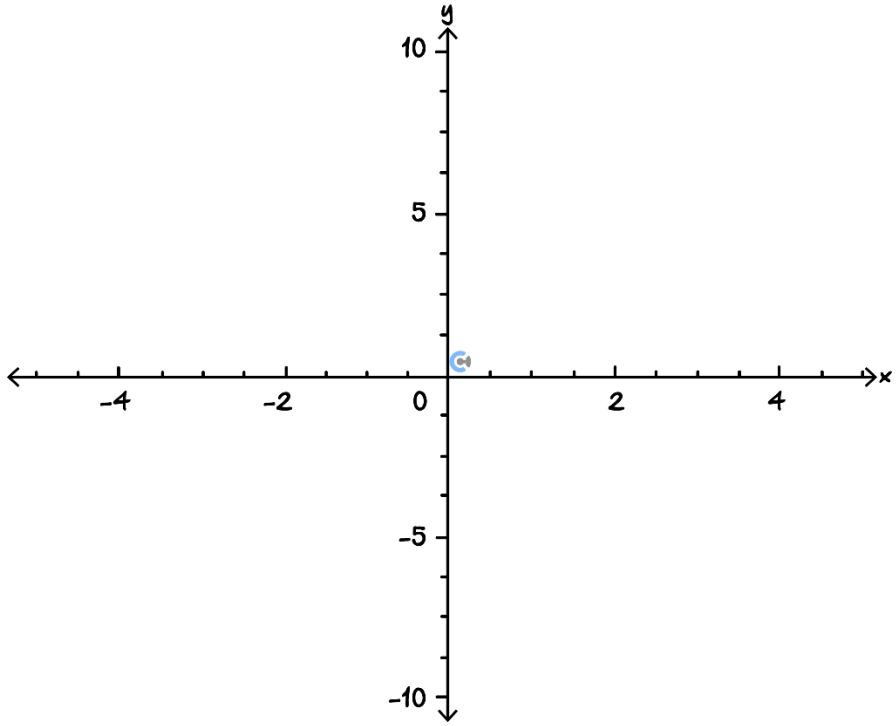
$$y = f(-|x|)$$



Question 5 Walkthrough.

Sketch the graph below.

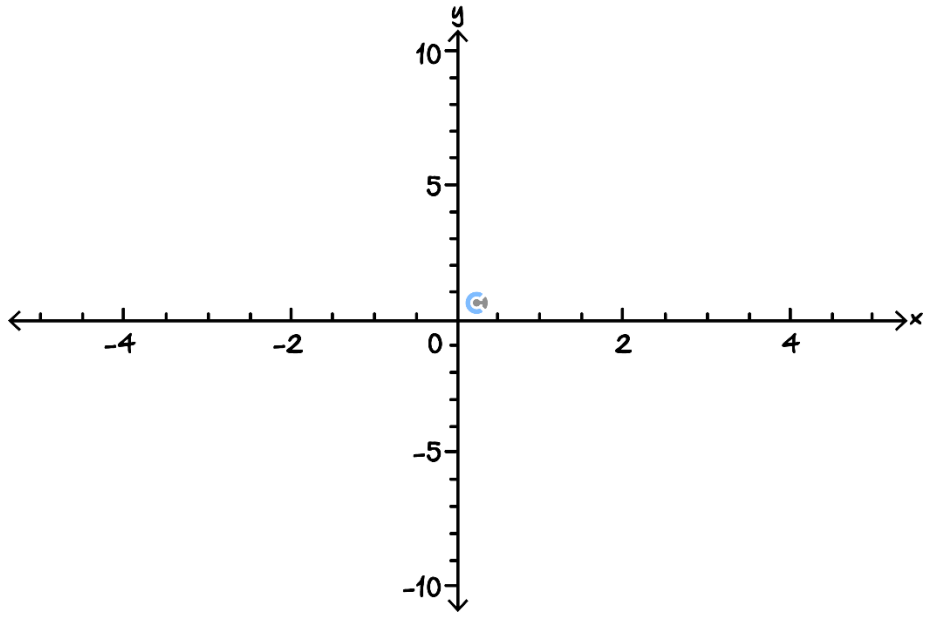
$y = f(|x|)$, where $f(x) = (x-1)^2 + 2$



Question 6

Sketch the graph below.

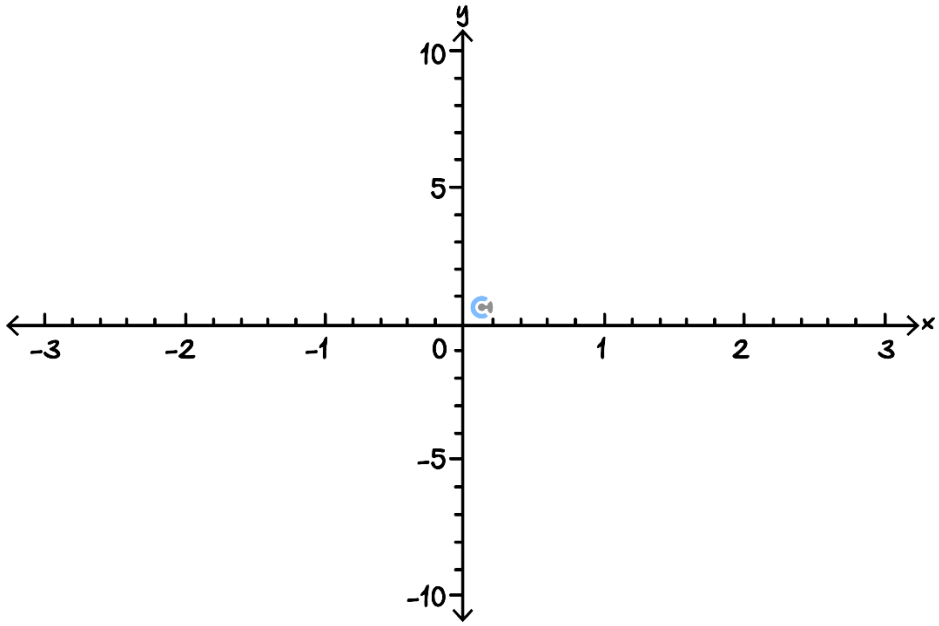
$y = f(-|x|)$, where $f(x) = x + 1$.



Question 7 Additional Question.

Sketch the graph below.

$y = f(|x|)$, where $f(x) = x^2 + x + 1$.



Section B: Partial Fractions

Sub-Section: Introduction to Partial Fractions

Partial Fractions

- The rules for partial fractions:

For every factor of this form in the denominator of the function...	There will be a partial fraction(s) of this form:
Linear factors: $\frac{1}{(ax+b)(cx+d)}$	
Repeated linear factor: $\frac{1}{(cx+d)^n}$	
Irreducible quadratic: $\frac{1}{(ax^2+bx+c)}$	

- Must do long division before using any of the rules above.

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Sub-Section: Case 1

Let's consider when we have two linear factors in the denominator!

Question 8 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{2x+9}{(x-3)(x+2)}$.

NOTE: ALWAYS factorise the denominator by its factors first!

Question 9

Perform partial fraction decomposition for the following function.

$$\frac{2x+8}{(x-1)(x-5)}$$

Sub-Section: Case 2

How about repeated linear factors?

Question 10 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{x^2-5x+8}{x(x-3)^2}$.

NOTE: When a linear factor is repeated, we repeat the splitting by that power.

Question 11

Perform partial fraction decomposition for the following function.

$$\frac{8}{(x-1)(x+1)^2}$$

Sub-Section: Case 3

Finally, non-factorisable quadratic factors!

Question 12 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{x^2+3}{(x-1)(x^2+1)}$.

NOTE: For quadratic factors that cannot be factorised, we split it as it is.

Question 13

Perform partial fraction decomposition for the following function.

$$\frac{2x-4}{(x+2)(x^2+4)}$$

Section C: Modulus and Partial Fractions Exam Skills

Sub-Section: Solving Advanced Modulus Equations and Inequalities

How do we find an intersection between two modulus functions?

Misconception

"To solve $|x-a|+b=-|x-h|+k$, we just remove the modulus and put $\pm$!"

TRUTH: We must check which equation will give us a valid solution. (Case Check)

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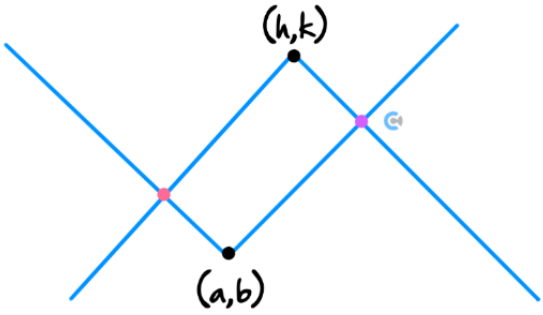
Exploration: Case Checking for Finding Intersections Between Two Modulus Functions

- Consider the following equation which finds an intersection between two modulus functions.



$$|x-a|+b=-|x-h|+k$$

- As previously mentioned, if we remove the modulus and change it to $\pm$, we get four equations.

$$(x-a)+b=(x-h)+k$$

$$-(x-a)+b=(x-h)+k$$

$$(x-a)+b=-(x-h)+k$$

$$-(x-a)+b=-(x-h)+k$$

- How many different solutions would we get from the four equations above?

- Looking at the graph above, how many solutions should we get?

- Look at the graph above, and think about which of the above equations will give us a valid solution.

  Highlight the two equations above!

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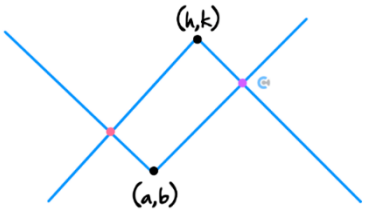
Solving Advanced Modulus Equations



$$|x-a|+b=-|x-h|+k$$

- Two corresponding equations are:

$$-(x-a)+b=(x-h)+k$$

$$(x-a)+b=-(x-h)+k$$

- Steps:

1. _____ the two modulus functions
2. Find the correct equation by looking at _____ linear lines.

Question 14 Walkthrough.

Solve $|5-2x|=-|x+3|+14$ for x.

Active Recall: Steps for Solving Advanced Modulus Equations

1. _____ the two modulus functions
2. Find the correct equation by looking at _____ linear lines.

Question 15

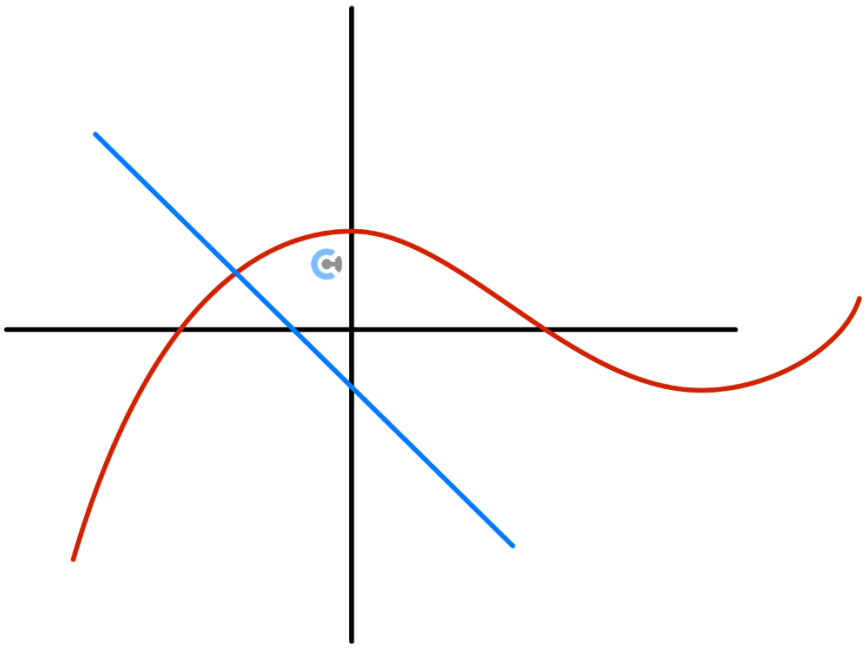
Find the value(s) of x for the following equations:

$$|x-4| = 7 - |x+1|$$

What about modulus inequalities now?

Modulus Inequalities

$f(x) > g(x)$



- **Steps:**

1. _____ either side of the inequality.
2. Find the _____ when one side is higher than the other.

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Question 16 Walkthrough.

Solve the inequality $x - 1 \leq |x^2 - 5x + 4|$ for x.

Active Recall: Steps for Solving Modulus Inequality

1. _____ either side of the inequality.
2. Find the _____ when one side is higher than the other.

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Question 17

Solve the inequality $x + 1 \leq |x^2 - 3|$ for x.

Key Takeaways

- ☑ Solving an intersection between two modulus functions requires graphing to eliminate invalid solutions.
- ☑ Finding inequalities is best done using graphs.

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Section D: Exam 1 Questions (15 Marks)

INSTRUCTION: 15 Marks. 18.5 Minutes Writing.

Question 18 Tech-Active.

Solve the inequality $|x-2| < |x^2-4|$.

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Question 19 (5 marks)

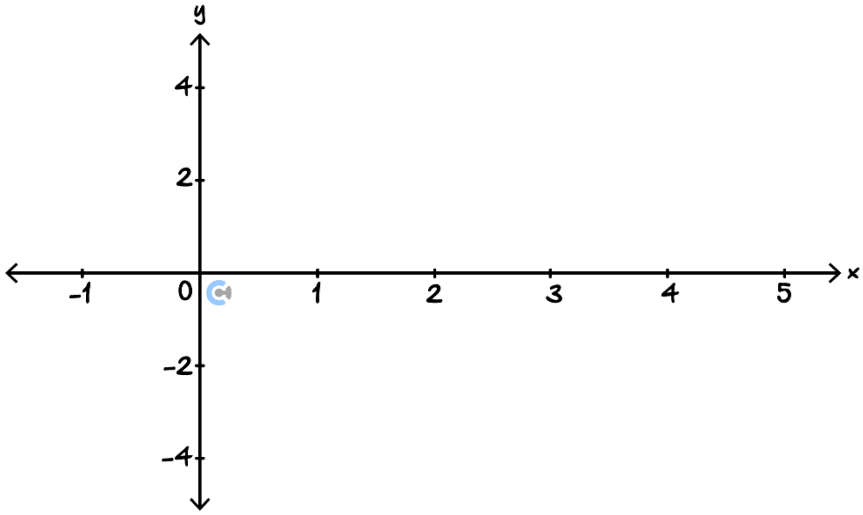
a. Perform partial fraction decomposition for $f(x) = \frac{2x^2+2}{(x+1)^2(x-1)}$. (3 marks)

b. Express $\frac{x^3-x^2-4x+7}{(x-2).(x+1)}$ in the form $Ax + \frac{B}{x-2} + \frac{C}{x+1}$. (2 marks)

Question 20 (7 marks)

Let $f(x) = -x^2 + 4x - 3$.

a. Sketch the graph of $y = |f(x)|$ on the axis below. Label all axes intercepts. (3 marks)



b. Find the value of k such that $|f(x)| = k$ has exactly 3 solutions. (1 marks)

c. Solve the inequality $x + 3 > |-x^2 + 4x - 3|$ for $x \in \mathbb{R}$. (3 marks)

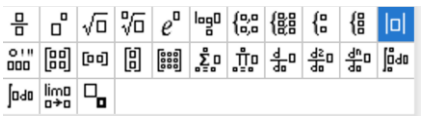
Section E: Tech Active Exam Skills

Calculator Commands: Solving Modulus Equations and Inequalities

Mathematica

- Abs[].

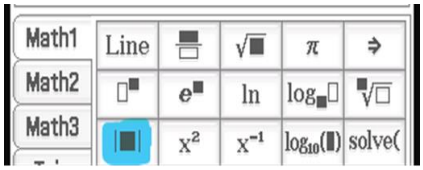
TI-Nspire



- Under button situated next to the book button.

Casio Classpad

- Under Math1.



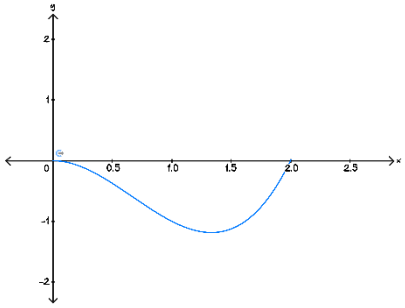
Question 21 (3 marks)

Solve the equation $3|2x + 2| - 1 = -2|3x + 6| + 5$ for $x \in \mathbb{R}$.

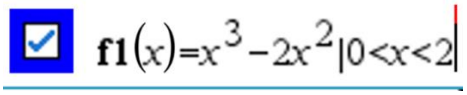
Calculator Commands: Graphing

Mathematica

- Plot[function,{x,$x$min,$x$max}, PlotRange→{$y$min, $y$max}].
- PlotRange is optional but makes the scale appropriate for the question.

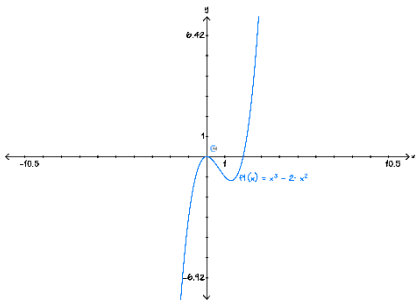


- Menu→ 6 (Analyse) to find min/max x and y intercepts
- Restrict domain to $0 < x < 2$ use the bar can get it from Ctrl+ =

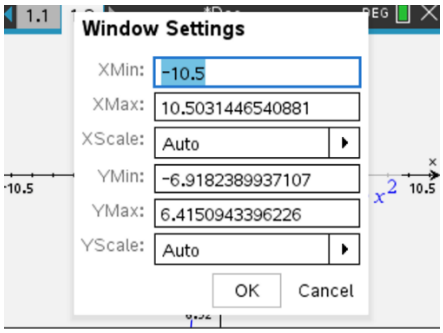


TI-Nspire

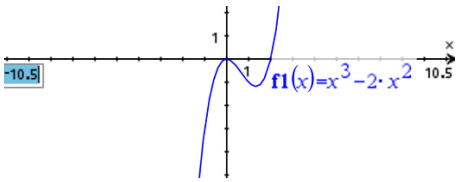
- Open a graph page and plot your function.



- Zoom settings: Menu→ 4 (window/zoom)→ 1 enter your x and y ranges.

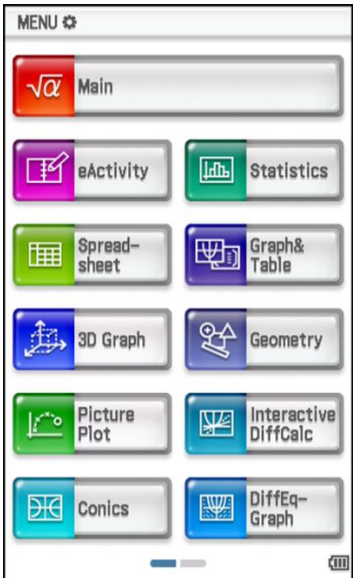


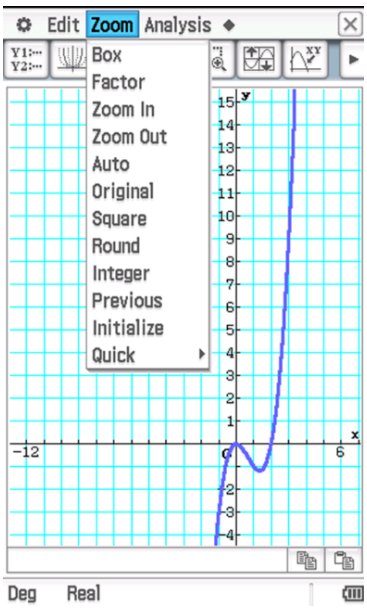
- Can also click the axis numbers on the graph and alter them directly.



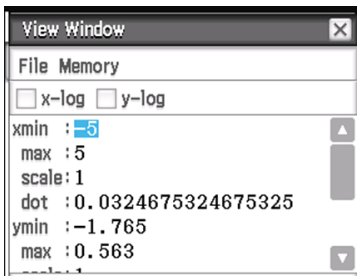
Casio Classpad

- Click Graph & Table, and enter the function.

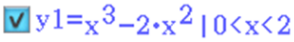




- Analysis→G-Solve to find intercepts.
- Use this button to set the view window.

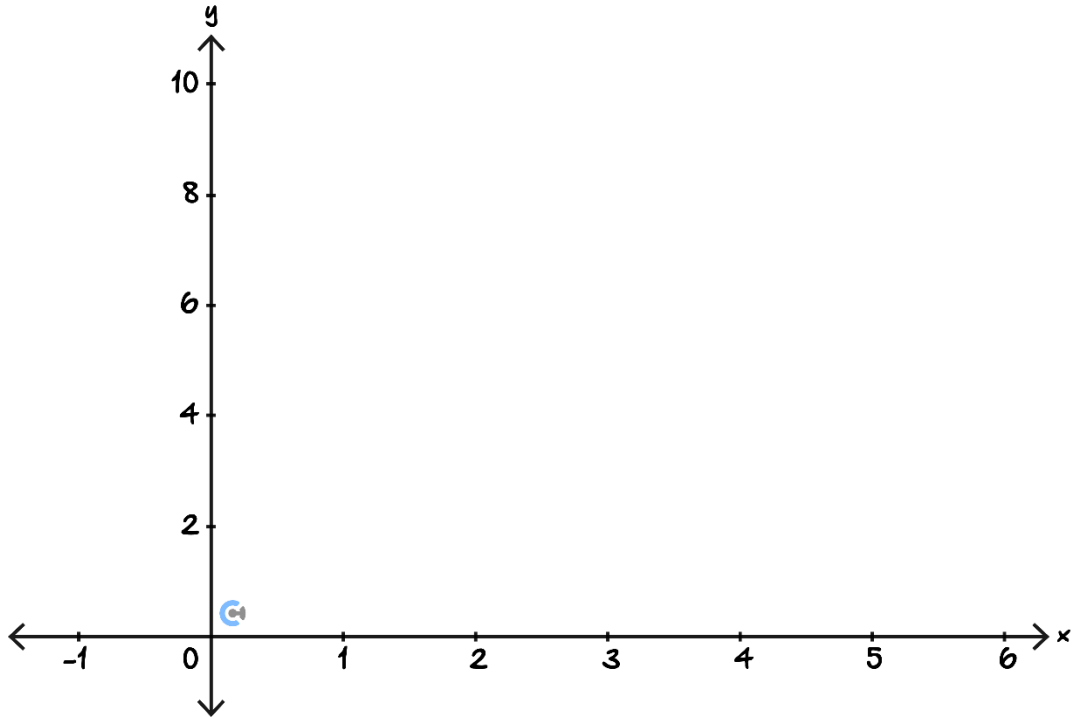


- Use | to restrict domain → find it in Math 3



Question 22 Tech-Active.

Let $f(x) = x^2 - 5x + 4$. Sketch the graph of $y = |f(x)|$ on the axis below.



Calculator Commands: Partial Fractions

Mathematica	TI-Nspire	Casio Classpad
Apart[].	Expand.	expand(func, x) or Interactive → Transformation → expand → Partial Fraction.

Question 23 Tech-Active.

Find the partial fraction decomposition of $f(x) = \frac{3}{x^2-5x+4}$.

Section F: Exam 2 Questions (18 Marks)

INSTRUCTION: 18 Marks. 4.5 Marks Reading. 22.5 Minutes Writing.

Question 24 (1 mark)

Which one of the following, where A, B, C, and D are non-zero real numbers, is a partial fraction form for the expression?

$$\frac{x-3}{(x^2+1)(x-2)^2}$$

A. $\frac{A}{x^2+1}-\frac{B}{(x-2)^2}$

B. $\frac{Ax+B}{x^2+1}+\frac{C}{(x-2)^2}+\frac{D}{x-2}$

C. $\frac{Ax+B}{x^2+1}+\frac{C}{(x-2)^2}+\frac{Dx}{x-2}$

D. $\frac{A}{x^2+1}+\frac{C}{(x-2)^2}+\frac{D}{x-4}$

Question 25 (1 mark)

For the interval $\frac{1}{2} \leq x \leq 3$, the graph of $y = |2x-1| + |x-3|$ is the same as the graph of:

A. $y = -x - 2$

B. $y = 3x - 4$

C. $y = x + 2$

D. $y = 3x + 2$

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Question 26 (1 mark)

The equation $|2x - 3| = -|x + 2| + 6$, where $x \in \mathbb{R}$, has solution(s):

A. $x = -1, \frac{7}{3}$

B. $x = \frac{7}{3}$

C. $x = -1$

D. $1, \frac{7}{3}$

Question 27 (1 mark)

The equation $|x^2 - 2x - 3| = k$, where $k \in R\backslash 0$ has exactly two solutions for:

A. $k = 4$

B. $k > 4$

C. $k = 3$

D. $k < 3$

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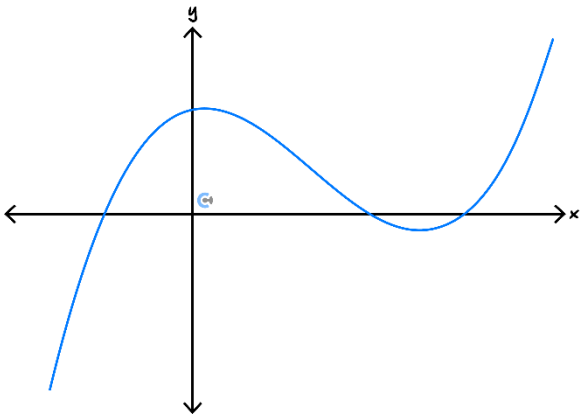
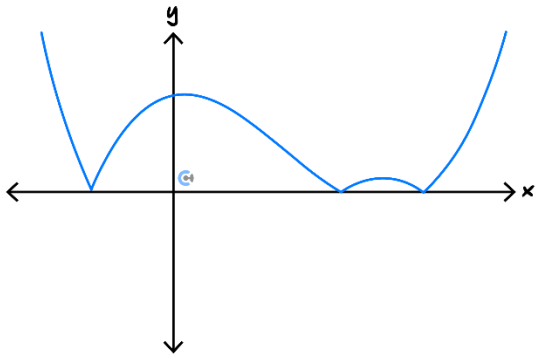
Question 28 (1 mark)

Part of the graph of $y = f(x)$ is shown below.

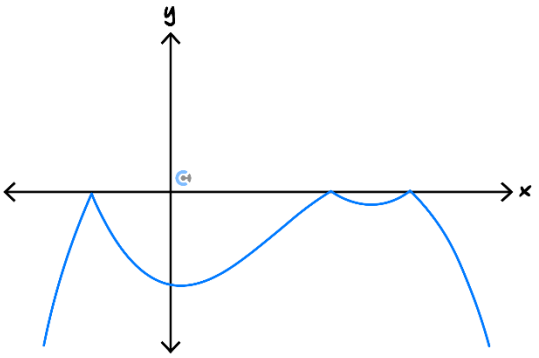


The function $f(-|x|)$ is best represented by:

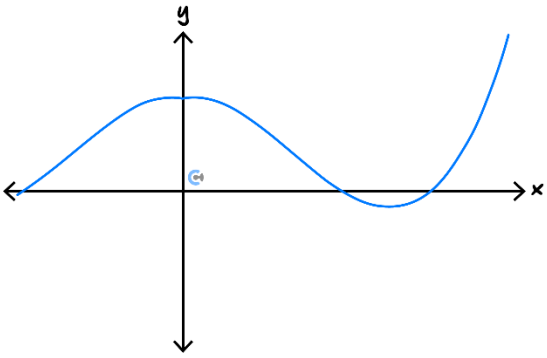
A.



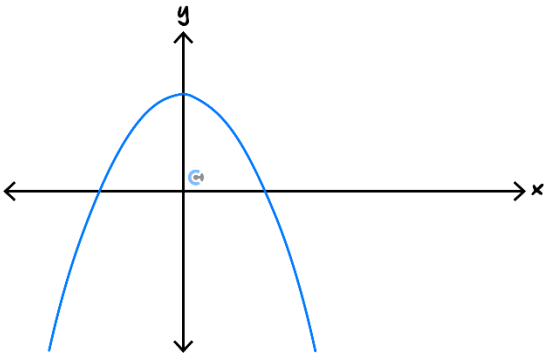
B.



C.



D.



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Question 29 (13 marks)

a. Consider the functions $f(x) = |x - 3| - 1$ and $g(x) = -|x - 3| + 3$.

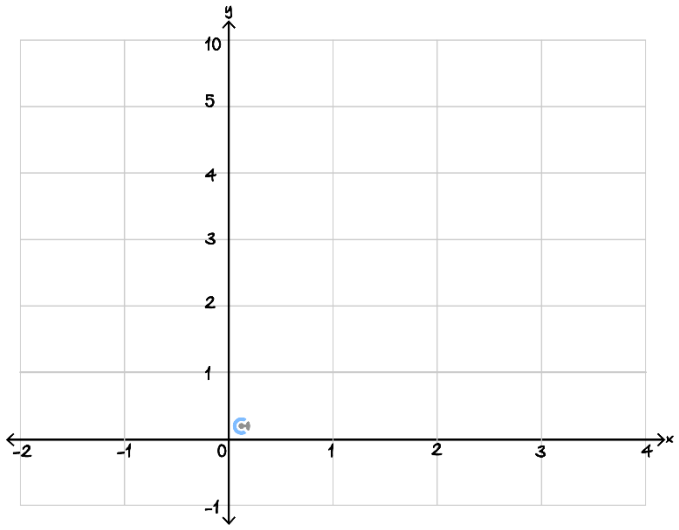
i. Let A be the vertex point of f and let B be the vertex point of g. State the coordinates of A and B. (1 mark)

ii. Let C and D be the points of intersection of $f(x)$ and $g(x)$. State the coordinates of C and D. (2 marks)

iii. Find the area of the square $ABCD$. (2 marks)

Consider the function $h(x) = |(x-3)(x+1)|$.

b. Sketch the graph of $y = h(x)$ on the axis below. Label all axes intercepts and turning points. (3 marks)



c. Solve the inequality $x + 1 > h(x)$ for $x \in \mathbb{R}$. (2 marks)

d. The equation $k - x = h(x)$, where k is a real number, has 4 real solutions. Find the possible value(s) of k. (3 marks)

Contour Check

Learning Objective: [1.1.1]

Study Design

Graphs of sum, difference, product, and composite functions involving functions of the types specified above (not including composite functions that result in reciprocal or quotient functions).

Key Takeaways

- Modulus finds a _____ of things.
- $|a-b|$ is a _____ between a and b.
- $\sqrt{x^2} =$ _____.
- For simple modulus equations, remove modulus and put _____.

Learning Objective: [1.1.2]

Study Design

Graphs of sum, difference, product, and composite functions involving functions of the types specified above (not including composite functions that result in reciprocal or quotient functions).

Key Takeaways

- Graph of a simple modulus graph $a|x-h|+k$ is like a straightened _____.
- Wrapping modulus around the function makes the y value always non-_____.
- Wrapping the modulus around the x value makes the function symmetrical around the _____ axis.
- $f(|x|)$ take the RHS and make it symmetrical about the _____ axis.
- $f(-|x|)$ take the _____ _____ and make it symmetrical about the y-axis.

Learning Objective: [1.1.3]

Key Takeaways

- Partial fractions are the process of _____.
- Must _____ before doing partial fractions.
- Must do _____ before doing partial fractions.
- Linear factors always have a _____ at the top.
- Irreducible quadratic factors have a _____ function at the top.

Learning Objective: [1.2.1]

Key Takeaways

- To solve modulus inequalities, we should:

1. _____ either side of the inequality.
2. Find the _____ when one side is higher than the other.

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