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VCE Specialist Mathematics ½

Modulus & Partial Fractions Exam Skills [1.2]

Homework Solutions

Homework Outline:

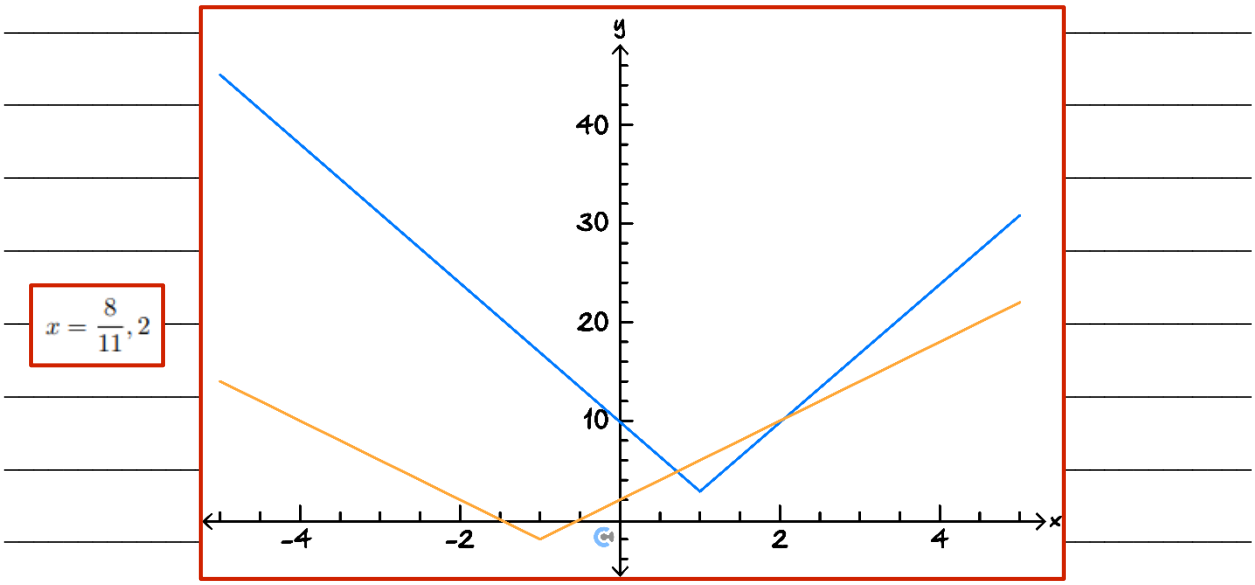
Compulsory Questions	Pg 2 - Pg 15
Supplementary Questions	Pg 16 - Pg 29

Section A: Compulsory Questions

Sub-Section [1.2.1]: Solving Advanced Algebra and Inequalities

Question 1

Solve the equation $7|x - 1| + 3 = 4|x + 1| - 2$ for $x \in \mathbb{R}$.



$x = \frac{8}{11}, 2$

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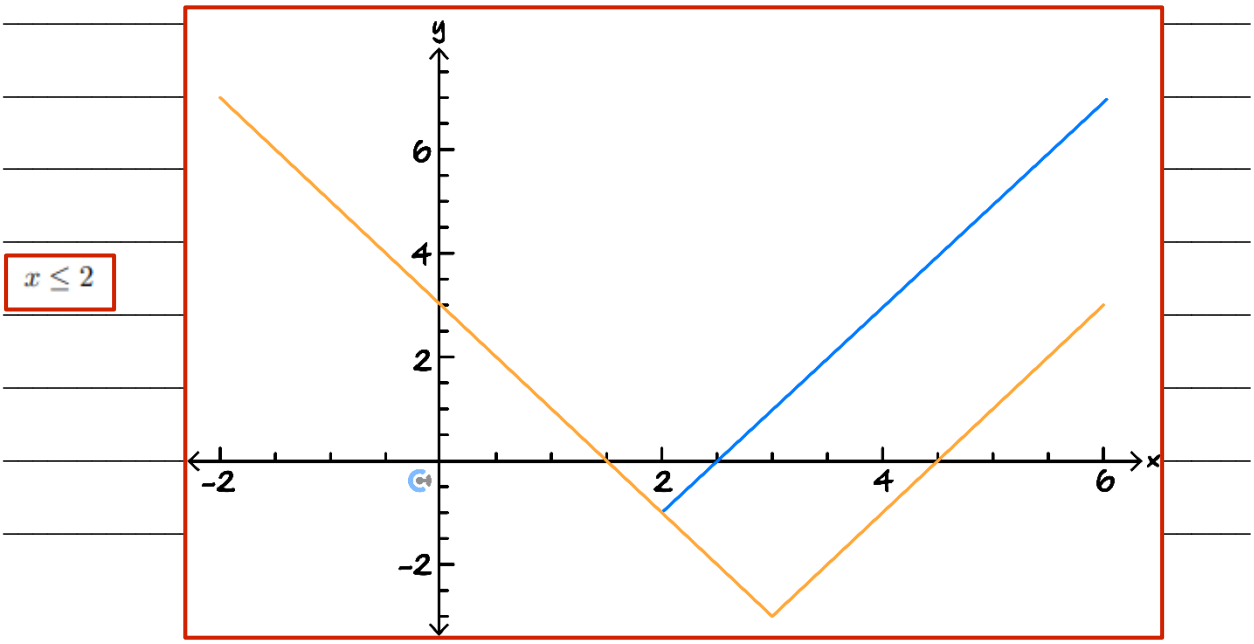
Question 2

Solve the equation $|2x - 4| - 1 = 2|x - 3| - 3$ for $x \in \mathbb{R}$.

$x \leq 2$



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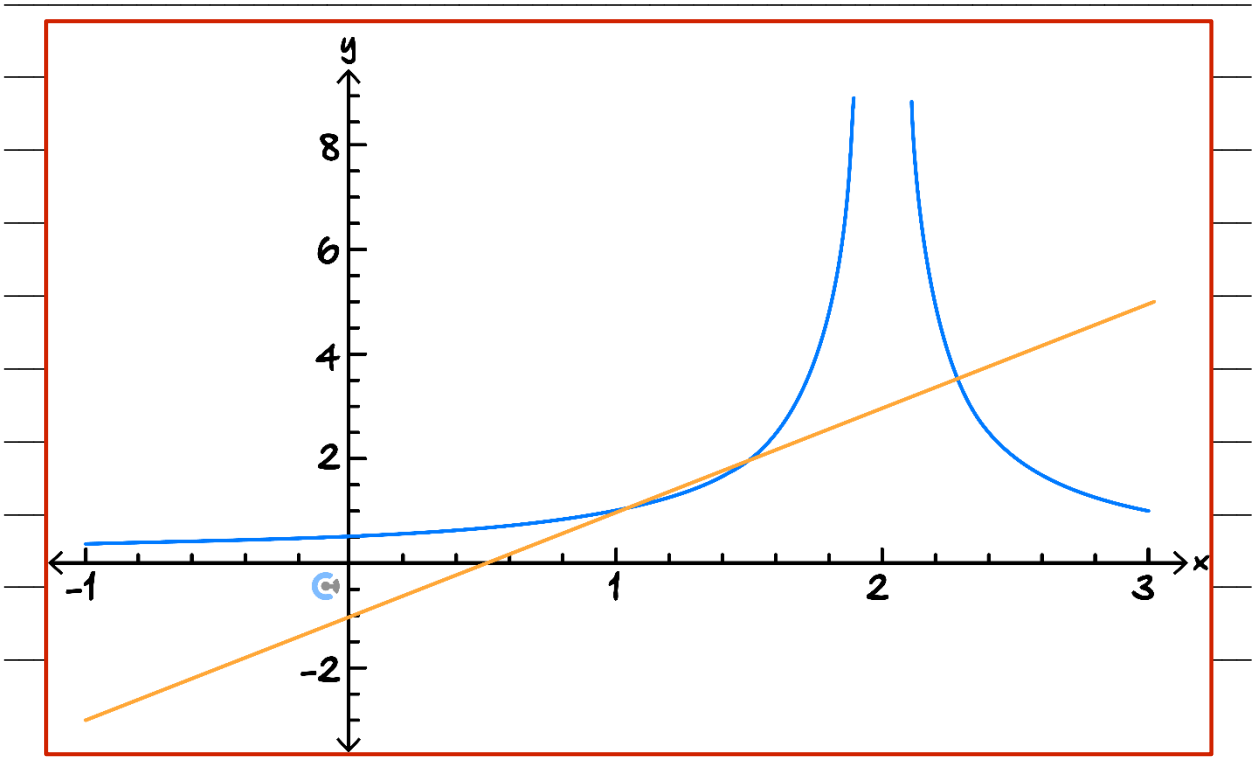
Question 3

Solve the inequality $\frac{1}{|x-2|} > 2x - 1$ for $x \in \mathbb{R}$.

$$x < 1 \text{ or } \frac{3}{2} < x < 2 \text{ or } 2 < x < \frac{1}{4}(5 + \sqrt{17})$$



Question 4 Tech-Active.

Solve the inequality $|x^2 - 4x + 5| > x$.

$$x < \frac{1}{2}(5 - \sqrt{5}) \text{ or } x > \frac{1}{2}(5 + \sqrt{5})$$

TI:

$\text{solve}(|x^2-4\cdot x+5|<x,x)$

$\frac{-(\sqrt{5}-5)}{2}<x<\frac{\sqrt{5}+5}{2}$

Mathematica:

In[122]:= Reduce[Abs[x^2 - 4 x + 5] < x, x]

Out[122]= $\frac{1}{2}(5-\sqrt{5}) < x < \frac{1}{2}(5+\sqrt{5})$

Casio:

solve(|x^2−4x+5|>x, x)

$\left\{x<\frac{-\sqrt{5}}{2}+\frac{5}{2}, \frac{\sqrt{5}}{2}+\frac{5}{2}<x\right\}$

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Sub-Section: Exam 1 Questions

Question 5

Solve the equation $|x - 4| = \frac{x}{2} + 5$.

$$x = -\frac{2}{3}, 18$$

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Question 6

Solve the inequality $4 - x > \frac{1}{|x-2|}$ for $x \in \mathbb{R}$.

$x < 3 - \sqrt{2}$

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Question 7

Consider the function f with rule $f(x) = \frac{x^2+x-6}{x+2}$.

a. Show that the rule for the function f can be written as $f(x) = x - 1 - \frac{4}{x+2}$

$$f(x) = \frac{(x+2)(x-1)-4}{x+2} = x - 1 - \frac{4}{x+2}$$

Or, using long division:

$$x+2 \overline{)\, x^2+x-6}$$

$$x - 1$$

$$-(x^2+2x)$$

$$-x-6$$

$$-(-x-2)$$

$$-4$$

$$\therefore f(x) = x - 1 - \frac{4}{x+2}$$

b. Solve the inequality $f(x) > x + 3$ for $x \in \mathbb{R}$.

$$-3 < x < -2$$

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Question 8

a. Perform partial fraction decomposition for $f(x) = \frac{3x^2}{(x-1)^2(x+2)}$.

$$\frac{1}{(x-1)^2} + \frac{4}{3(x+2)} + \frac{5}{3(x-1)}$$

b. Express $g(x) = \frac{x^3-27}{(x-3)(x^2+2x+1)}$ in the form $\frac{A}{(x+1)^2} + \frac{B}{x+1} + C$ for real numbers A, B and C.

$$\frac{7}{(x+1)^2} + \frac{1}{x+1} + 1$$

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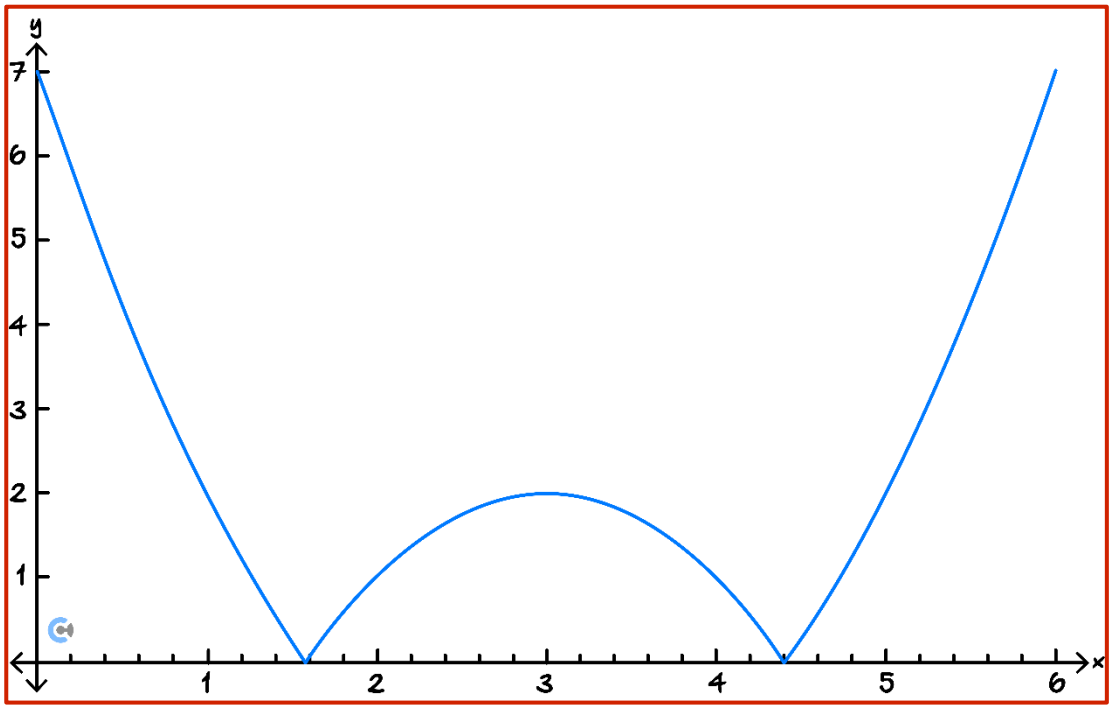
Question 9

Let $f(x) = (x - 3)^2 - 2$.

Sketch the graph of $y = |f(x)|$ on the axis below. Label all axes intercepts and turning points.



$x-$ intercepts : $(3-\sqrt{2},0),(3+\sqrt{2},0)$ and y-intercept $(0,7)$ and TP $(3,2)$

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Sub-Section: Exam 2 Questions

Question 10

Which one of the following, where A, B, C, D and E are non-zero real numbers, is a partial fraction form for the expression?

$$\frac{2}{(x^2-4)(x-2)^3}$$

A. $\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{x+2}$

B. $\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}+\frac{E}{x+2}$

C. $\frac{Ax+B}{x^2-4}+\frac{C}{x+2}+D$

D. $\frac{Ax}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}+\frac{E}{x+2}$

Question 11

The equation $x + 3 = |x - 4| + 2$, where $x \in \mathbb{R}$, has solution(s):

A. $x = 1, -\frac{3}{2}$

B. $x = \frac{3}{2}$

C. $x = -1$

D. $x = -1, \frac{3}{2}$

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Question 12

The graph of $y = |2x - 1| - |x - 3|$ is the same as the graph of $y = 3x - 4$ for which of the following ranges of x-values:

A. $x > \frac{1}{2}$

B. $x \leq \frac{1}{2}$

C. $\frac{1}{2} \leq x \leq 3$

D. $x \geq 3$

Question 13

The equation $|x^2 + 4x - 6| = k$, where k is a real number has exactly three solutions for:

A. $k = 10$

B. $0 < k < 10$

C. $k > 10$

D. $k > 0$

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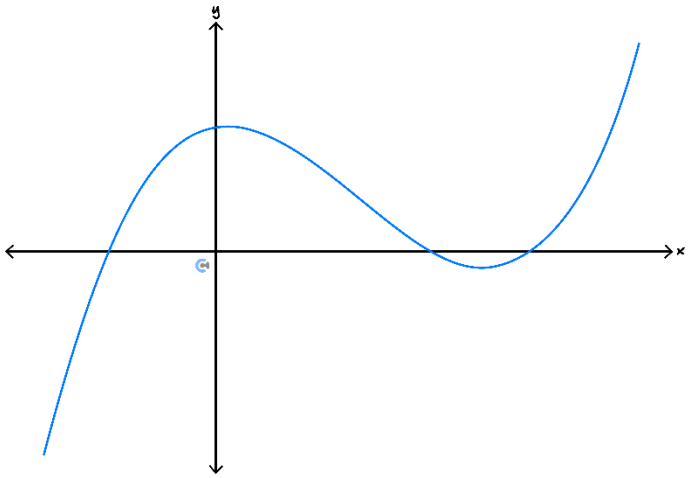
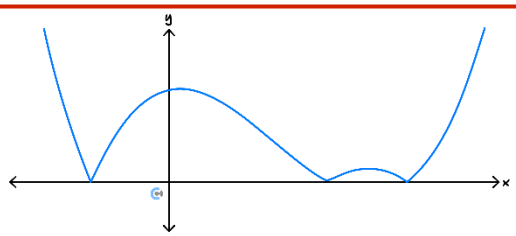
Question 14

Part of the graph of $y = f(x)$ is shown below.

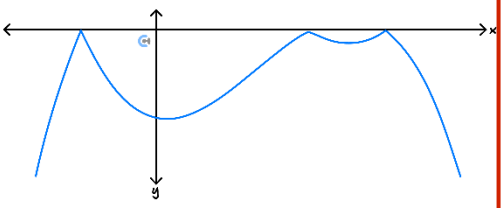


The function $-|f(x)|$ is best represented by:

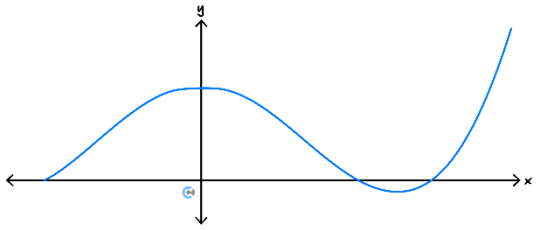
A.



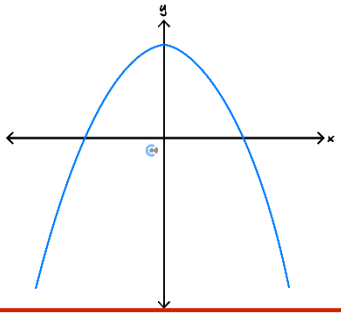
B.



C.



D.



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Question 15

Consider the function $f(x) = |x - 1| + \left|\frac{x}{2} - 2\right|$

a. Explain why the graph of $y = f(x)$ has no x-intercepts.

Both $|x - 1|$ and $\left|\frac{x}{2} - 2\right|$ are always ≥ 0, so $f(x)$ can never equal zero because it is the sum of two quantities that are ≥ 0 and are both never zero at the same time.

b. State the minimum value of $f(x)$ and the x-value where this occurs.

Minimum of $\frac{3}{2}$ when $x = 1$.

c.

i. Find the range of x-values for which $f(x) = \frac{1}{2}x + 1$.

$1 \leq x \leq 4$

ii. When $x < -6$, $f(x)$ may be written as $f(x) = mx + c$. Find the values of m and c.

$f(x) = -\frac{3}{2}x + 3 \implies m = -\frac{3}{2}$ and $c = 3$

iii. When $x > 6$, $f(x)$ may be written as $f(x) = nx + d$. Find the values of n and d.

$f(x) = \frac{3}{2}x - 3 \implies n = \frac{3}{2}$ and $d = -3$

d. Solve the inequality $f(x) < x + 3$.

$0 < x < 12$

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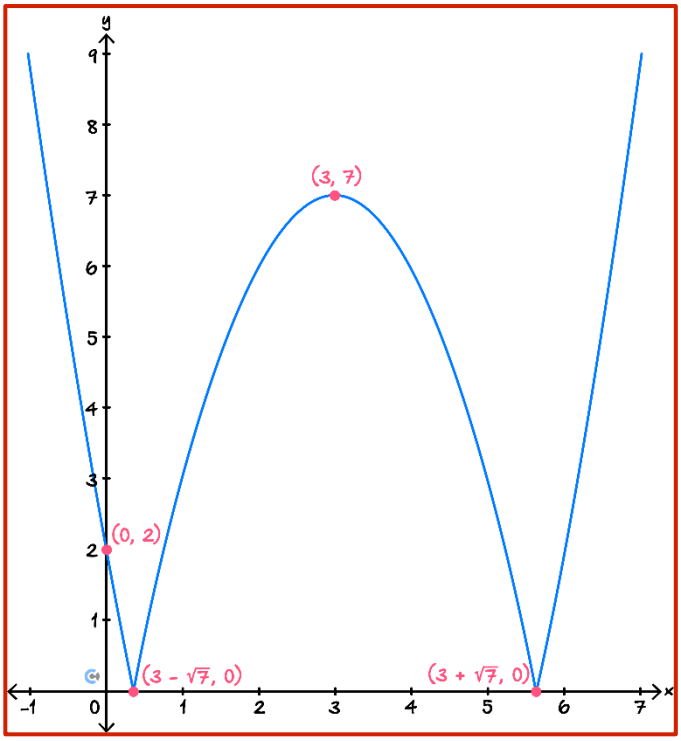
Question 16

Consider the function $h(x) = |x^2 - 6x + 2|$.

a. Sketch the graph of $y = h(x)$ on the axis below. Label all axes intercepts and turning points.



b. Solve the inequality $x + 2 > h(x)$ for $x \in \mathbb{R}$.

$0 < x < 1$ or $4 < x < 7$

c. The equation $h(x) = k$, where k is a real number, has 4 real solutions. Find the possible value(s) of k.

$0 < k < 7$.

Section B: Supplementary Questions

Question 17

Solve the equation $|x - 1| + 3 = |3x + 1| - 2$ for $x \in \mathbb{R}$.

$$x = -\frac{7}{2}, \frac{3}{2}$$

Question 18

Solve the equation $|2x - 3| = -2|x + 1| + 5$ for $x \in \mathbb{R}$.

$$-1 \leq x \leq \frac{3}{2}$$

Question 19

Solve the inequality $\frac{1}{|x-4|} + 2 < x + 6$ for $x \in \mathbb{R}$.

$$-\sqrt{15} < x < \sqrt{15} \text{ or } x > \sqrt{17}$$

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Question 20

Solve the inequality $\left|\frac{x-4}{x+1}\right| - 3 > |x+2|$ for $x \in \mathbb{R}$.

$$x \in \left(\frac{1}{2}(-1-\sqrt{21}), \frac{1}{2}(3\sqrt{5}-7)\right) \setminus \{-1\}$$

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Sub-Section: Exam 1 Questions

Question 21

Solve the equation $|x - 4| = 2|x + 8|$.

$x = -20, -4$

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Question 22

Solve the inequality $x + 2 > \frac{1}{\sqrt{x^2-4x+4}}$ for $x \in \mathbb{R}$.

Equivalent to $x + 2 > \dfrac{1}{|x-2|}$

$-\sqrt{3} < x < \sqrt{3}$ or $x > \sqrt{5}$

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Question 23

a. Perform partial fraction decomposition for $f(x) = \frac{6x}{(x-1)(x+2)}$.

$$\frac{2}{x-1} + \frac{4}{x+2}$$

b. Express $g(x) = \frac{x^3+8}{(x+2)(x^2+4x+4)}$ in the form $\frac{A}{(x+2)^2} + \frac{B}{x+2} + C$ for real numbers A, B and C.

$$\frac{12}{(x+2)^2} - \frac{6}{x+2} + 1$$

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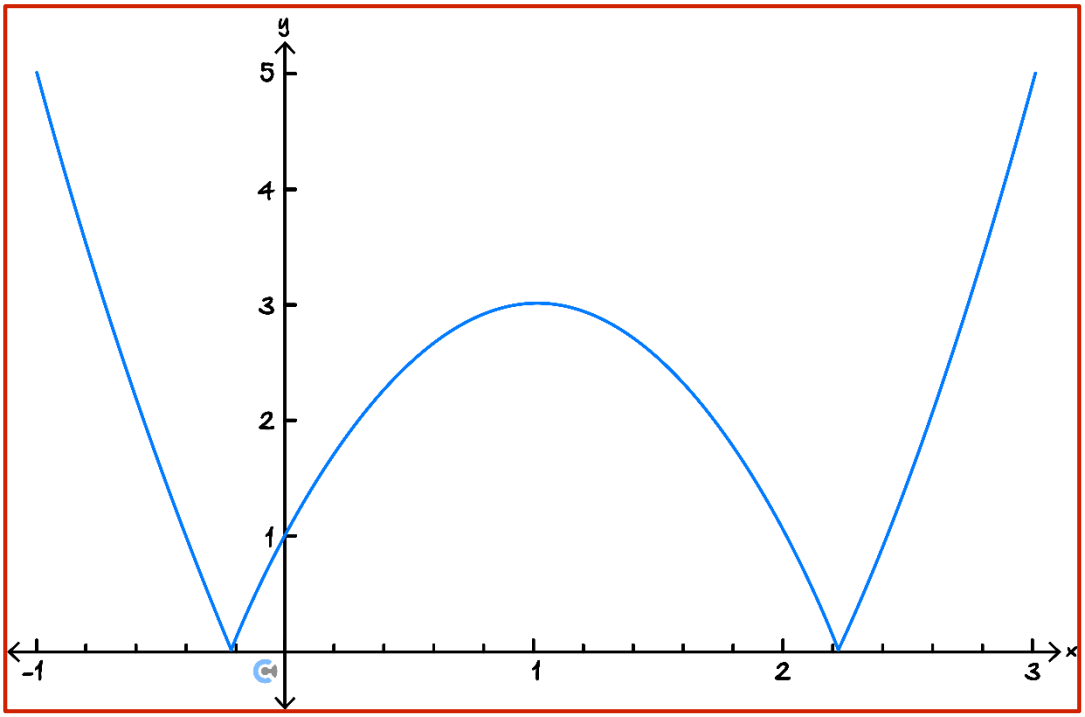
Question 24

Let $f(x) = 2x^2 - 4x - 1$.

Sketch the graph of $y = |f(x)|$ on the axis below. Label all axes intercepts and turning points.



$x-$ intercepts : $(1-\frac{\sqrt{6}}{2},0),(1+\frac{\sqrt{6}}{2},0)$ and y-intercept $(0,1)$ and TP $(1,3)$

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Question 25

Consider the function f with rule $f(x) = \frac{x^2+x+4}{x+1}$.

a. Show that the rule for the function f can be written as $f(x) = x + \frac{4}{x+1}$.

$$f(x) = \frac{x(x+1)+4}{x+1} = x + \frac{4}{x+1}$$

Long division : $x+1 \overline{)\, x^2 + x + 4}$ ... x

$-(x^2 + x)$

$0 + 4$

$f(x) = x + \frac{4}{x+1}$

b. Solve the inequality $f(x) > x + 5$ for $x \in \mathbb{R}$.

$$-1 < x < -\frac{1}{5}$$

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Sub-Section: Exam 2 Questions

Question 26

The equation $|2x - 3| = -|x + 2| + 6$, where $x \in \mathbb{R}$, has solution(s):

A. $x = -1, \frac{7}{3}$

B. $x = \frac{5}{3}$

C. $x = -1$

D. $x = 7, \frac{5}{3}$

Question 27

The graph of $y = |2x - 1| - |x - 3|$ is the same as the graph of $y = -2 - x$ for which of the following ranges of x values:

A. $x > \frac{1}{2}$

B. $x \leq \frac{1}{2}$

C. $\frac{1}{2} \leq x \leq 3$

D. $x \geq 3$

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Question 28 (1 mark)

Which one of the following, where A, B, C, and D are non-zero real numbers, is a partial fraction form for the expression?

$$\frac{x-3}{(x^2-1)(x-2)}$$

A. $\frac{A}{x^2-1}-\frac{B}{(x-2)^2}$

B. $\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-2}$

C. $\frac{Ax+B}{x^2-1}+\frac{C}{x-2}+\frac{Dx}{x-2}$

D. $\frac{A}{x^2-1}+\frac{C}{x-2}+\frac{D}{x-4}$

Question 29

The equation $|x^2+2x-8|=k$, where k is a real number has exactly four solutions for:

A. $k=9$

B. $0<k<9$

C. $k>9$

D. $k>0$

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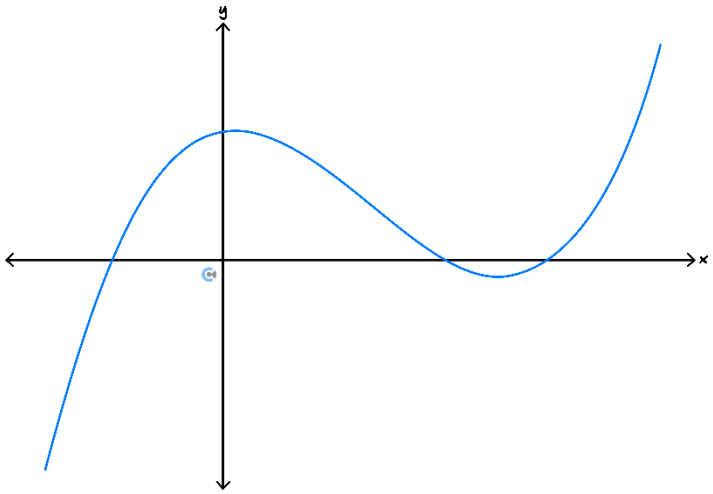
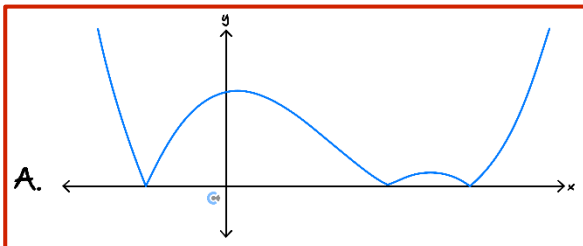
Question 30

Part of the graph of $y = f(x)$ is shown below.

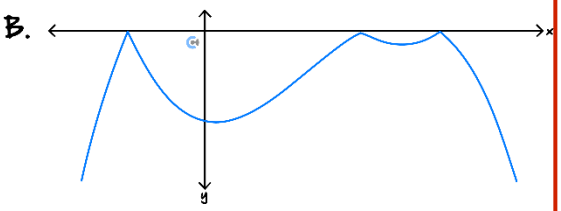


The function $f(|x|)$ is best represented by

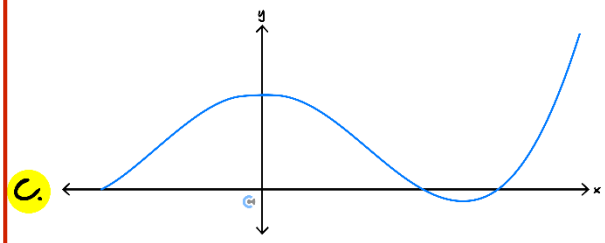
A.



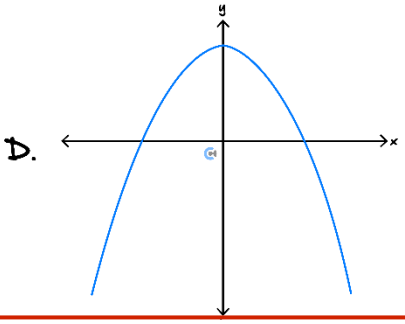
B.



C.



D.



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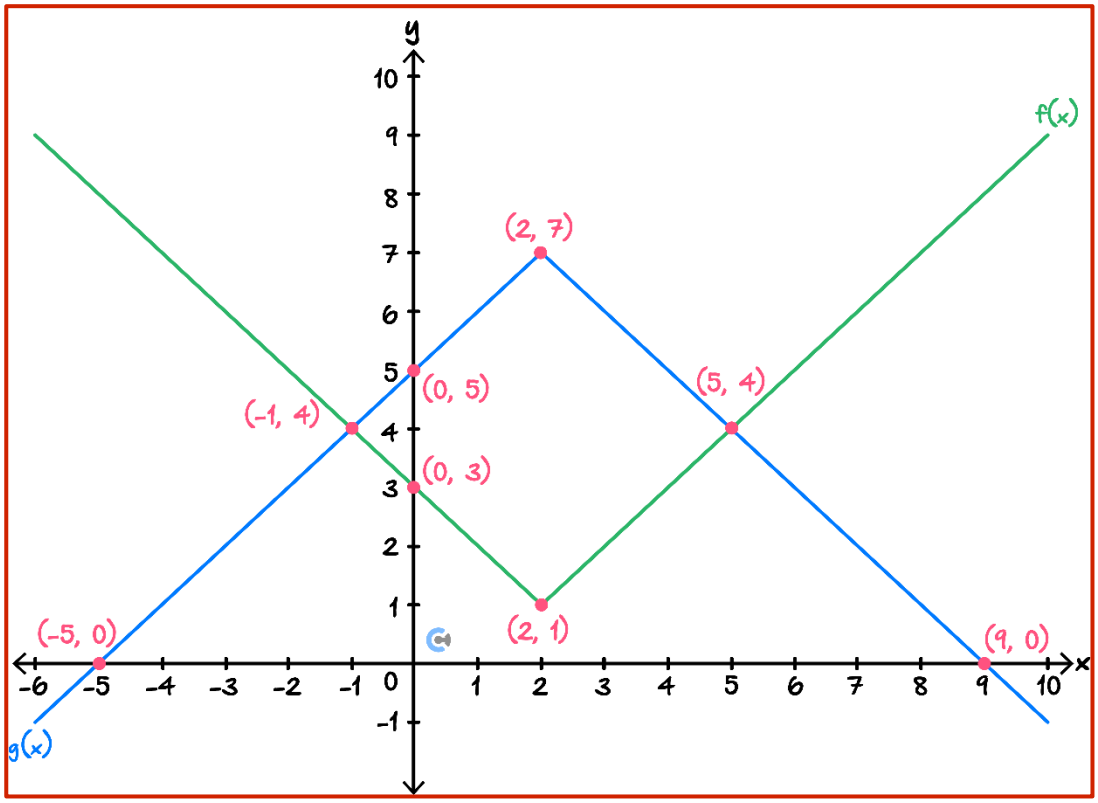
Question 31

Consider the functions $f(x) = |x - 2| + 1$ and $g(x) = -|x - 2| + 7$

a. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Label all points of intersection, axes intercepts, and vertex points with coordinates.



b. Solve the inequality $f(x) < g(x)$.

$-1 < x < 5$

c.

i. Find the value(s) of k for which the line $y = k - x$ never intersects the graph of $y = g(x)$.

$k > 9$

ii. Find the value(s) of k for which $k - x = g(x)$ has infinitely many solutions.

$k = 9$

d. Find the area of the region bounded between the graphs of $y = f(x)$ and $y = g(x)$

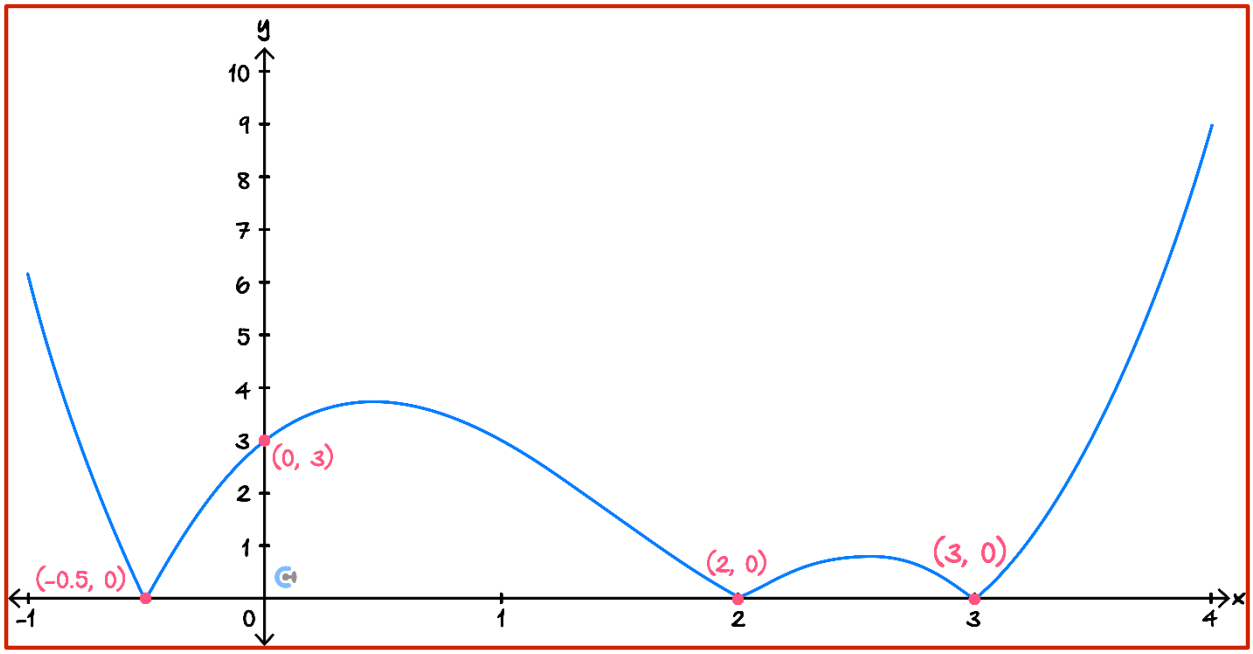
$$\text{Area} = 2 \times \frac{1}{2} \times 6 \times 3 = 18$$

Space for Personal Notes

Question 32

Consider the function $h(x) = \left|x^3 - \frac{9x^2}{2} + \frac{7x}{2} + 3\right|$.

a. Sketch the graph of $y = h(x)$ on the axis below. Label all axes intercepts.



b. Solve the inequality $x + 5 > h(x)$ for $x \in \mathbb{R}$. Give your answer correct to two decimal places.

$-0.87 < x < 4$

c. The equation $h(x) = k$, where k is a real number, has 6 real solutions. Find the possible value(s) of k. Give your answer correct to three decimal places.

The maximum value of $h(x)$ for $2 < x < 3$ is ≈ 0.755
Therefore, $0 < k < 0.755$.

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