CONTOUREDUCATION

VCE Specialist Mathematics ½
Modulus & Partial Fractions Exam Skills [1.2]
Homework

Homework Outline:

Compulsory Questions	Pg 2 - Pg 15
Supplementary Questions	Pg 16 - Pg 29

Section A: Compulsory Questions

Sub-Section [1.2.1]: Solving Advanced Algebra and Inequalities
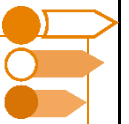
Question 1

Solve the equation $7|x - 1| + 3 = 4|x + 1| - 2$ for $x \in \mathbb{R}$.

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Question 2

Solve the equation $|2x - 4| - 1 = 2|x - 3| - 3$ for $x \in \mathbb{R}$.

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Question 3

Solve the inequality $\frac{1}{|x-2|} > 2x - 1$ for $x \in \mathbb{R}$.

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Question 4 Tech-Active.

Solve the inequality $|x^2 - 4x + 5| > x$.

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Sub-Section: Exam 1 Questions

Question 5

Solve the equation $|x - 4| = \frac{x}{2} + 5$.

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Question 6

Solve the inequality $4 - x > \frac{1}{|x-2|}$ for $x \in \mathbb{R}$.

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Question 7

Consider the function f with rule $f(x) = \frac{x^2+x-6}{x+2}$.

a. Show that the rule for the function f can be written as $f(x) = x - 1 - \frac{4}{x+2}$

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b. Solve the inequality $f(x) > x + 3$ for $x \in \mathbb{R}$.

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Question 8

a. Perform partial fraction decomposition for $f(x) = \dfrac{3x^2}{(x-1)^2(x+2)}$.
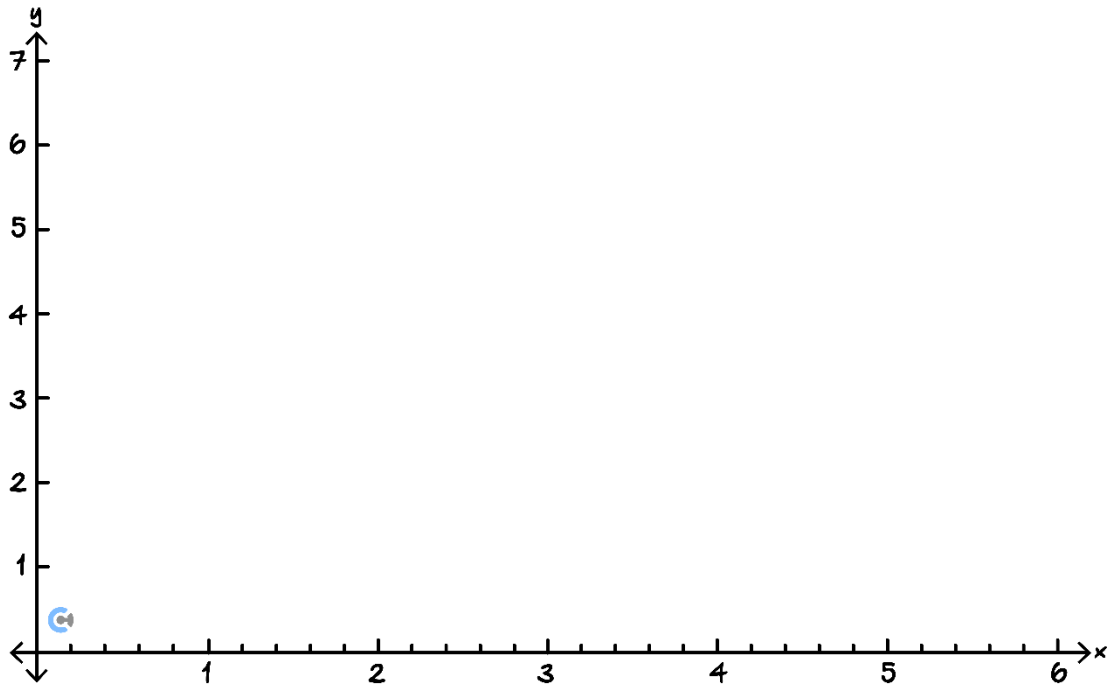
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b. Express $g(x) = \dfrac{x^3-27}{(x-3)(x^2+2x+1)}$ in the form $\dfrac{A}{(x+1)^2} + \dfrac{B}{x+1} + C$ for real numbers A, B and C.

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Question 9

Let $f(x) = (x - 3)^2 - 2$.

Sketch the graph of $y = |f(x)|$ on the axis below. Label all axes intercepts and turning points.

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Sub-Section: Exam 2 Questions

Question 10

Which one of the following, where A, B, C, D and E are non-zero real numbers, is a partial fraction form for the expression?

$$\frac{2}{(x^2 - 4)(x - 2)^3}$$

A. $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{D}{x+2}$

B. $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{D}{(x-2)^4} + \frac{E}{x+2}$

C. $\frac{Ax+B}{x^2-4} + \frac{C}{x+2} + D$

D. $\frac{Ax}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{D}{(x-2)^4} + \frac{E}{x+2}$

Question 11

The equation $x + 3 = |x - 4| + 2$, where $x \in \mathbb{R}$, has solution(s):

A. $x = 1, -\frac{3}{2}$

B. $x = \frac{3}{2}$

C. $x = -1$

D. $x = -1, \frac{3}{2}$

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Question 12

The graph of $y = |2x - 1| - |x - 3|$ is the same as the graph of $y = 3x - 4$ for which of the following ranges of x-values:

A. $x > \frac{1}{2}$

B. $x \leq \frac{1}{2}$

C. $\frac{1}{2} \leq x \leq 3$
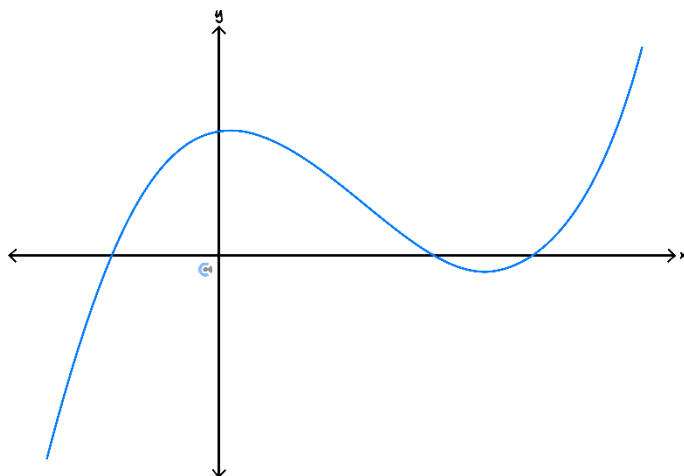
D. $x \geq 3$

Question 13

The equation $|x^2 + 4x - 6| = k$, where k is a real number has exactly three solutions for:
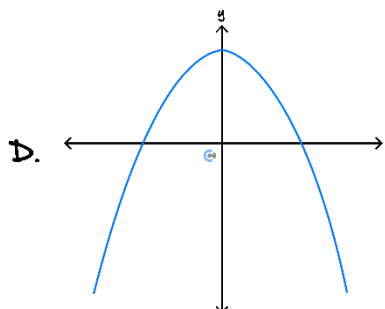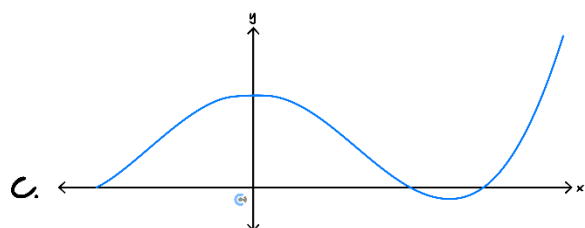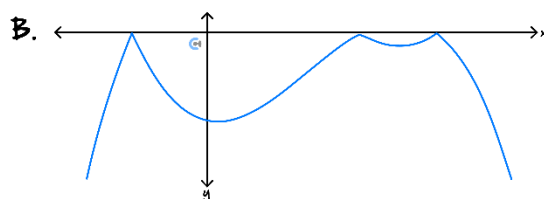
A. $k = 10$

B. $0 < k < 10$

C. $k > 10$
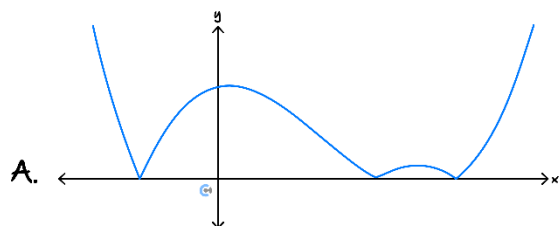
D. $k > 0$

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Question 14

Part of the graph of $y = f(x)$ is shown below.

The function $-|f(x)|$ is best represented by:

A.

B.

C.

D.

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Question 15

Consider the function $f(x) = |x - 1| + \left|\frac{x}{2} - 2\right|$

a. Explain why the graph of $y = f(x)$ has no x-intercepts.

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b. State the minimum value of $f(x)$ and the x-value where this occurs.

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i. Find the range of x-values for which $f(x) = \frac{1}{2}x + 1$.

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ii. When $x < -6$, $f(x)$ may be written as $f(x) = mx + c$. Find the values of m and c.

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iii. When $x > 6$, $f(x)$ may be written as $f(x) = nx + d$. Find the values of n and d.

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d. Solve the inequality $f(x) < x + 3$.

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Question 16

Consider the function $h(x) = |x^2 - 6x + 2|$.

a. Sketch the graph of $y = h(x)$ on the axis below. Label all axes intercepts and turning points.
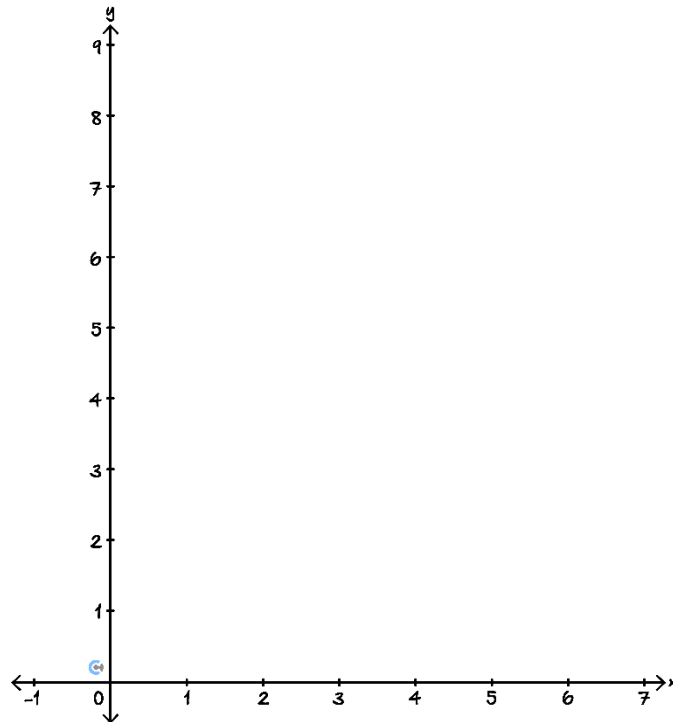
b. Solve the inequality $x + 2 > h(x)$ for $x \in \mathbb{R}$.

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c. The equation $h(x) = k$, where k is a real number, has 4 real solutions. Find the possible value(s) of k.

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Section B: Supplementary Questions

Question 17

Solve the equation $|x - 1| + 3 = |3x + 1| - 2$ for $x \in \mathbb{R}$.

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Question 18

Solve the equation $|2x - 3| = -2|x + 1| + 5$ for $x \in \mathbb{R}$.

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Question 19

Solve the inequality $\frac{1}{|x-4|} + 2 < x + 6$ for $x \in \mathbb{R}$.

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Question 20

Solve the inequality $\left|\frac{x-4}{x+1}\right| - 3 > |x + 2|$ for $x \in \mathbb{R}$.

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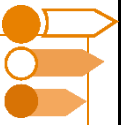
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Question 21

Solve the equation $|x - 4| = 2|x + 8|$.

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Question 22

Solve the inequality $x + 2 > \dfrac{1}{\sqrt{x^2-4x+4}}$ for $x \in \mathbb{R}$.

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Question 23

a. Perform partial fraction decomposition for $f(x) = \frac{6x}{(x-1)(x+2)}$.

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b. Express $g(x) = \frac{x^3+8}{(x+2)(x^2+4x+4)}$ in the form $\frac{A}{(x+2)^2} + \frac{B}{x+2} + C$ for real numbers A, B and C.

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Question 24

Let $f(x) = 2x^2 - 4x - 1$.

Sketch the graph of $y = |f(x)|$ on the axis below. Label all axes intercepts and turning points.

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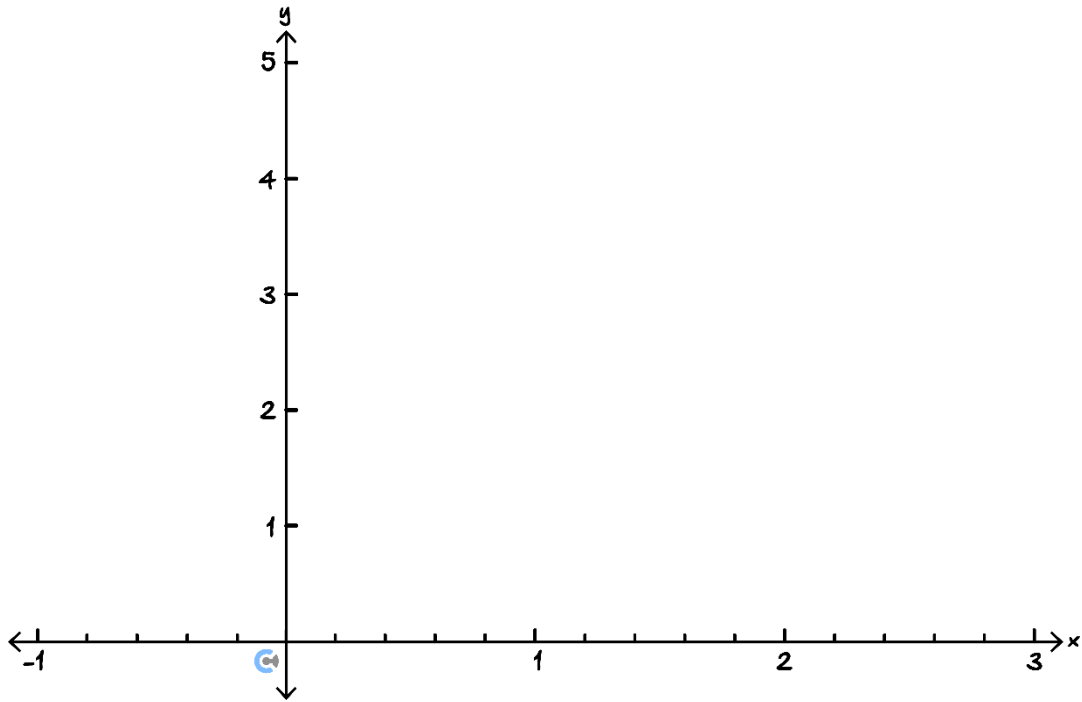
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Question 25

Consider the function f with rule $f(x) = \frac{x^2+x+4}{x+1}$.

a. Show that the rule for the function f can be written as $f(x) = x + \frac{4}{x+1}$.

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b. Solve the inequality $f(x) > x + 5$ for $x \in \mathbb{R}$.

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Sub-Section: Exam 2 Questions

Question 26

The equation $|2x - 3| = -|x + 2| + 6$, where $x \in \mathbb{R}$, has solution(s):

A. $x = -1, \frac{7}{3}$

B. $x = \frac{5}{3}$

C. $x = -1$

D. $x = 7, \frac{5}{3}$

Question 27

The graph of $y = |2x - 1| - |x - 3|$ is the same as the graph of $y = -2 - x$ for which of the following ranges of x values:

A. $x > \frac{1}{2}$

B. $x \leq \frac{1}{2}$

C. $\frac{1}{2} \leq x \leq 3$

D. $x \geq 3$

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Question 28 (1 mark)

Which one of the following, where A, B, C, and D are non-zero real numbers, is a partial fraction form for the expression?

$$\frac{x - 3}{(x^2 - 1)(x - 2)}$$

A. $\dfrac{A}{x^2-1} - \dfrac{B}{(x-2)^2}$

B. $\dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{x-2}$

C. $\dfrac{Ax+B}{x^2-1} + \dfrac{C}{x-2} + \dfrac{Dx}{x-2}$

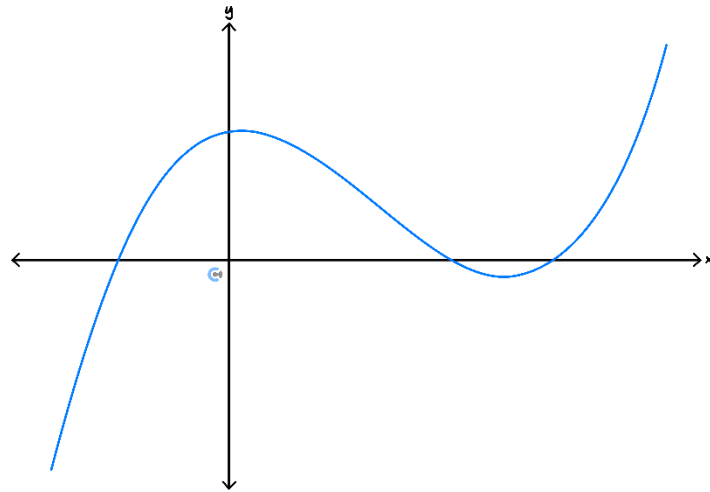**D.** $\dfrac{A}{x^2-1} + \dfrac{C}{x-2} + \dfrac{D}{x-4}$

Question 29

The equation $|x^2 + 2x - 8| = k$, where k is a real number has exactly four solutions for:

A. $k = 9$

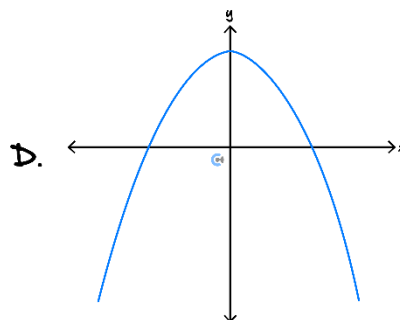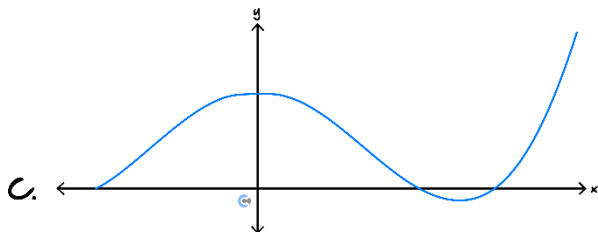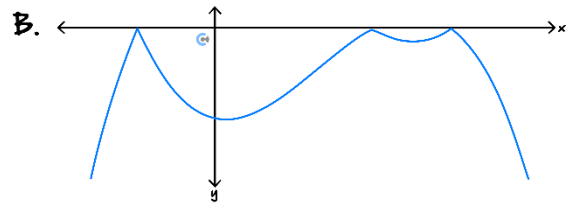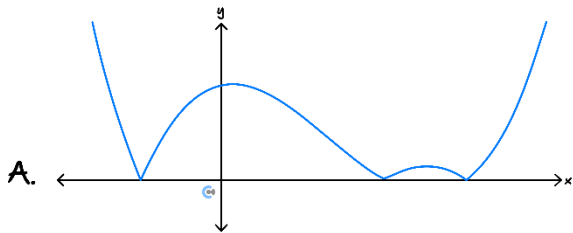**B.** $0 < k < 9$

C. $k > 9$

D. $k > 0$

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Question 30

Part of the graph of $y = f(x)$ is shown below.

The function $f(|x|)$ is best represented by
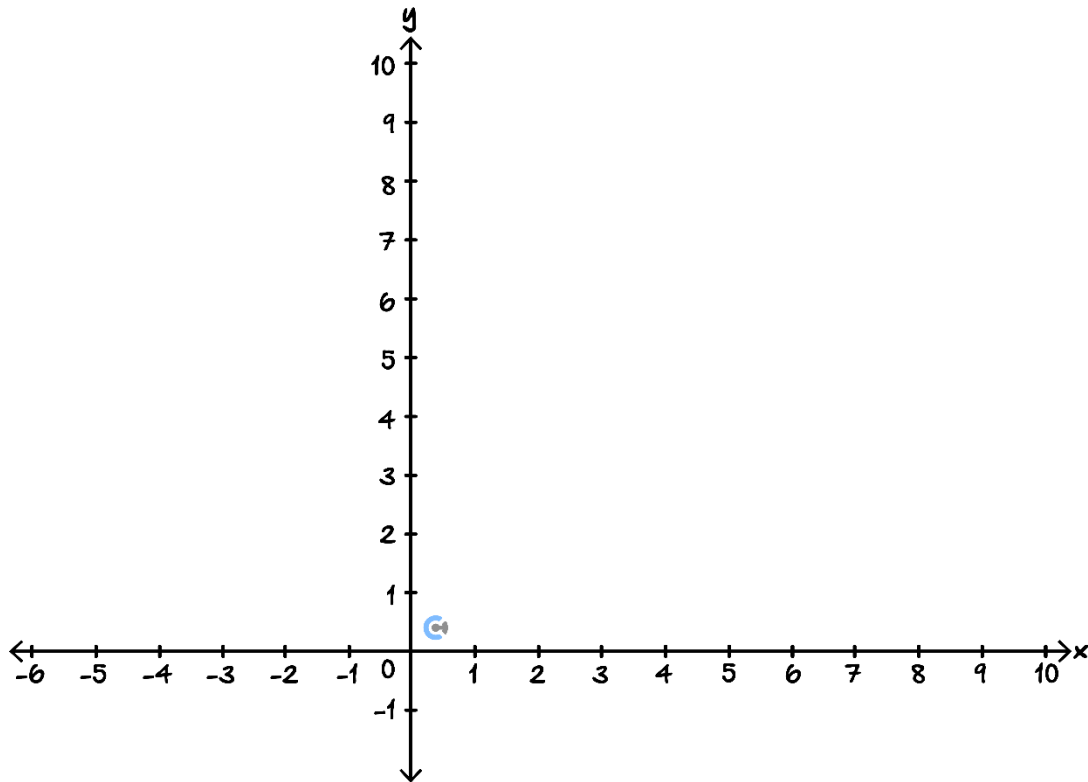
A.

B.

C.

D.

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Question 31

Consider the functions $f(x) = |x - 2| + 1$ and $g(x) = -|x - 2| + 7$

a. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Label all points of intersection, axes intercepts, and vertex points with coordinates.

b. Solve the inequality $f(x) < g(x)$.

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c.

    i.   Find the value(s) of k for which the line $y = k - x$ never intersects the graph of $y = g(x)$.

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    ii. Find the value(s) of k for which $k - x = g(x)$ has infinitely many solutions.

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d. Find the area of the region bounded between the graphs of $y = f(x)$ and $y = g(x)$

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Question 32

Consider the function $h(x) = \left| x^3 - \frac{9x^2}{2} + \frac{7x}{2} + 3 \right|$.

a. Sketch the graph of $y = h(x)$ on the axis below. Label all axes intercepts.
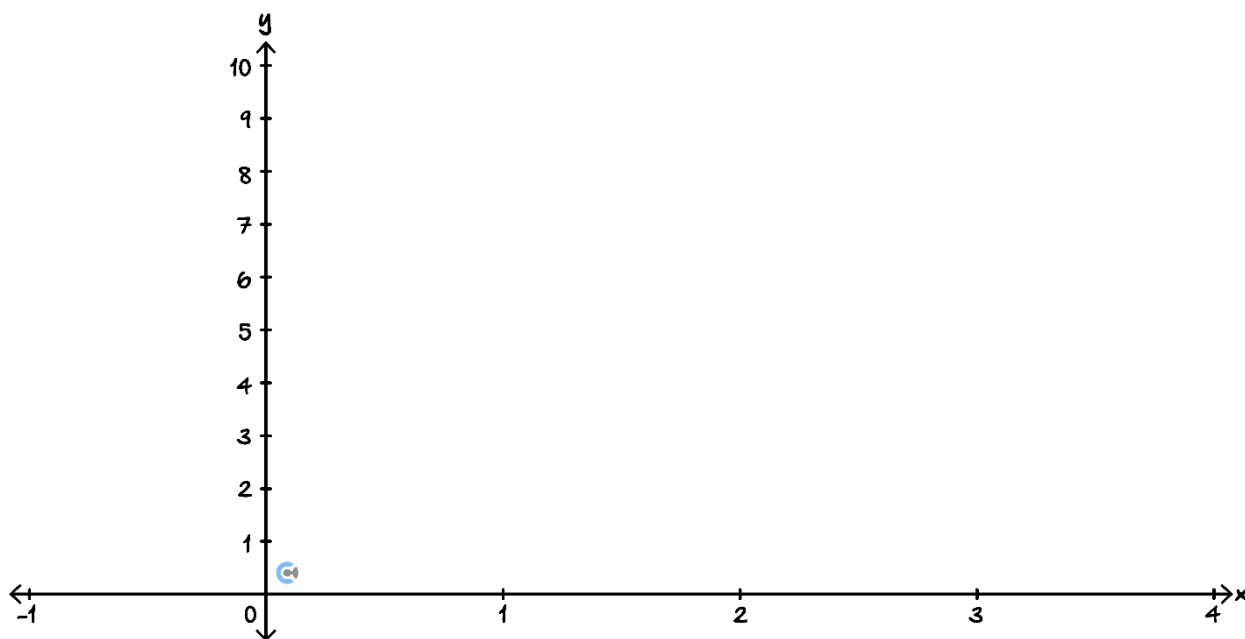
b. Solve the inequality $x + 5 > h(x)$ for $x \in \mathbb{R}$. Give your answer correct to two decimal places.

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c. The equation $h(x) = k$, where k is a real number, has 6 real solutions. Find the possible value(s) of k. Give your answer correct to three decimal places.

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