Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
Modulus & Partial Fractions [1.1]
Workbook

Outline:

Section A: Modulus Functions

What is a Modulus Function?

Modulus functions

- **Definition:**

$$f(x) = |x| = \begin{cases} x \ \ if \ x \geq 0 \\ -x \ \ if \ x < 0 \end{cases}$$

- Is a **hybrid function.**
- **Purpose:** Always return a non-negative number.
- **Range:** $[0, \infty)$.

Question 1

Evaluate the following:

a. $|-7|$

b. $|3^2 - 2(3) - 2|$

Discussion: What's the point of modulus?

Exploration: Alternative definition of modulus functions

- What happens when you square -2 and square root it?

$$-2 \rightarrow \_\_\_\_\_ \rightarrow \_\_\_\_\_$$

- So, what is squaring a number and then square rooting it the same as?

$$\sqrt{x^2} = \_\_\_\_\_$$

Alternative definition of Modulus Functions

$$\sqrt{x^2} = |x|$$

NOTE: Important not to forget the modulus in the exams!

Space for Personal Notes

Sub-Section: Solving Modulus Equations

Discussion: How do we solve for modulus equations like $|f(x)| = 2$?

Solving equations involving modulus functions

$$|f(x)| = b$$

$$f(x) = \pm b$$

- Interpretation:
  - The _____ $f(x)$ equals to b.
  - $f(x)$ can be either _____.

Question 2 Walkthrough.

Solve the following equation below.

$$|x - 3| = 2$$

Your turn!

Question 3

Solve the following equations for x:

a. $|x + 2| = 5$

b. $|3 - \sqrt{x}| = 1$

TIP: Check your solutions by substituting it back into the equation!

Sub-Section: Solving Modulus Inequalities

How far is the number 5 from 2?

Discussion: What does $|5-2|$ equal to? What does this mean?

Exploration: Solving modulus inequalities

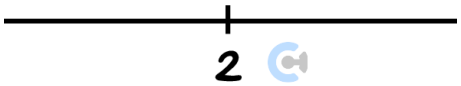
- Understanding:

$$|x-2| = \text{distance between } x \text{ and } 2$$

- Interpretation:

$|x-2| \leq 1 \rightarrow$ distance between x and 2 is [less] / [greater] than or equal to 1

- Solving: Use the number line below to solve $|x-2| \leq 1$.



Space for Personal Notes

Definition

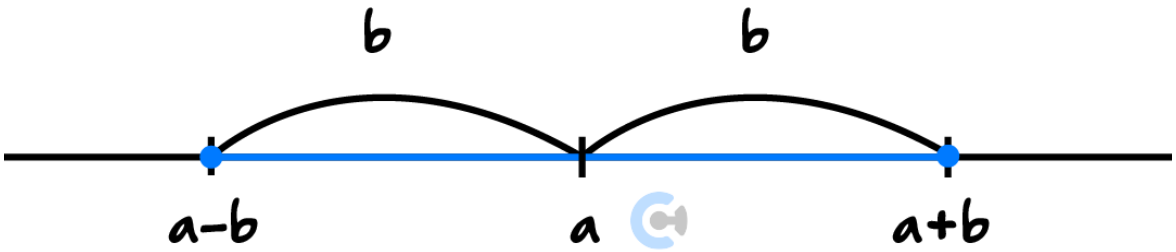
Solving modulus inequalities

$$|x - a| \leq b$$

- Interpretation:

x has a distance from 'a' that is less than or equal to 'b'

- Visualise:



$$a - b \leq x \leq a + b$$

TIP: Always sketch a number line!

Question 4 Walkthrough.

Solve the following inequality.

$$|x + 3| < 3$$

Question 5

Solve each of the following inequalities for x:

a. $|x| \geq 3$

b. $|2x - 4| - 3 \geq 5$

Key Takeaways

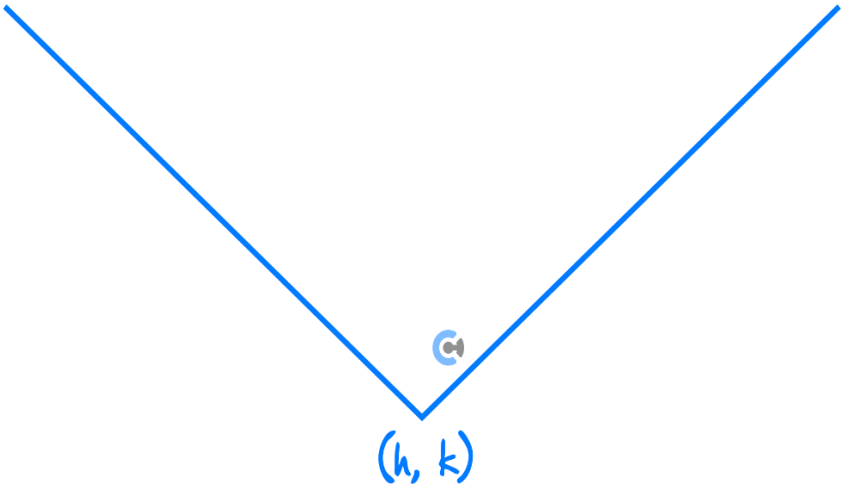
- Modulus finds a size of things.
- $|a - b|$ is a distance between a and b.
- $\sqrt{x^2} = |x|$
- For simple modulus equations, remove modulus and put $\pm$.

Section B: Graphing Modulus Functions

Sub-Section: Sketching Modulus Functions

Let's now consider the graph of Modulus Functions!

Graph of the modulus function



- General form:

$$y = a|x - h| + k$$

  - Vertex is at (h, k).

- Hybrid form:

$$y = \begin{cases} a(x-h)+k, & x \geq h \\ -a(x-h)+k, & x < h \end{cases}$$

Space for Personal Notes

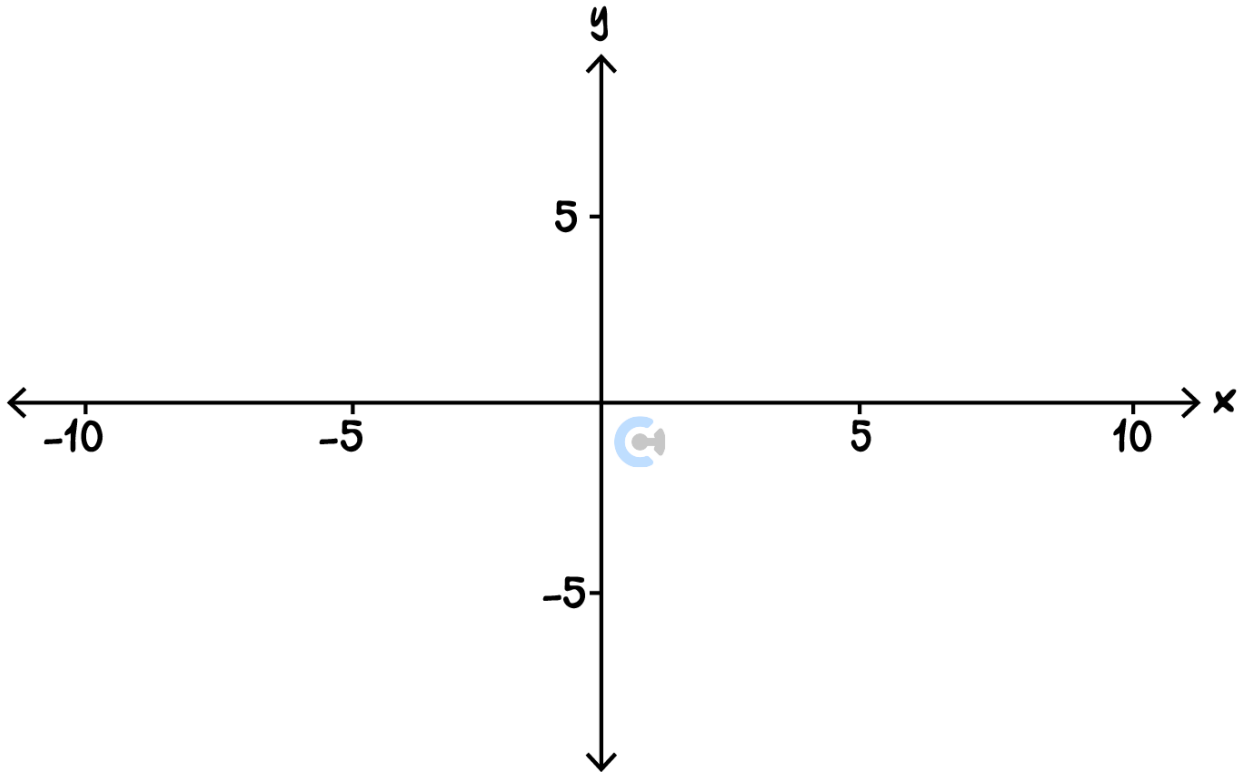
Question 6 Walkthrough.

Consider the function $f: \mathbb{R} \to \mathbb{R},\ f(x) = 2|x-1| - 2$.

a. Find the vertex.

b. Find the axes intercepts.

c. Sketch the graph of $y = f(x)$. Label all axes intercepts and the vertex.



TIP: Think of modulus functions as a "**Straightened quadratic**".

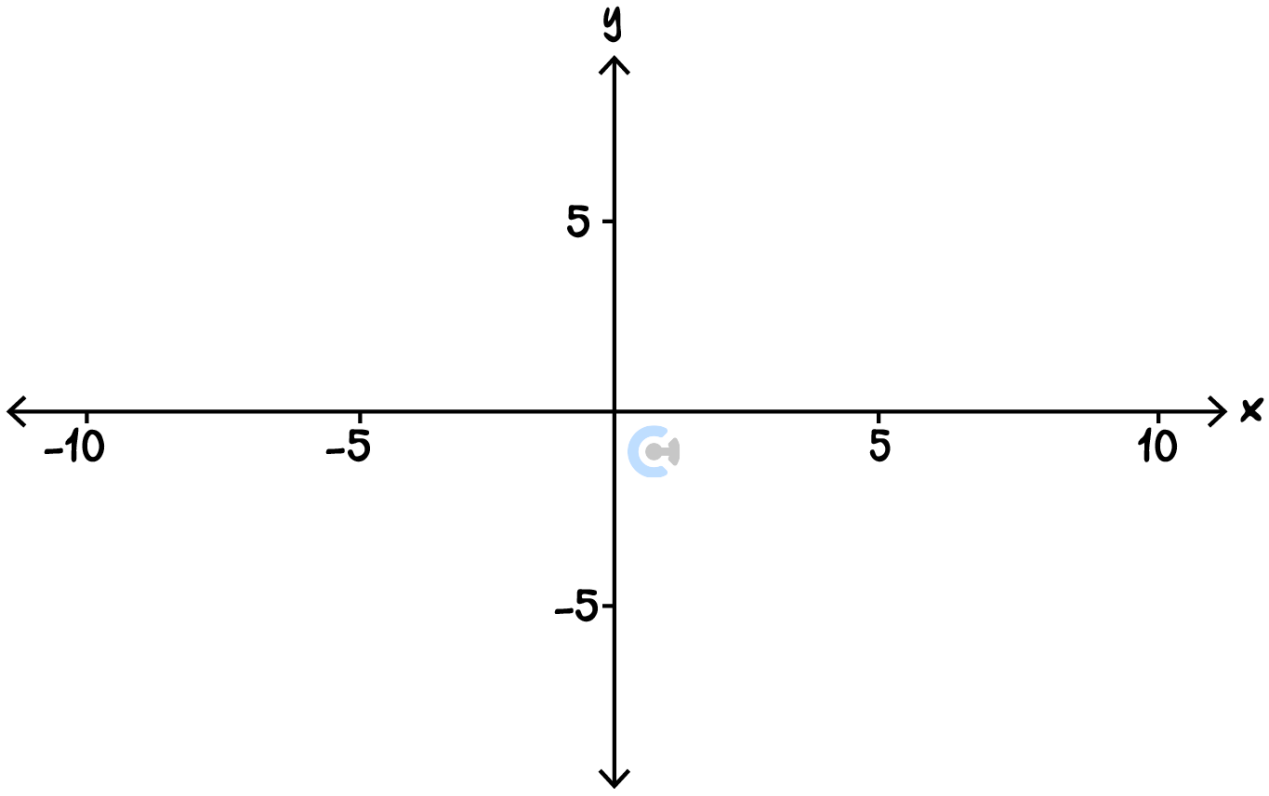
Question 7

Consider the function $g: \mathbb{R} \to \mathbb{R},\ g(x) = -3|x + 2| + 6$.

a. Find the vertex.

b. Find the axes intercept(s).

c. Sketch the graph of $y = g(x)$. Label all axes intercepts and the vertex.



Sub-Section: Graphing Composite of Modulus Functions

REMINDER: Don't forget!

- $|-6| = 6$
- $|6| = 6$

Discussion: Could $|f(x)|$ be negative? Hence what does the graph of $|f(x)|$ look like?

- The output of $y = |f(x)|$ must be: _____.

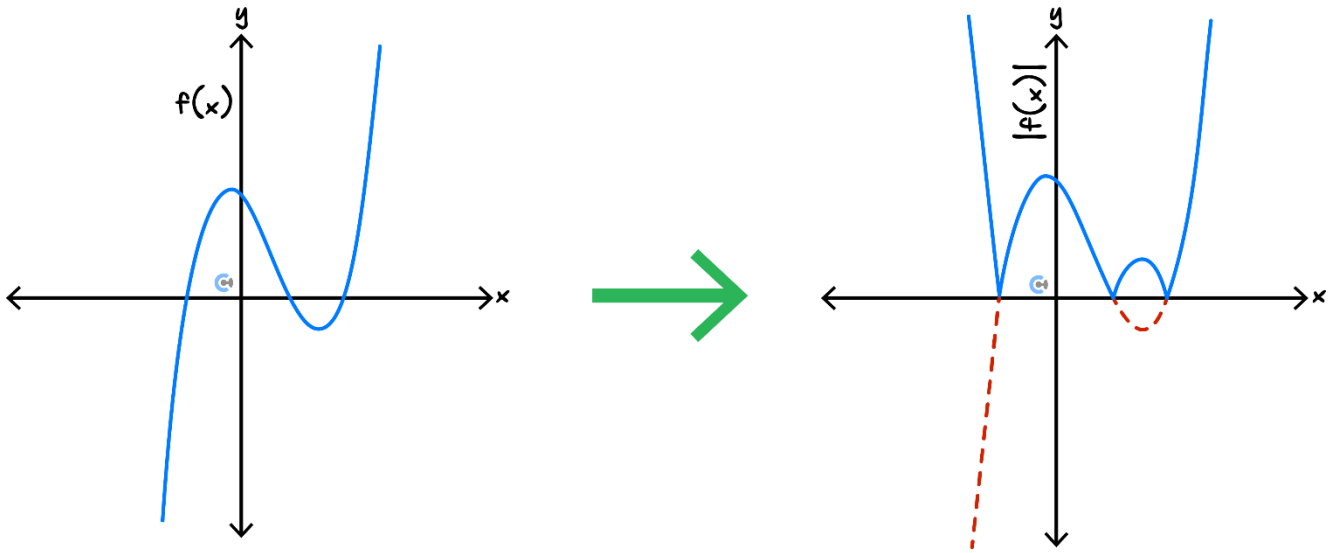
$$|f(x)| = \{$$

Graphs of composite modulus functions

- Modulus is the Outer Function.

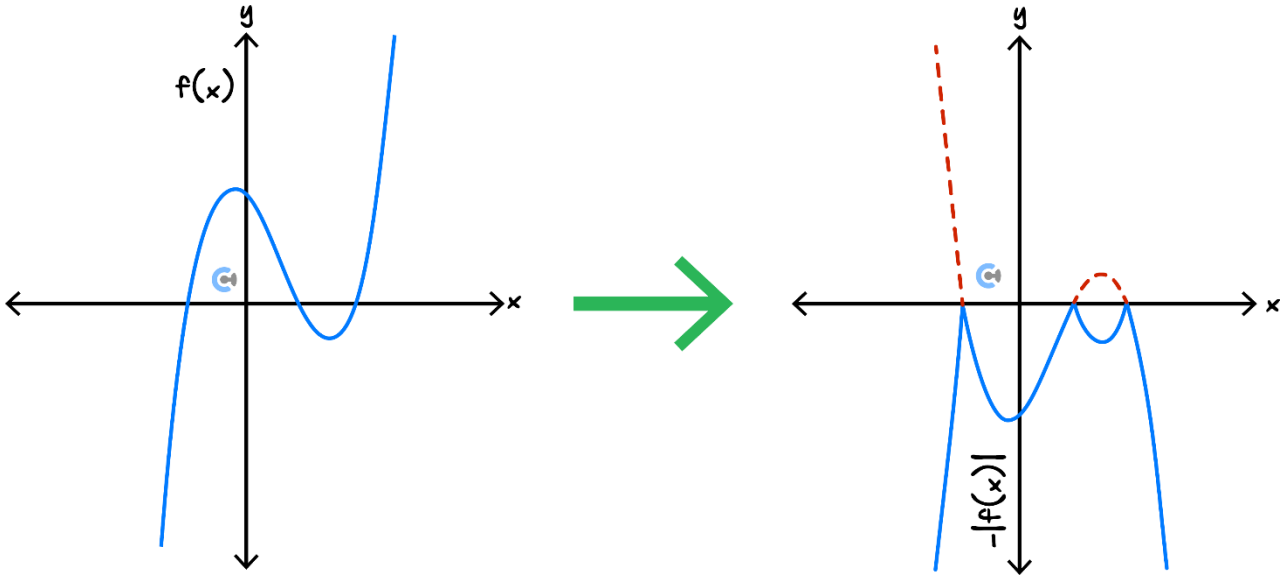
$$y = |f(x)|$$

  - All negative y values are flipped to be _____.



$y = -|f(x)|$

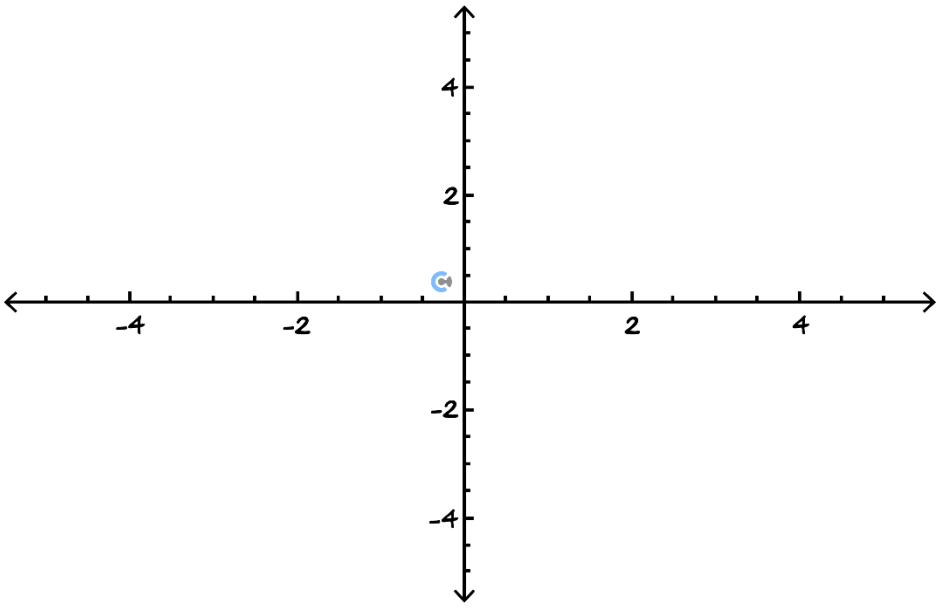
The graph of $y = |f(x)|$ has undergone a reflection in the [x-axis] / [y-axis]



Question 8 Walkthrough.

Sketch the following graph over the specified domain. Label all key intercepts.

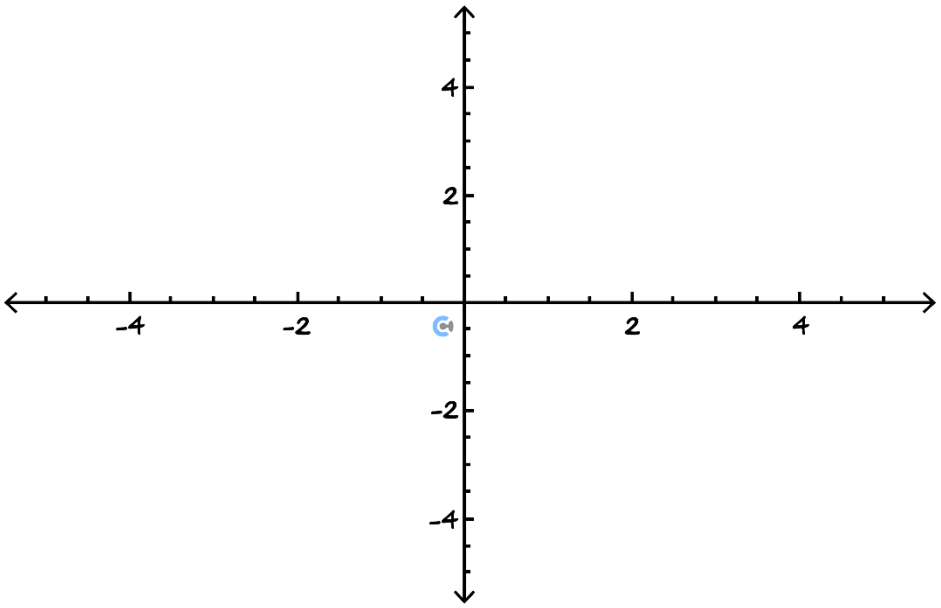
$$y = |(x-1)(x+2)|$$



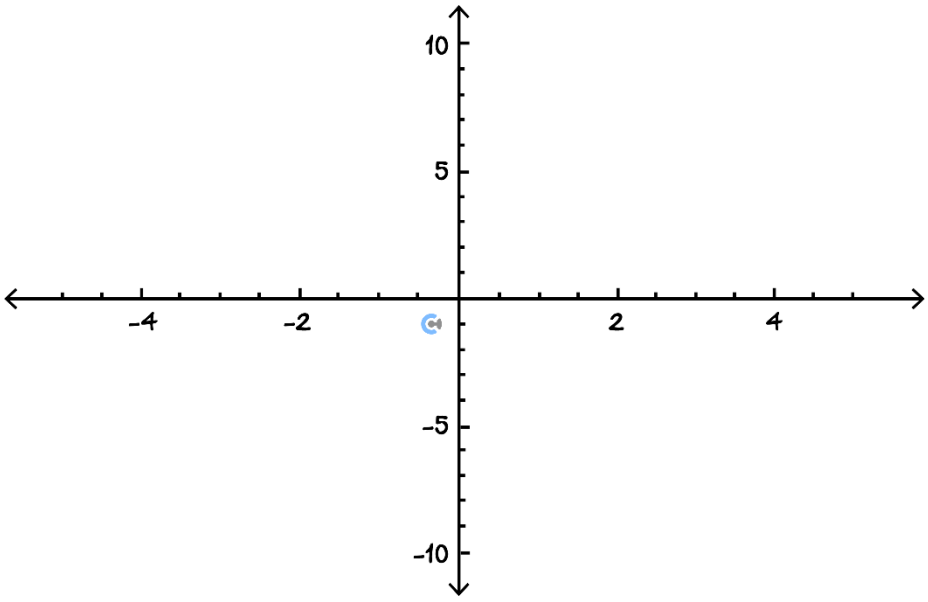
Question 9

Sketch the following graphs over the specified domain. Label all key intercepts.

a. $y = |(x-1)^2 - 1|$



b. $y = -|(x+2)(x-2)|$



Discussion: What would happen if $f(x)$ turned into $f(|x|)$?

- f will always take a [positive] / [negative] value, even if the x value is negative.

$\rightarrow$ $\rightarrow$

- At:
  - $x = -2$: _____
  - $x = 2$: _____

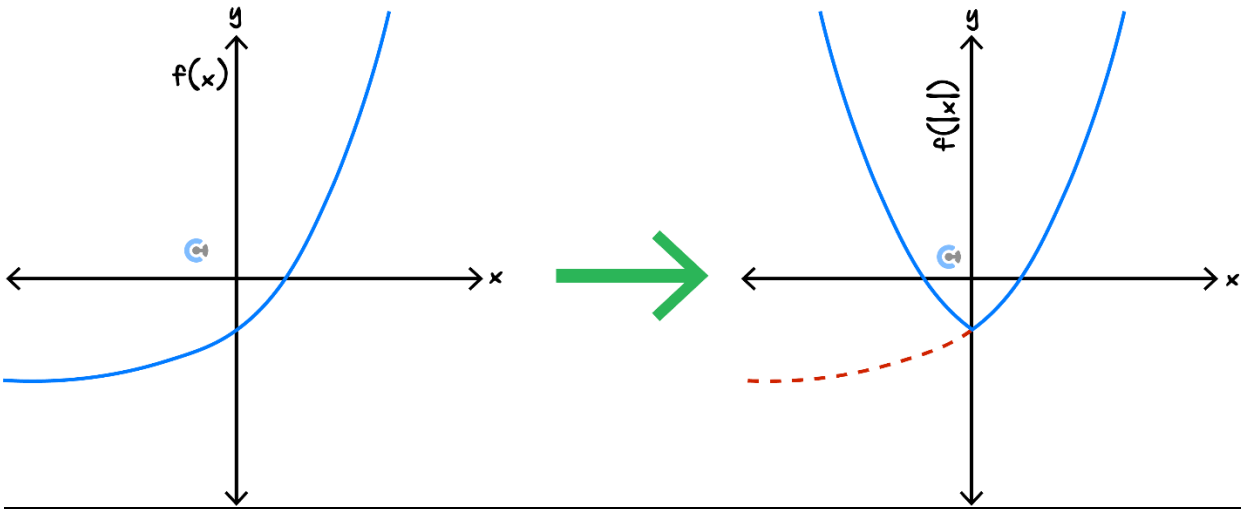
Discussion: Since $f(|-2|) = f(|2|)$, where is $f(|x|)$ symmetrical about?

Graphs of composite modulus functions

- Modulus is the Inside Function

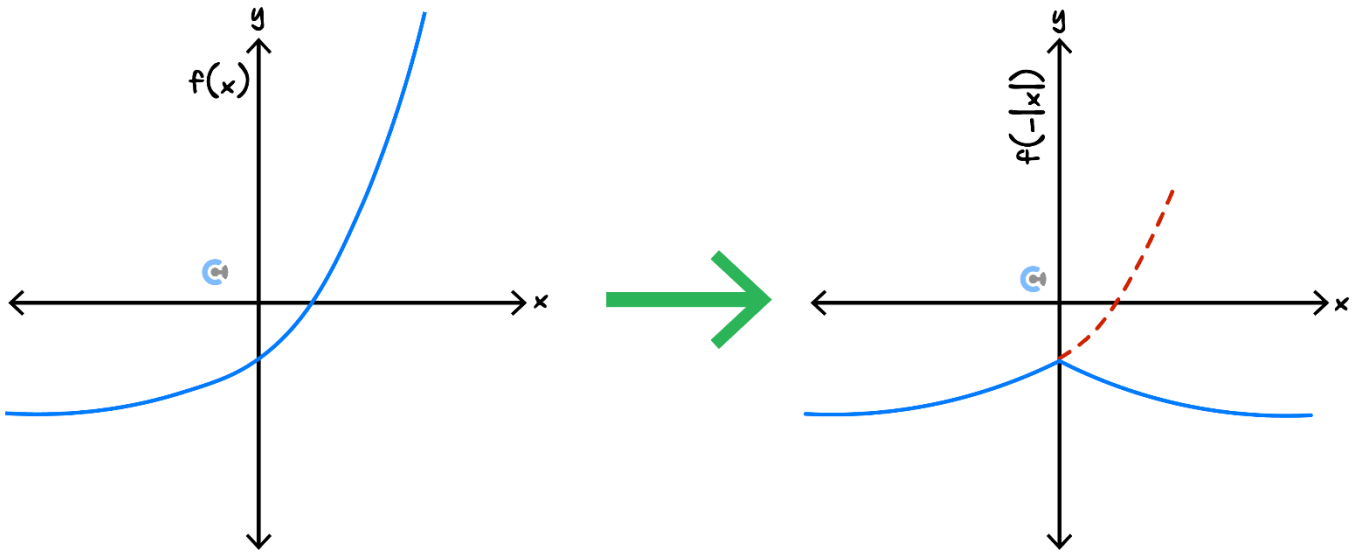
$$y = f(|x|)$$

- Take the positive side and flip it to the other side.



- Take the negative side and flip it to the other side.

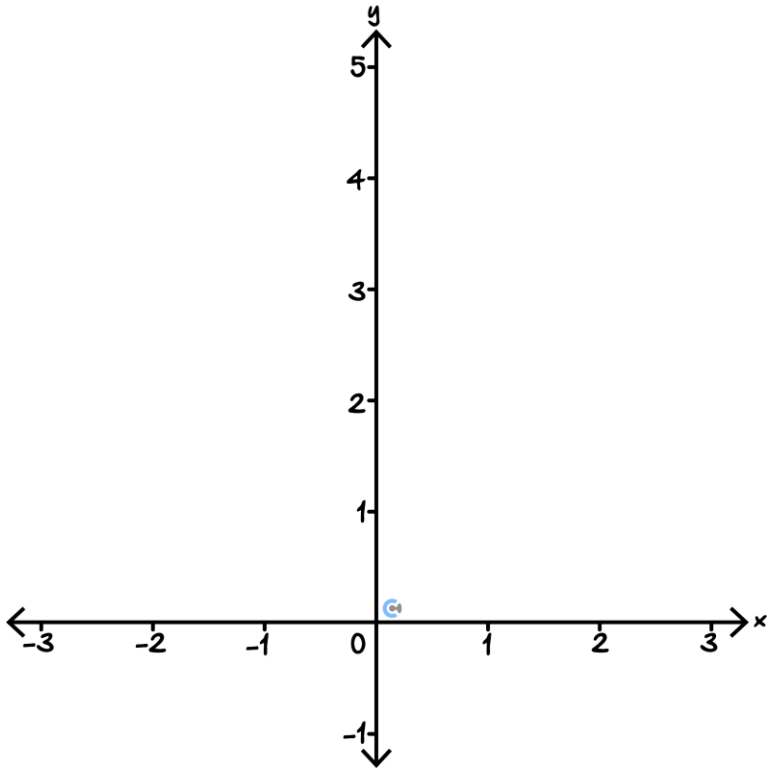
$$y = f(-|x|)$$



Question 10 Walkthrough.

Sketch the graph below.

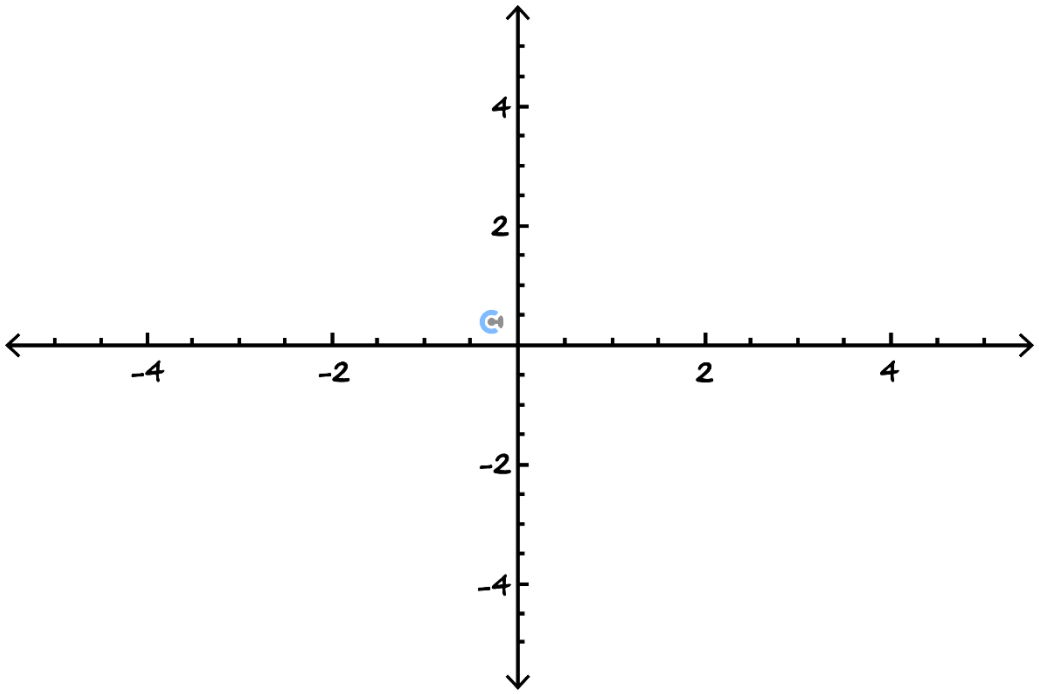
$$y = f(|x|), \text{ where } f(x) = (x+1)(x+2)$$



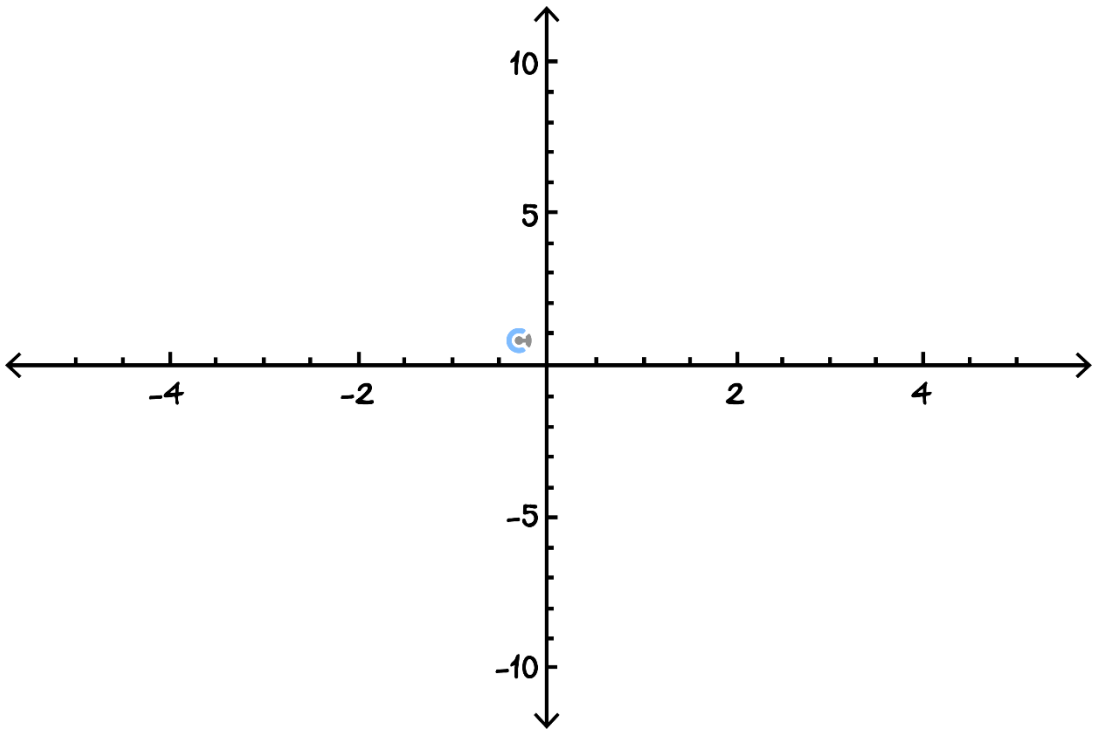
Question 11

Sketch the graph below.

a. $y = f(-|x|)$, where $f(x) = 2x - 1$.



b. $y = f(|x|)$, where $f(x) = (x + 1)^2 + 1$.



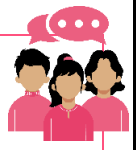
Key Takeaways

- ☑ Graph of a simple modulus graph $a|x-h|+k$ is like a straightened quadratic.
- ☑ Wrapping modulus around the function makes the y value always non negative.
- ☑ Wrapping the modulus around the x value makes the function symmetrical around the y-axis.
- ☑ $f(|x|)$ take the RHS and make it symmetrical about the y-axis.
- ☑ $f(-|x|)$ take the LHS and make it symmetrical about the y-axis.

Space for Personal Notes

Section C: Partial Fractions

Sub-Section: Introduction to Partial Fractions



Discussion: What are partial fractions?

Partial fractions

- The rules for partial fractions:

For every factor of this form in the denominator of the function...	There will be partial fraction(s) of this form:
Linear factors: $\frac{1}{(ax+b)(cx+d)}$	
Repeated linear factor: $\frac{1}{(cx+d)^n}$	
Irreducible quadratic: $\frac{1}{(ax^2+bx+c)}$	

- Must do long division before using any of the rules above.

Sub-Section: Case 1

Let's consider when we have two linear factors in the denominator!

Question 12 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{2x-1}{(x-3)(x+2)}$.

NOTE: ALWAYS factorise the denominator by its factors first!

Question 13

Perform partial fraction decomposition for the following functions.

a. $\frac{4x+2}{(x-1)(x-5)}$

b. $\frac{5x+6}{x(x-3)}$

Sub-Section: Case 2

How about repeated linear factors?

Question 14 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{3x^2-7x+4}{x(x-2)^2}$.

NOTE: When a linear factor is repeated, we repeat the splitting by that power.

Question 15

Perform partial fraction decomposition for the following functions.

a. $\frac{4}{(x-1)(x+1)^2}$

b. $\frac{8x+8}{(x-1)(x+3)^2}$

Sub-Section: Case 3

Finally, non-factorisable quadratic factors!

Question 16 Walkthrough.

Perform partial fraction decomposition for $f(x) = \frac{2x^2}{(x-1)(x^2+1)}$.

NOTE: For quadratic factors that cannot be factorised, we split it as it is.

Question 17

Perform partial fraction decomposition for the following functions.

a. $\frac{2x-2}{(x+1)(x^2+1)}$

b. $\frac{15}{(x+1)(x^2+9)}$

Key Takeaways

- ☑ Partial fractions is the process of splitting the factors in the denominator.
- ☑ Must factorise fully before doing partial fractions.
- ☑ Must do long division before doing partial fractions.
- ☑ Linear factors always have a constant at the top.
- ☑ Irreducible quadratic factors have a linear function at the top.

Contour Check

Learning Objective: [1.1.1]

Study Design

Graphs of sum, difference, product and composite functions involving functions of the types specified above (not including composite functions that result in reciprocal or quotient functions).

Key Takeaways

- Modulus finds a _____ of things.
- $|a - b|$ is a _____ between a and b.
- $\sqrt{x^2} =$ _____.
- For simple modulus equations, remove modulus and put _____.

Learning Objective: [1.1.2]

Study Design

Graphs of sum, difference, product and composite functions involving functions of the types specified above (not including composite functions that result in reciprocal or quotient functions).

Key Takeaways

- Graph of a simple modulus graph $a|x - h| + k$ is like a straightened _____ _____ .
- Wrapping modulus around the function makes the y value always non _____ .
- Wrapping the modulus around the x value makes the function symmetrica l around the _____ axis.
- $f(|x|)$ take the RHS and make it symmetrical about the _____-axis.
- $f(-|x|)$ take the _____ _____ and make it symmetrical about the y-axis .

Learning Objective: [1.1.3]

Key Takeaways

- Partial fractions is the process of _____.
- Must _____ before doing partial fractions.
- Must do _____ before doing partial fractions.
- Linear factors always have a _____ at the top.
- Irreducible quadratic factors have a _____ function at the top.

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Specialist Mathematics ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45+ raw scores and 99+ ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after school weekdays, and all day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-specialist-consult-2025