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VCE Specialist Mathematics ½

AOS 1 SAC 1 [1.0]

SAC 1 Solutions

40 Marks. 5 Minutes Reading. 40 Minutes Writing.

Section A: SAC Questions (40 Marks)

Question 1 (6 marks)

a. If $|x| = 5$, find the possible values of $|2x + 3|$. (1 mark)

13 and 7

b. Solve the equation $2|x| - 3 = 2$. (1 mark)

$x = \pm\frac{5}{2}$

c. Solve the inequality $|2x + 1| < 5$. (1 mark)

$-3 < x < 2$

d. Solve the equation $|x^2 - 4x + 1| = 4$. (3 marks)

$x=-(\sqrt{5}-2)$ or $x=1$ or $x=3$ or $x=\sqrt{5}+2$

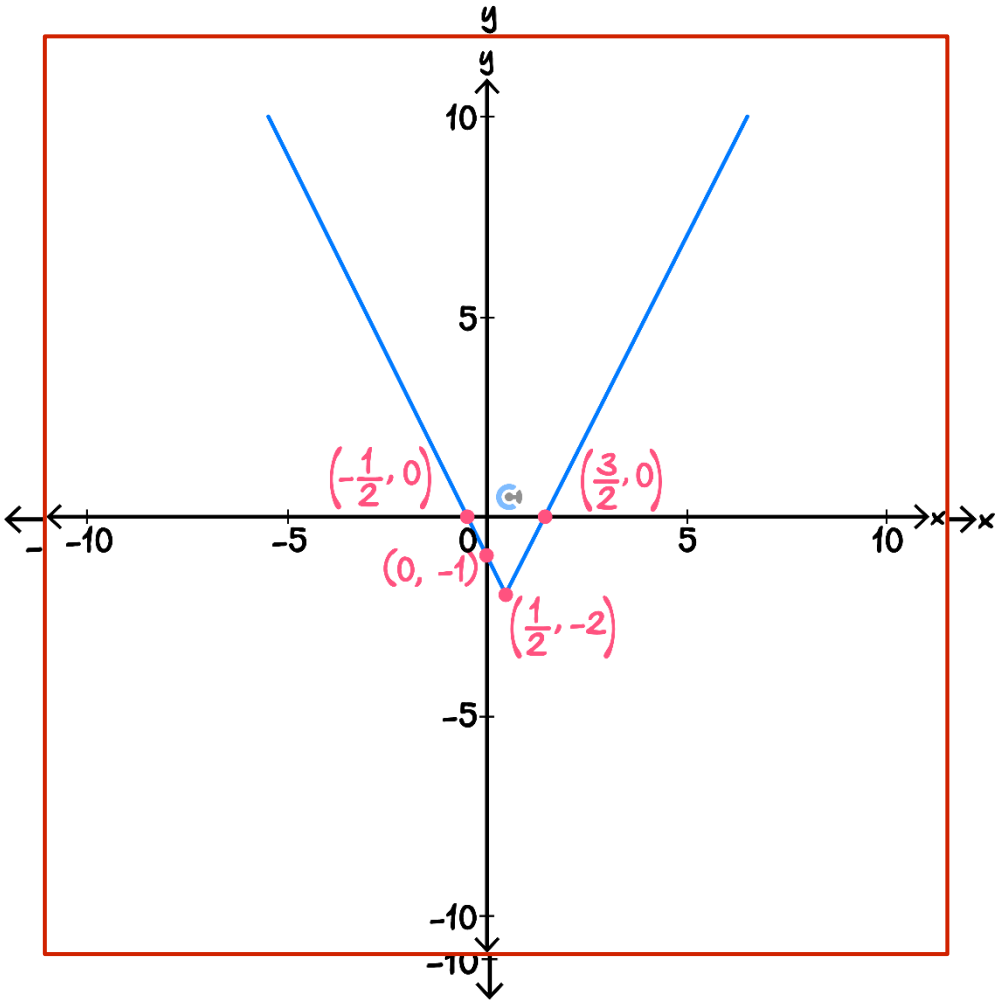
Space for Personal Notes

Question 2 (4 marks)

You are given that:

$$f(x) = |2x - 1| - 2$$

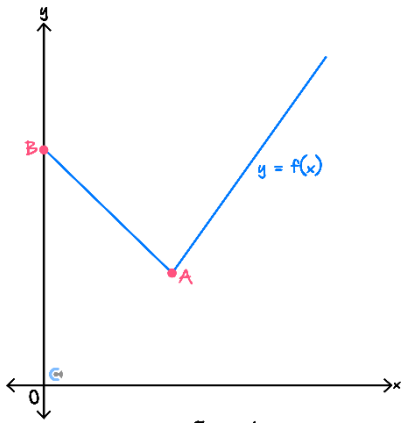
a. Sketch the graph of $f(x)$ on the set of axes below, clearly indicating the coordinates of any points where the graph of $f(x)$ meets the coordinate axes. (2 marks)



b. Using your graph, or otherwise, find the values of x for which $f(x) \geq 3$. (2 marks)

$x \geq 3$ or $x \leq -2$

Question 3 (6 marks)



The above figure shows a sketch of part of the graph of $y = f(x)$, where:

$$f(x) = a|h - x| + b$$

a. You are given that the coordinates of A are $(3,5)$ and the coordinates of B are $(0,11)$.
Find the values of a, h and b. (2 marks)

$a = 2, h = 3, b = 5$

b. Solve the equation $f(x) = \frac{1}{2}x + 30$ for x. (3 marks)

$x = -\frac{38}{5}$ or $x = \frac{62}{3}$

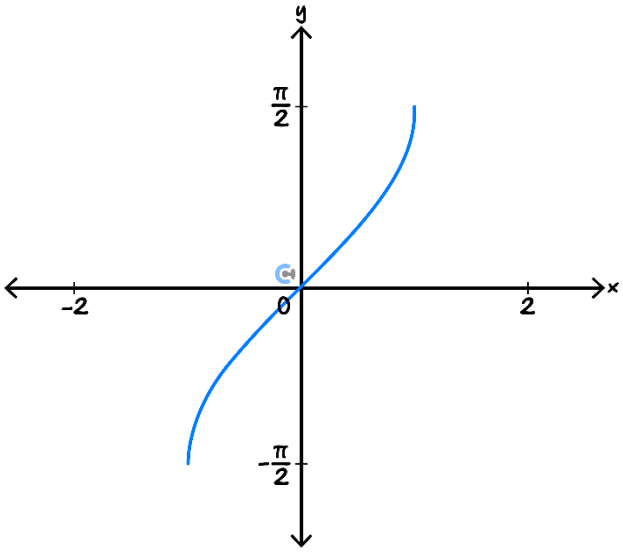
Given that the equation $f(x) = k$, where k is a constant, has two distinct roots.

c. State the set of possible values for k. (1 mark)

$k \geq 5$

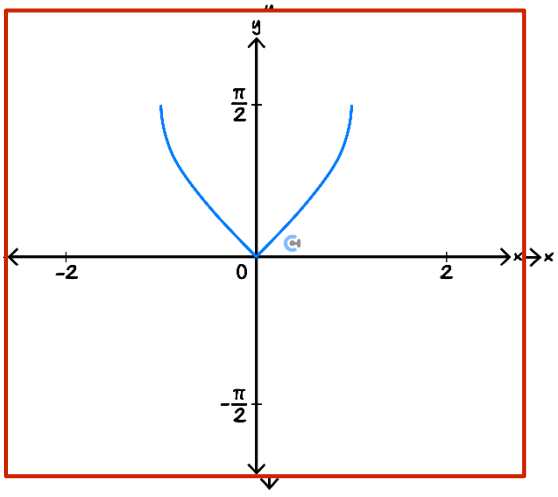
Question 4 (4 marks)

The graph of $y = f(x)$ is shown on the set of axes below:

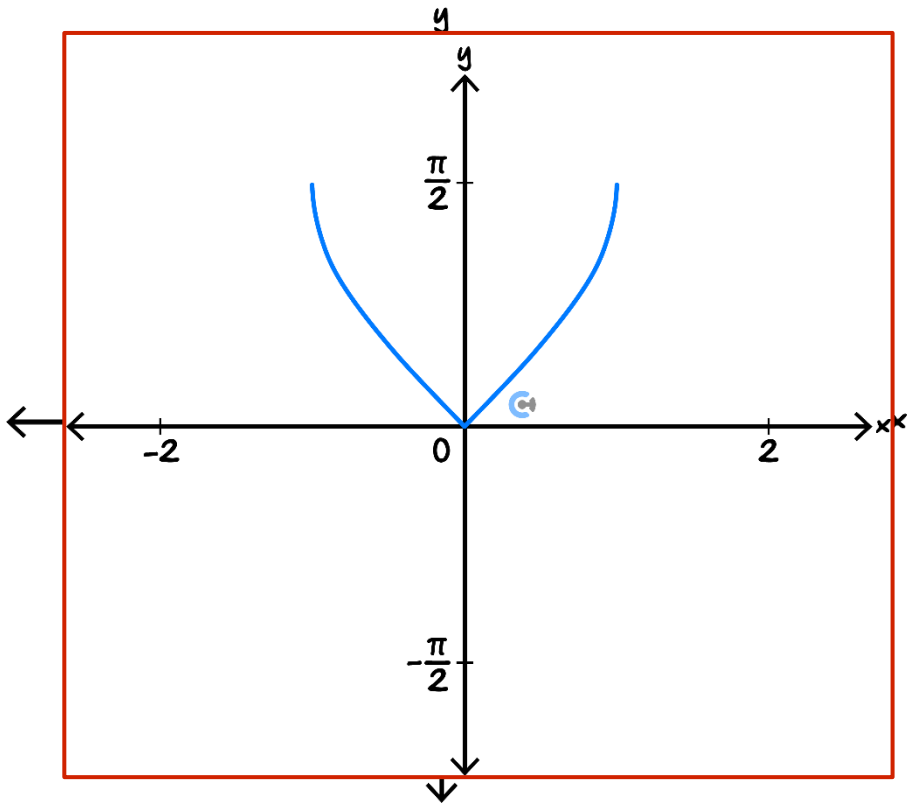


On the set of axes below, sketch the graphs of:

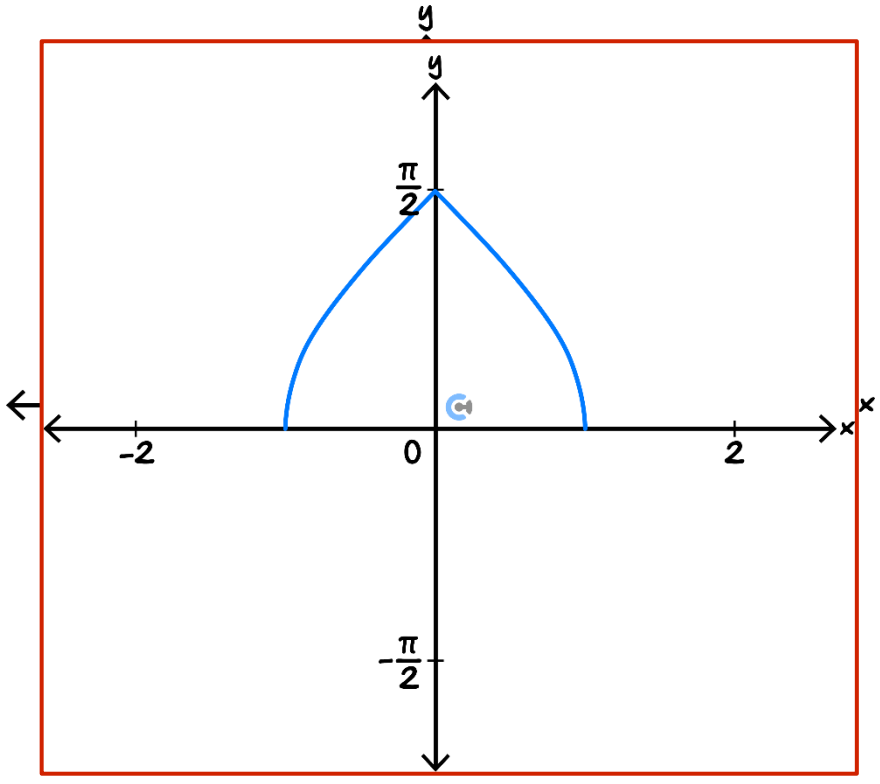
a. $y = |f(x)|$. (1 mark)



b. $y = f(|x|)$. (1 mark)



c. $y = f(-|x|) + \frac{\pi}{2}$. (2 marks)



Question 5 (3 marks)

Solve the following inequality for x:

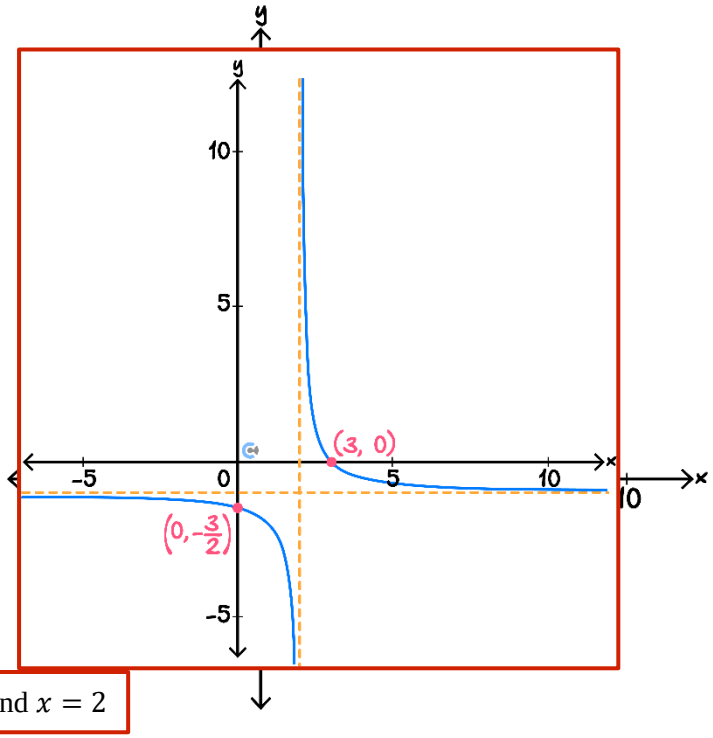
$$|2x - 1| - 1 < |x - 1|$$

$x \in (-1,1)$ or $-1 < x < 1$

Question 6 (8 marks)

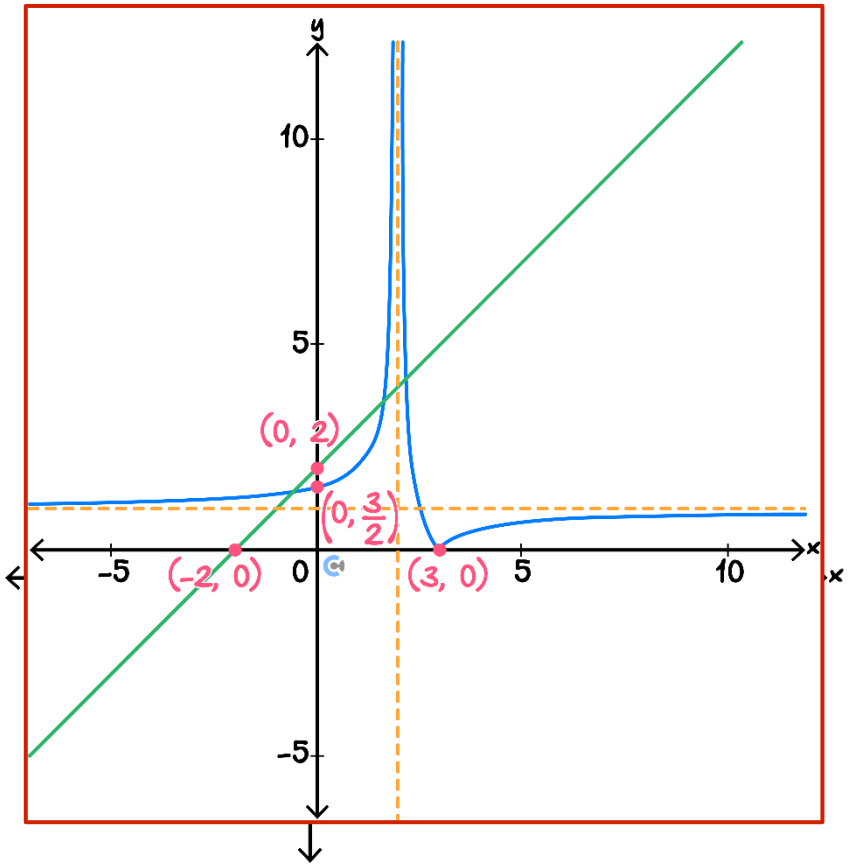
Consider the function $f : R\backslash\{2\} \to R, f(x) = \frac{1}{x-2} - 1$.

a. Sketch the graph of $y = f(x)$ on the set of axes below, labelling all asymptotes and axial intercepts. (2 marks)



Asymptote at $y = 1$ and $x = 2$

b. Hence, on the set of axes below, sketch the graph of $y = |f(x)|$, labelling all asymptotes and axial intercepts. (2 marks)



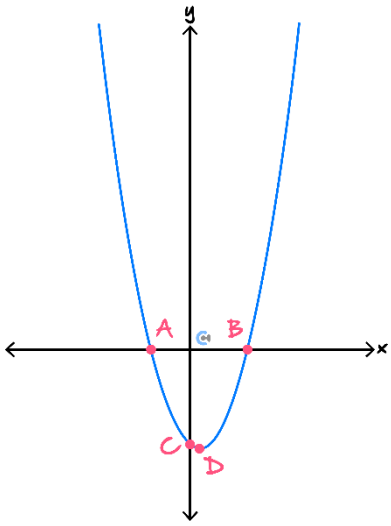
c. On the same set of axes above in **part b.**, sketch the line $y = x + 2$, labelling the axial intercepts. (1 mark)

d. Hence, solve the inequality $|f(x)| \leq x + 2$. (3 marks)

$$\frac{-(\sqrt{5}-1)}{2} \leq x \leq \frac{\sqrt{5}+1}{2} \text{ or } x \geq \frac{\sqrt{29}-1}{2}$$

Question 7 (9 marks)

The graph of $y = g(x)$, where $g(x) = x^2 - x - 6$ is shown below:



a. Write down the coordinates of A, B, C and D. (2 marks)

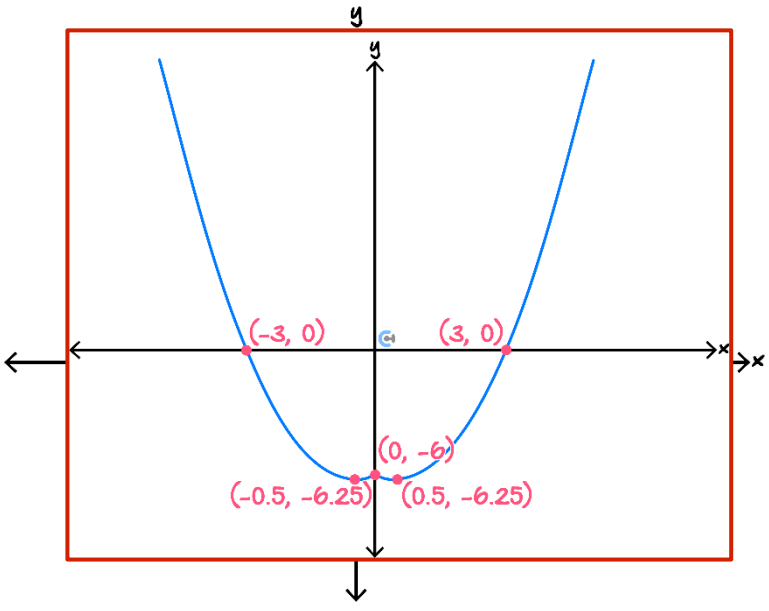
$(-2,0)$ Label: A

$(3,0)$ Label: B

$(0,-6)$ Label: C

$\left(\frac{1}{2}, -\frac{25}{4}\right)$ Label: D

b. On the set of axes below, sketch the graph of $y = g(|x|)$, labelling all turning points and axial intercepts. (3 marks)



c. Find all the values of k for which the equation $g(|x|) + k = 0$ has:

i. Three distinct solutions. (1 mark)

$k = 6$

ii. Four distinct solutions. (1 mark)

$6 < k < \frac{25}{4}$

iii. Two distinct solutions. (2 marks)

$k < 6$ or $k = \frac{25}{4}$

Space for Personal Notes