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VCE Specialist Mathematics ½

AOS 1 Revision [1.0]

SAC 2 Solutions

40 Marks. 5 Minutes Reading. 35 Minutes Writing.

Section A: SAC Questions (40 Marks)

Question 1 (6 marks)

Only one of the following four sequences is arithmetic, and only one of them is geometric.

$$a_n = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$$

$$c_n = 3, 1, \frac{1}{3}, \frac{1}{9}, \dots$$

$$b_n = 2.5, 5, 7.5, 10, \dots$$

$$d_n = 1, 3, 6, 10, \dots$$

- a. State which sequence is arithmetic and find the common difference of the sequence. (2 marks)

b_n is an arithmetic sequence (1)
and common difference is 2.5 (1).

- b. State which sequence is geometric and find the common ratio of the sequence. (2 marks)

c_n is a geometric sequence (1)
and the common ratio is $\frac{1}{3}$ (1).

- c. For the **geometric** sequence, find the **exact** value of the sixth term. Give your answer as a fraction. (2 marks)

$$c_6 = 3 \times \left(\frac{1}{3}\right)^{6-1} = 3 \times \left(\frac{1}{3}\right)^5 = \frac{1}{3^4} = \frac{1}{81} \text{ [1M, 1A]}$$

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Question 2 (3 marks)

Given a geometric sequence as the second term $t_2 = 6$ and $S_2 = 8$, find t_6 .

$$\begin{aligned} ar &= 6 \\ \frac{a(1-r^2)}{1-r} &= 8 \\ \frac{a(1-r)(1+r)}{1-r} &= 8 \Rightarrow a(1+r) = 8 \text{ [1M]} \\ &\Rightarrow a + ar = 8 \\ &\Rightarrow a + 6 = 8 \\ &\Rightarrow a = 2 \\ r &= 3 \text{ [1M]} \\ t_6 &= 2 \times (3)^5 = 486 \text{ [1A]} \end{aligned}$$

Question 3 (6 marks)

An arithmetic sequence has $t_1 = 40$ and $t_5 = 26$.

a. Find the common difference, d . (1 mark)

$$d = \frac{26 - 40}{4} = -\frac{7}{2} \text{ [1A]}$$

b. Find t_9 . (1 mark)

$$t_9 = 40 - \frac{7}{2}(8) = 12 \text{ [1A]}$$

c. Find S_9 . (2 marks)

$$\begin{aligned} S_9 &= \frac{9}{2} \left(2(40) + (9-1) \times -\frac{7}{2} \right) \text{ [1M]} \\ &= \frac{9}{2} (80 - 28) = 9(26) = 234 \text{ [1A]} \end{aligned}$$

- d. Find the smallest possible value of k , where $k \in \mathbb{N}$, such that $S_k < 0$. (2 marks)

$$S_k = \frac{k}{2} \left(2(40) + (k-1) \times -\frac{7}{2} \right)$$

$$= \frac{k}{2} \left(80 + \frac{7}{2} - \frac{7}{2}k \right) = \frac{1}{4}k(167 - 7k) \text{ [1M]}$$

$$S_k < 0 \text{ for } k < 0 \text{ or } k > \frac{167}{7}$$

$$\frac{167}{7} = 23\frac{6}{7} \text{ therefore, the smallest value is } k = 24 \text{ [1A].}$$

Question 4 (3 marks)

An arithmetic has $u_1 = 41, u_2 = 37, u_3 = 33$ and continues in this way.

- a. What is the largest value of n such that $u_n > -200$? (2 marks)

$$d = -4$$

$$t_n = 41 - 4(n-1)$$

$$\text{Solve: } 45 - 4n > -200$$

$$4n < 245$$

$$n < 61.25$$

$$\text{Largest value of } n \text{ is } 61.$$

- b. For the value of n found in **part a.**, calculate $u_{n+7} - u_n$. (1 mark)

$$7(d) = -28 \text{ [1A]}$$

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Question 5 (7 marks)

The first three terms of a geometric sequence are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

- a. Show that $k^2 - 7k - 60 = 0$. (2 marks)

$$\begin{aligned}\frac{2k - 15}{k} &= \frac{k}{k + 4} (= r) [1M] \\ (2k - 15)(k + 4) &= k^2 \\ 2k^2 - 7k - 60 &= k^2 \\ k^2 - 7k - 60 &= 0, \text{ as required } [1A].\end{aligned}$$

- b. Hence, solve for the value of k . (1 mark)

$$(k - 12)(k + 5) = 0, k = 12 \text{ as } k > 0 [1A].$$

- c. Find the common ratio of this geometric sequence. (1 mark)

$$r = \frac{k}{k + 4} = \frac{12}{16} = \frac{3}{4} [1A]$$

- d. The sum of the first four terms in the geometric series can be written as a fully reduced fraction of the form $\frac{p}{q}$ where $p, q \in \mathbb{N}$. State the value of p and the value of q . (2 marks)

$$\begin{aligned}S_4 &= \frac{16 \left(1 - \left(\frac{3}{4} \right)^4 \right)}{1 - \frac{3}{4}} = 64 \times \left(1 - \frac{81}{256} \right) = 64 \left(\frac{175}{256} \right) = \frac{175}{4} [1M] \\ p &= 175, q = 4 [1A]\end{aligned}$$

- e. Find the sum to infinity of this series. (1 mark)

$$S_{\infty} = \frac{a}{1 - r} = \frac{16}{1 - \frac{3}{4}} = 64 [1A]$$

Question 6 (12 marks)

Let $\{u_1, u_2, \dots, u_n\}$ where $n \in \mathbb{N}$ be an arithmetic sequence with the first term equal to a and common difference equal to d .

Let another sequence $\{v_1, v_2, \dots, v_n\}$ where $n \in \mathbb{N}$ be defined such that $v_k = 2^{u_k}$, $1 \leq k \leq n$ and $k \in \mathbb{N}$.

a.

- i.** Show that $\frac{v_{n+1}}{v_n}$ is a constant. (2 marks)

$$\frac{v_{n+1}}{v_n} = \frac{2^{u_{n+1}}}{2^{u_n}} = 2^{u_{n+1}-u_n} = 2^d,$$

which is a constant, since d is a constant [1M, 1A].

- ii.** Write down the first term of the sequence $\{v_n\}$. (1 mark)

$$v_1 = 2^{u_1} = 2^a \text{ [1A]}$$

- iii.** Write down the formula for v_n in terms of a , d and n . (1 mark)

$$v_n = 2^{u_n} = 2^{a+(n-1)d} \text{ [1A]}$$

b. Let S_n be the sum of the first n terms in the sequence $\{v_n\}$.

i. Express S_n in terms of a , d and n . (2 marks)

$$S_n = \frac{v_1(1-r^n)}{1-r}$$

$$v_1 = 2^a, r = 2^d \text{ from part a. [1M]}$$

$$S_n = \frac{2^a(1-2^{d \times n})}{1-2^d} \text{ OR } \frac{2^a(2^{d \times n}-1)}{2^d-1} \text{ [1A]}$$

ii. Find the values of d for which S_∞ exists. (2 marks)

$$\text{Need } |r| < 1, \text{ so need } 2^d < 1 \text{ [1M]}$$

$$d < \log_2(1)$$

$$d < 0 \text{ [1A]}$$

You are now told that S_∞ exists.

iii. Write down S_∞ in terms of a and d . (2 marks)

$$S_n = \frac{2^a(1-2^{d \times n})}{1-2^d}$$

$$S_\infty = \lim_{n \rightarrow \infty} \left(\frac{2^a(1-2^{d \times n})}{1-2^d} \right) = \frac{2^a}{1-2^d} \text{ as } 2^{dn} \rightarrow 0$$

$$\text{If } d < 0 \text{ and } n \rightarrow \infty \text{ [1M]}$$

$$S_\infty = \frac{2^a}{1-2^d} \text{ [1A]}$$

iv. Given that $S_\infty = 2^{a+1}$, find the value of d . (2 marks)

$$\frac{2^a}{1-2^d} = 2^{a+1}$$

$$1-2^d = \frac{1}{2} \text{ [1M]}$$

$$2^d = \frac{1}{2}$$

$$d = -1 \text{ [1M]}$$

Question 7 (3 marks)

A sequence $t_1, t_2, \dots, t_n$ is given by:

$$t_{n+1} = (3 - t_n)^2 \text{ with } t_1 = 4$$

- a.** Find t_2, t_3 , and t_4 . (1 mark)

$$t_2 = 1, t_3 = 4, t_4 = 1$$

- b.** Find t_{10} . (1 mark)

$$1$$

- c.** What do you notice about this sequence? (1 mark)

Oscillating, 'even' terms = 1, 'odd' terms = 3.

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