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VCE Specialist Mathematics ½
AOS 1 Revision [1.0]
SAC 2

40 Marks. 5 Minutes Reading. 35 Minutes Writing.

Section A: SAC Questions (40 Marks)

Question 1 (6 marks)

Only one of the following four sequences is arithmetic, and only one of them is geometric.

$$a_n = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots \qquad c_n = 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$$

$$b_n = 2.5, 5, 7.5, 10, \ldots \qquad d_n = 1, 3, 6, 10, \ldots$$

a. State which sequence is arithmetic and find the common difference of the sequence. (2 marks)

b. State which sequence is geometric and find the common ratio of the sequence. (2 marks)

c. For the **geometric** sequence, find the **exact** value of the sixth term. Give your answer as a fraction. (2 marks)

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Question 2 (3 marks)

Given a geometric sequence as the second term $t_2 = 6$ and $S_2 = 8$, find t_6.

Question 3 (6 marks)

An arithmetic sequence has $t_1 = 40$ and $t_5 = 26$.

a. Find the common difference, d. (1 mark)

b. Find t_9. (1 mark)

c. Find S_9. (2 marks)

d. Find the smallest possible value of k, where $k \in N$, such that $S_k < 0$. (2 marks)

Question 4 (3 marks)

An arithmetic has $u_1 = 41, u_2 = 37, u_3 = 33$ and continues in this way.

a. What is the largest value of n such that $u_n > -200$? (2 marks)

b. For the value of n found in **part a.**, calculate $u_{n+7} - u_n$. (1 mark)

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Question 5 (7 marks)

The first three terms of a geometric sequence are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

a. Show that $k^2 - 7k - 60 = 0$. (2 marks)

b. Hence, solve for the value of k. (1 mark)

c. Find the common ratio of this geometric sequence. (1 mark)

d. The sum of the first four terms in the geometric series can be written as a fully reduced fraction of the form $\frac{p}{q}$ where $p, q \in N$. State the value of p and the value of q. (2 marks)

e. Find the sum to infinity of this series. (1 mark)

Question 6 (12 marks)

Let $\{u_1, u_2, \ldots, u_n\}$ where $n \in N$ be an arithmetic sequence with the first term equal to a and common difference equal to d.

Let another sequence $\{v_1, v_2, \ldots, v_n\}$ where $n \in N$ be defined such that $v_k = 2^{u_k}$, $1 \le k \le n$ and $k \in N$.

a.

i. Show that $\frac{v_{n+1}}{v_n}$ is a constant. (2 marks)

ii. Write down the first term of the sequence $\{v_n\}$. (1 mark)

iii. Write down the formula for v_n in terms of a, d and n. (1 mark)

b. Let S_n be the sum of the first n terms in the sequence $\{v_n\}$.

i. Express S_n in terms of a, d and n. (2 marks)

ii. Find the values of d for which S_∞ exists. (2 marks)

You are now told that S_∞ exists.

iii. Write down S_∞ in terms of a and d. (2 marks)

iv. Given that $S_\infty = 2^{a+1}$, find the value of d. (2 marks)

Question 7 (3 marks)

A sequence $t_1, t_2, \dots, t_n$ is given by:

$$t_{n+1} = (3 - t_n)^2 \text{ with } t_1 = 4$$

a. Find t_2, t_3, and t_4. (1 mark)

b. Find t_{10}. (1 mark)

c. What do you notice about this sequence? (1 mark)

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