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# VCE Mathematical Methods ¾
## Discrete Random Variables II [5.2]
## Homework

## Admin Info & Homework Outline:

| | |
|---|---|
| **Student Name** | |
| **Questions You Need Help For** | |
| **Compulsory Questions** | Pg 2-Pg 16 |
| **Supplementary Questions** | Pg 17-Pg 30 |

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## Section A: Compulsory Questions

### Sub-Section [5.2.1]: Basics – Distribution of Discrete Random Variables

**Question 1**

Could the following be a valid probability mass function? Justify your answer in one clear sentence.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.3 | 0.3 | 0.4 |

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**Question 2**

A discrete random variable, $X$, takes the values shown in the table, along with its probabilities.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\Pr(X = x)$ | $4k$ | $4k$ | $2k$ | $k$ | $k$ |

Find:

**a.** The constant, $k$.

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**b.** $\Pr(X = 1 | X < 2)$

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**c.** $\Pr(X < 5 | X > 2)$

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**d.** $\Pr(X > 4 | X > 3)$

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**Question 3**

Given the probability distribution.

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $\Pr(X = x)$ | $\dfrac{1}{2k}$ | $\dfrac{k}{5}$ | $\dfrac{7}{10k}$ |

Find the value(s) of $k$.

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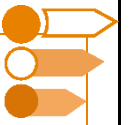
## <u>Sub-Section [5.2.2]</u>: Basics – Mean, Median and Mode

**Question 4**

Let the random variable $X$ have the following distribution:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.3 | 0.2 | 0.2 |

**a.** What is the mode of $X$?

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**b.** What is the median of $X$?

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**c.** Calculate the mean of $X$.

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**Question 5**

State True or False for each of the following, and explain briefly:

**a.** A data set can have more than one mode.

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**b.** The median is always one of the values in the data set.

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**c.** The mean is unaffected by extreme values (outliers).

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**Question 6**

The random variable $X$ has the following probability distribution:

| $x$ | $-3$ | $0$ | $3$ |
|---|---|---|---|
| $\Pr(X = x)$ | $2(0.1 - p)$ | $5p$ | $p - 0.2$ |

Find the mean of $X$.

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## Sub-Section [5.2.3]: Basics – Measures of Spread

**Question 7**

Let random variable $X$ represent the number of coding tasks completed by a student in one week. Its probability distribution is as follows:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $a$ | 0.3 | 0.4 | $b$ |

➤ The mean of $X$ is 1.8.

**a.** Using the total probability rule and the formula for the expected value, form a system of equations to solve for $a$ and $b$.

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**b.** Find variance and standard deviation of $X$.

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Now, consider another student's performance, represented by random variable $Y$, with the following distribution:

| $y$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(Y = y)$ | 0.1 | 0.4 | 0.3 | 0.2 |

**c.** Compare the spread of $X$ and $Y$.

Which student demonstrates more consistent task completion? Justify using standard deviations.

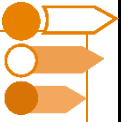_____

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## Sub-Section [5.2.4]: Basics – Linear Transformations of Random Variables

**Question 8**

If $\text{Var}(X) = 16, \text{E}[X] = 7$, find:

a. $\text{Var}(3X)$

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b. $\text{Var}(2 - X)$

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c. $\text{Sd}(2X)$

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d. $\text{E}(5X - 3)$

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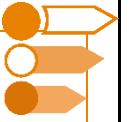
## Sub-Section: Problem Solving

**Question 9 Tech-Active.**

A company is trialling a new performance-based bonus scheme for its sales interns. The random variable $X$ represents the number of successful product pitches an intern makes in one day (from 0 to 3).

The probability distribution is:

| $x$ | 0 | 1 | 2 | 3 |
|------|------|------|------|------|
| $p(x)$ | 0.12 | 0.36 | 0.38 | 0.14 |

The intern's daily bonus (in points), denoted by $S$, is calculated using a non-linear rule:

$$S = 3X - X^2 + 2$$

This formula was designed to reward moderate success but discourage aggressive over-selling, which may result in poor customer experience.

**a.** Calculate the expected value of $X$.

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**b.** Calculate the variance and standard deviation of $X$, correct to 4 decimal places.

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**c.** Find median and mode of $X$.

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**d.** Use the transformation $S = 3X - X^2 + 2$ to compute the probability distribution of $S$.

| $s$ | | | | |
|---|---|---|---|---|
| $p(s)$ | | | | |

**e.** Calculate the expected value $E(S)$ using $E(S) = E[3X - X^2 + 2]$.

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**f.** Calculate the variance and standard deviation of $S$.

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**g.** Interpret your result: Does this bonus scheme reward interns fairly, or does it disproportionately reward or punish certain performance levels?

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**h.** Given that an intern scored at least 1 sale, what is the probability that they earned the maximum bonus?

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**Question 10**

A discrete random variable, $X$, takes the values shown in the table, along with its probabilities.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0 | $6k^2 - 3$ | $2 + 2k$ | $2k^2 - 1$ |

Find:

**a.** The constant, $k$.

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**b.** E$(X)$, the expected value of $X$.

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**c.** $\mathrm{Var}(X)$, the variance of $X$.

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**Question 11**

Let $X$ be a discrete random variable representing the number of successful experiments a student completes in a lab session. The distribution is:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $p(x)$ | $a$ | $b$ | $c$ | 0.25 | 0.15 |

➤ The distribution is valid (i.e. total probability is 1).

➤ The expected number of successes is $\mathrm{E}(X) = 2.05$.

➤ The variance of $X$ is 1.5475.

**a.** Find the relationship between $a, b$ and $c$ using the total probability rule.

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**b.** Find the values of $a, b$ and $c$.

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Define a new bonus function: $S = 5X - X^2$, representing a score system where extra success starts to penalise accuracy.

**c.** Compute the new distribution of $S$.

| $s$ | 0 | 4 | 6 | 6 | 4 |
|------|-----|-----|-----|------|------|
| $p(s)$ | 0.1 | 0.3 | 0.2 | 0.25 | 0.15 |

**d.** Given that a student had at least 2 successes, what is the probability they scored more than 4?

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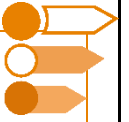**e.** Do you think the score function $S = 5X - X^2$, rewards balanced performance or peak performance? Justify your answer.

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## Sub-Section: The Tech-Free "Final Boss" [VCAA Level]

**Question 12**

A random variable $X$ has unknown mean $\mu = \mathrm{E}(X)$ and unknown variance $\sigma^2 = \mathrm{Var}(X)$.

Two statistics professors define linear transformations of $X$ as follows:

➤ Professor **A** defines $A = 3X + 4$ and tells you:

$$\mathrm{E}(A) = 25, \mathrm{Var}(A) = 36$$

➤ Professor **B** defines $B = 2X - 5$.

**a.** Find $\mathrm{E}(B)$ and $\mathrm{Var}(B)$.

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**b.** Define a new transformation $Y = X^2 - 2X + 1$.

Without knowing the full distribution of $X$, find $\mathrm{E}(Y)$.

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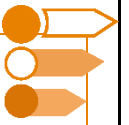
## Sub-Section: The Tech-Active "Final Boss" [VCAA Level]

**Question 13**

Let $X$ be the number of specialty smoothies machine sold by Fresh Blend Café on any given day. The probability distribution for this discrete random variable is as follows:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $Pr(X = x)$ | 0.15 | 0.45 | 0.2 | 0.2 |

Fresh Blend Café receives a profit of $275 for every smoothie machine sold. The cost of operating the café is $190 per day. The profit (in dollars) per day is a function of the random variable $X$ such that:
$Y = 275X - 190$.

**a.** Set up the probability distribution for the profit, $\$Y$, per day.

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**b.** Find the expected daily profit for Fresh Blend Café.

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**c.** What is the most likely daily profit?

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**d.** If the company increases the operating cost from $190 to $220, what is the new expected daily profit?

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**e.** Determine $\Pr(\mu - 2\sigma \leq Y \leq \mu + 2\sigma)$.

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Let the selling profit per smoothie machine be changed from $275 to an unknown value pp, keeping cost fixed at $190. Define profit as $Y_p = pX - 190$.

**f.** Find the minimum value of $p$ (rounded to the nearest dollar) such that there is less than 10% chance of making a loss.

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## Section B: Supplementary Questions

## Sub-Section: Exam 1 (Tech-Free)

**Question 14**

A discrete random variable, $X$, has the following probability distribution:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\Pr(X = x)$ | $t$ | $t$ | $2t$ | $3t$ | $2t^2$ | $4t^2$ | $4t^2 + 2t$ |

**a.** Find the value of $t$.

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**b.** Evaluate $\Pr(X \geq 6)$.

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**c.** If $\Pr(X \leq p) > 0.5$, find the minimum value of $p$.

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**Question 15**

The discrete random variable $X$ can take only the values $0, 1, 2, 3, 4, 5$. The probability distribution of $X$ is given by the following:

$$\Pr(X = 0) = \Pr(X = 1) = \Pr(X = 2) = a$$
$$\Pr(X = 3) = \Pr(X = 4) = \Pr(X = 5) = b$$
$$\Pr(X \geq 2) = 3\Pr(X < 2)$$

Where $a$ and $b$ are constants.

**a.** Determine the values of $a$ and $b$.

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**b.** Show that the expectation of $X$ is $\frac{23}{8}$.

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**c.** Determine the variance of $X$.

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**Question 16**

The random variable $X$ represents the number of custom coffee orders that Barista Serra prepares in a day. The probability distribution is as follows:

| $x$ | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| $\Pr(X = x)$ | 0.10 | 0.25 | 0.30 | 0.20 | 0.10 | 0.05 |

**a.** Calculate the mean number of custom coffee orders that Serra prepares in a day.

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Serra receives bonuses depending on how many custom coffees she serves each day.

➤ If Serra prepares fewer than 9 custom orders, she receives no bonus.

➤ If she serves between 9 and 11 customers (inclusive), she receives a **$150** bonus.

➤ If she serves 12 or more customers, she receives a **$270** bonus.

Let $Y$ be the bonus amount Serra receives each day.

**b.** Construct the probability distribution for $Y$.

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c. Find the expected value for $Y$.

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**Question 17**

A discrete random variable, $Y$, has the probability distribution shown:

| $y$ | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|------|------|------|------|------|------|
| $p(y)$ | 0.42 | 0.25 | 0.14 | 0.14 | 0.04 | 0.01 |

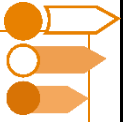If $\mu = 1.4$ and $\sigma = 1.2$, show that $\Pr(\mu - 2\sigma \leq Y \leq \mu + 2\sigma) \approx 0.95$.

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## Sub-Section: Exam 2 (Tech-Free)

**Question 18**

Consider the following table:

| $x$ | $-4$ | $-2$ | $0$ | $2$ | $4$ |
|---|---|---|---|---|---|
| $p(x)$ | $2k$ | $2k$ | $3k$ | $3k$ | $0.2$ |

For the table to represent a probability function, the value of $k$ is:

**A.** 0.08

**B.** 0.09

**C.** 0.1

**D.** 0.2

**E.** 0.5

**Question 19**

| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ |
|---|---|---|---|---|---|
| $p(x)$ | $0.05$ | $0.07$ | $0.12$ | $0.35$ | $0.41$ |

For this probability distribution, the median of $X$ is:

**A.** 1

**B.** 2

**C.** 3

**D.** 4

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**Question 20**

For this probability distribution, the mode of $X$ is:

A.  1

B.  3

C.  4

D.  5

**Question 21**

For this probability distribution, the expected value, $E(X)$, is:

A.  1.95

B.  3.86

C.  4.00

D.  4.50

**Question 22**

If $E(X) = 6.5$, then $E(2X - 1)$ is:

A.  6.5

B.  12

C.  13

D.  25

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**Question 23**

For a random variable, $X$, $E(X) = 1.45$ and $E(X^2) = 2.60$. The standard deviation of $X$ is closest to:

A.  1.15

B.  1.07

C.  5.31

D.  0.71

**Question 24**

The random variable $X$ has the following probability distribution:

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $\Pr(X = x)$ | $0.2$ | $m$ | $n$ |

If the mean of $X$ is 0.3, then:

A.  $m = 0.5, n = 0.5$

B.  $m = 0.2, n = 0.5$

C.  $m = 0.3, n = 0.5$

D.  $m = 0.5, n = 0.3$

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**Question 25**

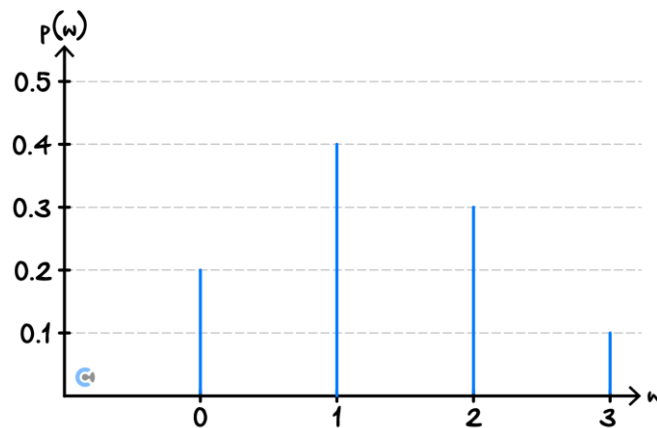The random variable $X$ has the following probability distribution:

| $x$ | $-3$ | $0$ | $3$ |
|---|---|---|---|
| $\Pr(X = x)$ | $0.5 - 2p$ | $p$ | $3p$ |

The mean of $X$ is:

**A.** $6p - 3$

**B.** $-9p$

**C.** $15p - 1.5$

**D.** $1.5 - 9p$

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**Question 26**

The graph of a probability distribution for the random variable $w$ is shown below.



The expected value of $w$ is equal to:

**A.** 0.25

**B.** 1

**C.** 1.3

**D.** 1.5

**E.** 2.5

**Question 27**

The random variable $X$ has the following probability distribution. If the mean of $X$ is 1.2, then the value of $a$ is:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $\Pr(X = x)$ | $a$ | $b$ | 0.5 |

**A.** 0

**B.** 0.2

**C.** 0.25

**D.** 0.3

**E.** 0.5

**Question 28**

Let $X$ be the number of specialty coffee machines sold by Brew Buzz on any given day. The probability distribution for this discrete random variable is as follows:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0.1 | 0.4 | 0.3 | 0.2 |

Brew Buzz makes a profit of **$250** on every coffee machine sold. The cost of operating the kiosk is **$170** per day. The profit (in dollars) per day is a function of the random variable $X$ such that:
$Y = 250X - 170$.

**a.** Set up the probability distribution for the profit, $Y$, per day.

| $Y$ | | | | |
|---|---|---|---|---|
| $\Pr(Y = y)$ | | | | |

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**b.** Find the expected daily profit for Brew Buzz.

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**c.** Determine $\Pr(\mu - \sigma \leq y \leq \mu + \sigma)$.

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**Question 29**

Two teams use different bonus formulas for their interns:

➤ Team $A$: $S_A = 3X + 2$

➤ Team $B$: $S_B = 5X - X^2$

Both teams have the same distribution for $X$:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0.1 | 0.3 | 0.4 | 0.2 |

**a.** Determine the mode and median of $X$. Explain your method clearly.

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**b.** Compute the mean and standard deviation for team *A*'s scheme without recalculating the distribution.

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**c.** Set up the probability distribution for team *B*'s scheme.

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**d.** Find the variance for team *B*'s scheme.

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**e.** Compare the spread (standard deviation) of Team *A* and Team *B*'s schemes.

Which one has greater variability? What does this say about the consistency of rewards?

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**f.** An intern is considered for promotion if their bonus on a given day is greater than or equal to the median of their team's bonus scheme.

Calculate the median of both bonus schemes and determine which team has more interns qualifying for promotion.

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**g.** Team $B$ considers changing their rule to $S'_B = 6X - X^2 - 1$ to improve fairness.

Do you expect the spread to increase or decrease compared to the current $S_B$? Explain without recalculating the full distribution.

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**Question 30**

Let $X$ be a discrete random variable representing the number of tasks a student completes in a collaborative project. The distribution is partially known:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\Pr(X = x)$ | $a$ | $b$ | 0.3 | 0.25 | $c$ |

You are told:

➤ The distribution is valid (i.e., total probability $= 1$).

➤ The expected number of tasks is $E(X) = 2.3$.

➤ The variance of $X$ is 1.21.

**a.** Use the total probability rule to write a relationship between $a$, $b$, and $c$.

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**b.** Use the expected value and variance information to form two more equations in $a$, $b$, and $c$. Then, solve for the values of $a$, $b$, and $c$.

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Define a scoring function:

$$S = 6X - X^2 + 1$$

This rule aims to reward productivity but control for overcommitment.

**c.** Compute the probability distribution of $S$ using the solved values of $a$, $b$, and $c$.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $s$ | | | | | |
| $\Pr(S = s)$ | | | | | |

A bonus of at least **8** is required for a student to earn a research opportunity.

**d.** Find the probability they earn it, given that they completed at least 2 projects.

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e. Explain how this relates to the function's goal of "rewarding productivity but controlling for overcommitment."

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