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VCE Mathematical Methods ¾
Discrete Random Variables I [5.1]
Workbook

Outline:

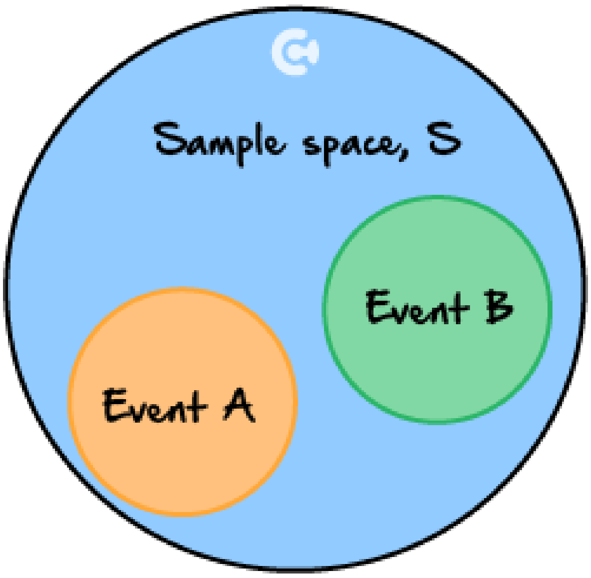
Section A: Introduction to Probability

Sub-Section: Sample Space and Uncertainty

What is probability?

Probability

- In probability, we are aiming to quantify ______________________________.



- Sample space (ε) = set of _________________________ outcomes.
- Probability of an event = proportion of ______________________ that the ________________________ takes up.
- The probability of an event must **always** take a value between ____________________, inclusive.

Space for Personal Notes

Definition

<u>Sample Space</u> (ε)

- The set of ____ in an experiment.
- For tossing two coins in a row, the sample space is:

$$\varepsilon =$$

- For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon =$$

- Total probability adds up to 1.

Try the following question!

Question 1

Using the sample space for tossing two coins in a row, find:

a. The probability of both tosses being tails.

b. The probability of tossing exactly one head and one tail.

c. The probability of tossing a tail, and **then** a head.

Sub-Section: Equally Likely Events

Discussion: How can we calculate probabilities if all events are equally likely? E.g., Each number on the dice or each side of a coin.

Calculating Probabilities for Equally Likely Outcomes

- When there is a number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(\text{success}) =$$

Have a go now!

Question 2

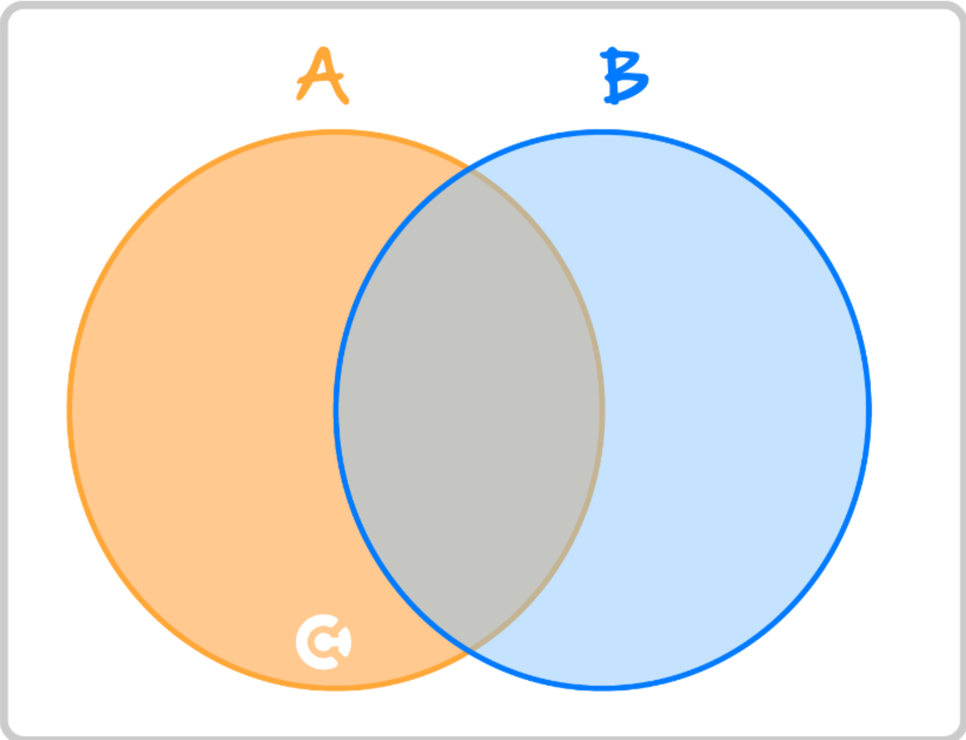
You have signed up for a trial of an experimental medicine, in which participants are randomly assigned, with 100 people in the test group (who receive the medication), and 200 in the control group (that receives a placebo).

What is the probability that you are receiving the actual medication?

Sub-Section: Union, Intersection and Complement

Definition

Union, Intersection, and Complement



- $A \cup B =$
  - ______________________________
  - ______________________________
- $A \cap B =$
  - ______________________________
  - ______________________________
- $A' =$
  - ______________________________
  - ______________________________

$$\Pr(A') =$$

Question 3

Ethan draws a card at random from a standard 52-card deck. What is the probability he draws a:

a. Spade?

b. King?

c. King of Spades?

d. A King, or a Spade?

Your turn!

Question 4

If 2 fair six-sided dice are tossed, what is the probability that they will show the same number?

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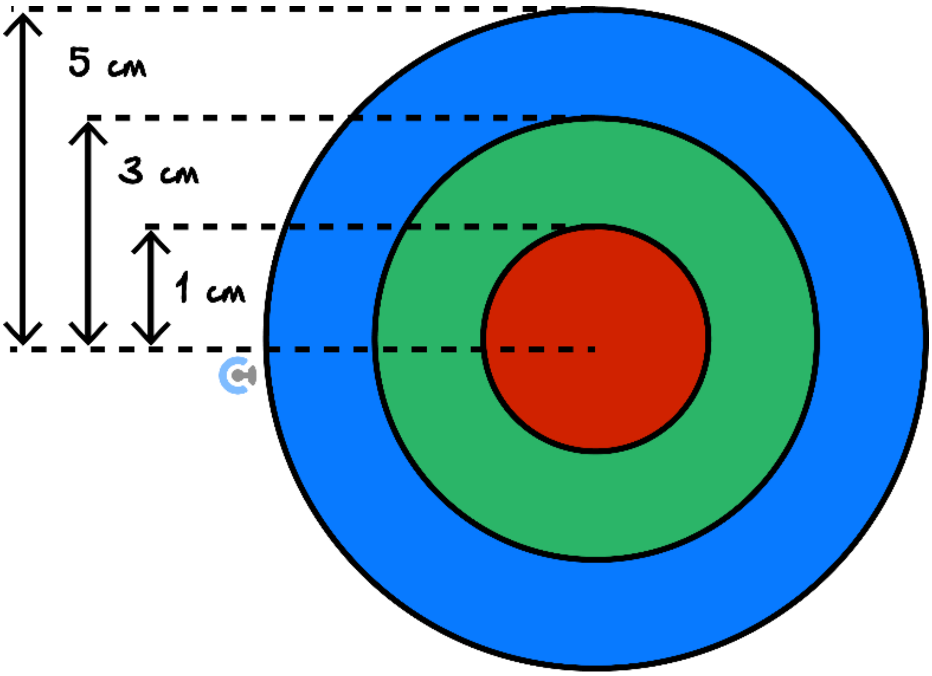
Question 5

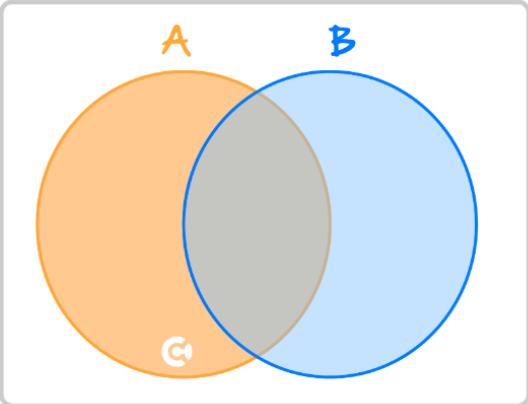
The dartboard shown is made up of three concentric circles with radii 1, 3, and 5. Assuming that a dart thrown will land randomly on the dartboard, what is the probability that it will land in the middle green region?



Section B: Methods of Finding Probability

Sub-Section: Venn Diagram and Karnaugh Tables

Venn Diagram



- Venn diagrams are useful to visualise the two events.

Is using Venn diagrams the only method?

Karnaugh Tables

- We can also represent probability problems using a Karnaugh map:

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

The rows and columns add up to the last cell value.

- Remember the **total** probability must always add to ________.

Question 6 Walkthrough.

Out of 250 consultants, 50 like Toyota as an automaker. Out of the same 250 consultants, 100 like Airbus as an aeroplane manufacturer. Exactly 50 consultants like both Toyota and Airbus.

a. How many consultants are neither like Toyota nor Airbus?

b. Find the probability that a randomly picked consultant from the group likes Toyota, but not Airbus.

c. Find the probability that a randomly picked consultant from the group does not like Toyota, but likes Airbus.

TIP: Drawing a Venn diagram can help a lot!

Your turn!

Question 7

At MHS, it is known that 40% of the cohort studies Specialist Maths, and 80% of the cohort studies Methods. If only 5% of the cohort studies, neither of these maths subjects, calculate the probability of a randomly picked MHS student studying both Specialist and Methods.

Question 8

The probability that a person is a student and goes to Contour is 0.4 and the probability of being neither is 0.1.

If the probability of being a student is 0.55, what is the probability of a person going to Contour?

Use the Karnaugh map.

Question 9 Extension.

At a school, students may enrol in Physics (P), Chemistry (C), or both. It is known that:

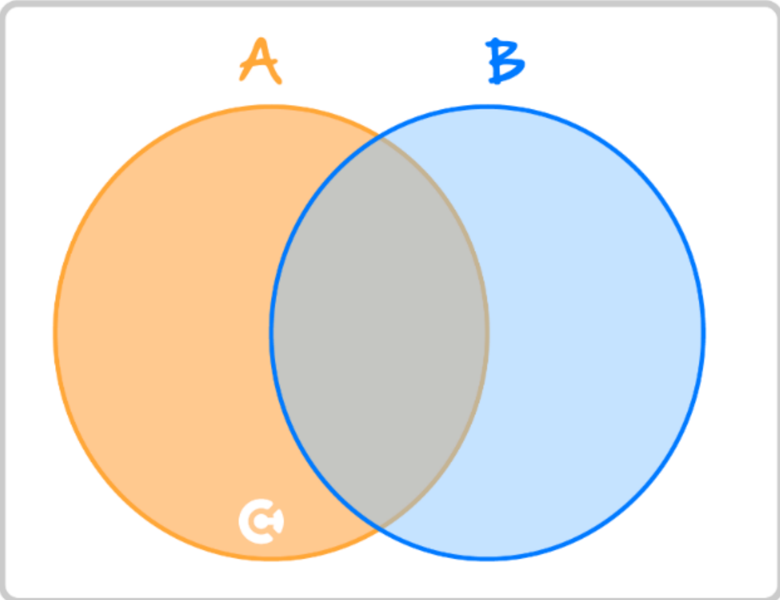
- 70% of students are enrolled in Physics.
- 65% of students are enrolled in Chemistry.
- q% of students are enrolled in neither subject.

a. Find the probability that a student studies only physics in terms of q.

b. Find the range of values that q can take.

Sub-Section: Addition Rule

The Addition Rule



- When we add the probabilities of A and B, we count the outcomes contained in $A \cap B$ **twice.**
- So, we must subtract one of them to get the probability of $A \cup B$.

$$\mathbf{Pr}(\boldsymbol{A} \cup \boldsymbol{B}) =$$

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Question 10 Walkthrough.

If A and B are events such that $\Pr(A) = \frac{1}{5}, \Pr(B) = \frac{1}{3}, \Pr(A \cap B) = \frac{1}{15}$, find:

a. $\Pr(A \cup B)$.

b. $\Pr(\text{neither } A \text{ nor } B)$.

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Your turn!

Question 11

A and B are events such that $\Pr(A) = 0.35$, $\Pr(B) = 0.55$ and $\Pr(A \cap B) = 0.20$.

Determine:

a. $\Pr(A')$.

b. $\Pr(B')$.

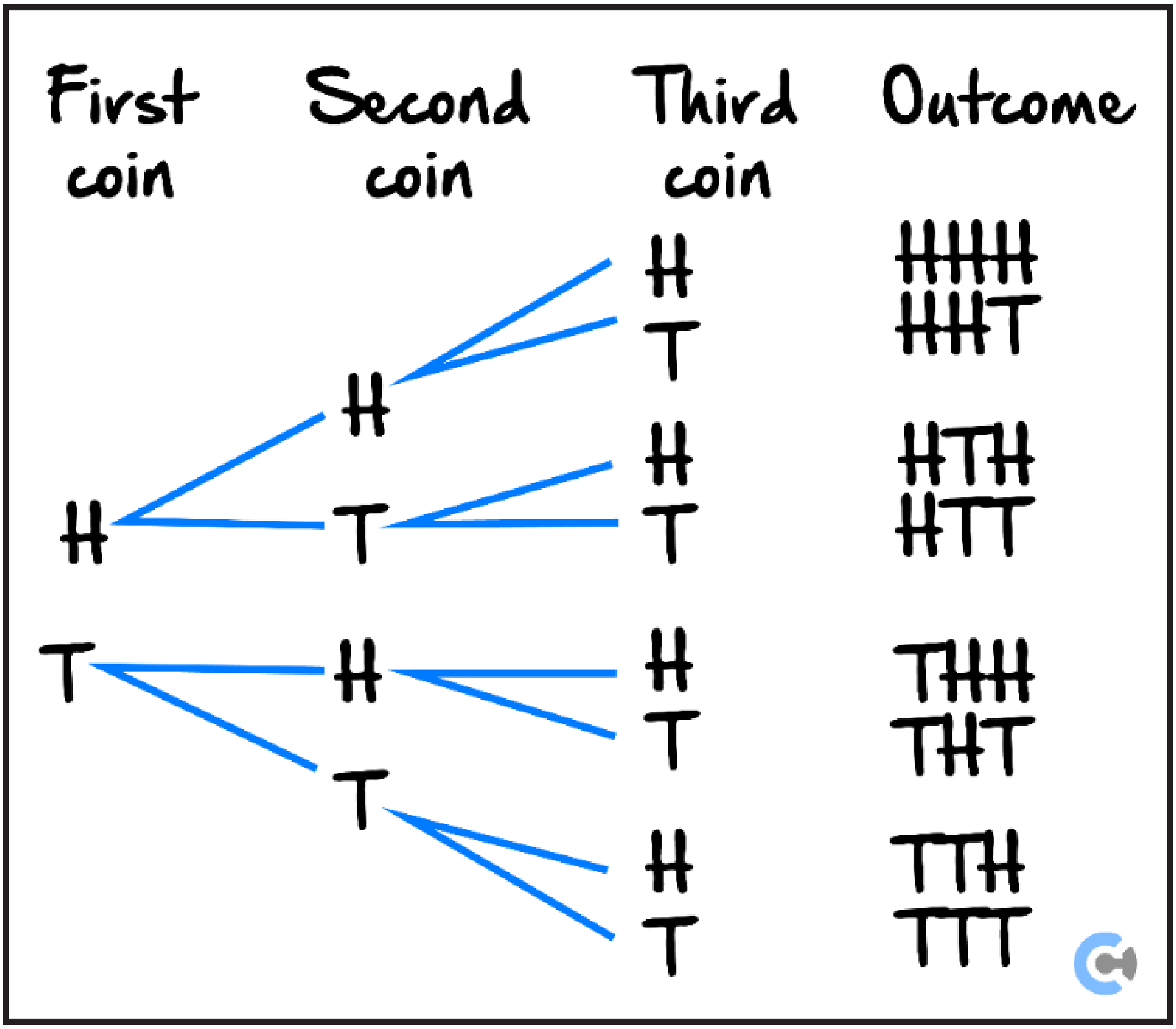
c. $\Pr(A \cup B)$.

Sub-Section: Tree Diagrams

What's another way to find probability?

Tree diagrams

- Useful for ____.
- To calculate the probability of a sequence, we ______________ **the probabilities along the relevant branches.**
- For instance, the following tree diagram shows the outcomes of three successive coin tosses.



Question 12 Walkthrough.

Anuk is tossing a biased coin that has a 60% chance of getting tails. He decides to toss it 3 times.

a. Construct a tree diagram for the three coin toss events.

b. Find the probability that Anuk tosses all heads.

c. Find the probability that Anuk will not have two heads or two tails in a row.

TIP: Even if the question doesn't say to draw a tree diagram, it's always useful to draw one!

Your turn!

Question 13

A box contains 6 red bags, 4 blue bags and 2 green bags.

2 bags are drawn from the box. What is the probability that:

a. All will be blue?

b. At least one will be green?

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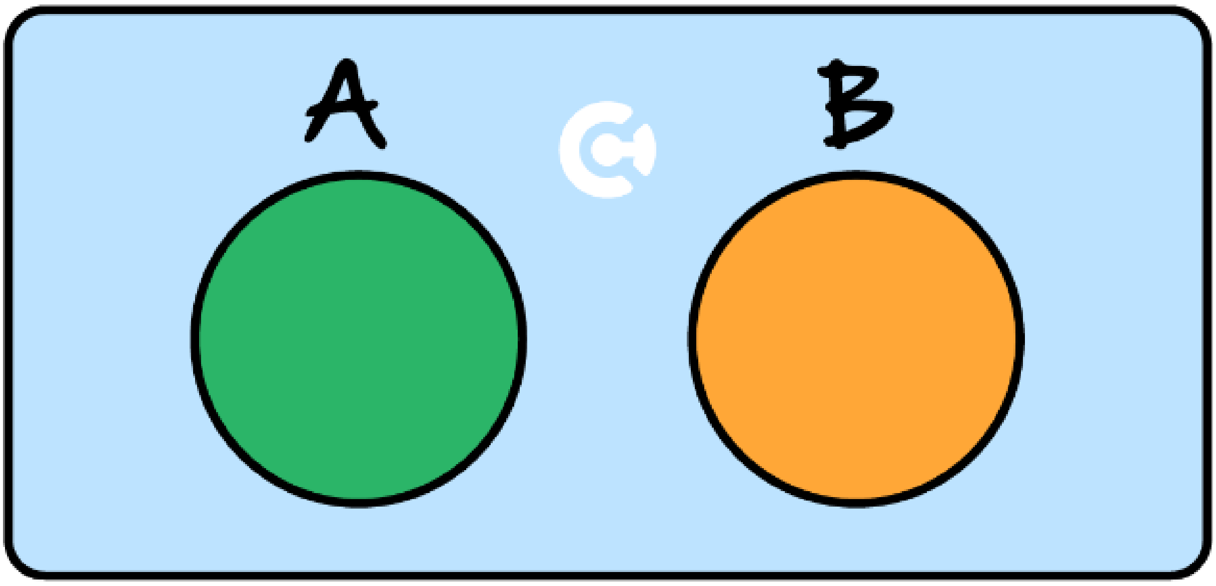
Section C: Mutually Exclusive and Independent Events

Sub-Section: Mutually Exclusive Events

What are mutually exclusive events?

Exploration: Mutually Exclusive Events



- Consider the event of getting 50 RAW in Methods and 25 RAW in Methods.
- Can these two events occur simultaneously? [Yes / No]

These two events are, hence, called mutually exclusive.

Mutually Exclusive Events

$$\Pr(A \cap B) = \_\_\_\_\_\_\_\_$$

- **Mutually Exclusive Events:** Events that cannot occur simultaneously.

Question 14 Walkthrough.

State whether each of the following events would be mutually exclusive or inclusive.

Event 1	Event 2	Exclusive / Inclusive
Getting 50 RAW in Methods.	Getting 45 RAW in Methods.	
Getting into Med.	Doing UCAT.	

Discussion: Pair up and come up with mutually exclusive events!

Question 15

Events E and F are such that $\Pr(E' \cup F') = 0.45$. State whether E and F are mutually exclusive.

Sub-Section: Independent Events

What about independent events?

Exploration: Independent Events

- Consider the event of Qing (Contour Tutor) wearing a swimsuit and James teaching an MM34 Class.
- Are these two things related? [Yes / No]
- If you saw Qing walking down the road wearing a swimsuit:

  How does it change the probability of James teaching the MM34 class?

  [Increases / Decreases / Doesn't change the probability.]

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These two events are called independent.

Independent Events

- When A and B are independent:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

- **Independent Events:** Do not affect the likelihood of each other.

Discussion: Pair up and come up with independent events!

Question 16 Walkthrough.

State whether each of the following events would be independent or dependent.

Event 1	Event 2	Independent / dependent
Wearing a swimsuit.	Going to the beach.	
Doing homework for Methods.	Getting 25 RAW in Methods.	
Rolling a 6 on the first dice roll.	Rolling a 6 on the second dice roll.	

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Question 17

Given that $\Pr(A) = 0.2$, $\Pr(B') = 0.6$ and $\Pr(A \cap B) = 0.03$, are the events A and B independent?

NOTE: Use $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ to algebraically check independence.

What is the difference between a mutually exclusive event and an independent event?

<u>Discussion:</u> What is the difference between a mutually exclusive event and an independent event?

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Question 18 Walkthrough.

Given that $\Pr(A) = 0.1$, $\Pr(B') = 0.7$ and $\Pr(A \cap B) = 0.03$, are the events A and B independent?

Question 19

In Contourville, the probability that it snows on any Wednesday is 0.6. If it snows on Wednesday, Thursday has a 45% chance of snowing, while if it doesn't snow on Wednesday, Thursday has only a 30% chance of snowing.

Let the event W represent snowing on Wednesday, and T represent snowing on Thursday.

a. Find the probability of it snowing on Thursday.

b. Are events W and T independent?

Question 20 Extension.

Let $\Pr(A) = p$ and $\Pr(B) = 2p$ and $\Pr(A \cup B) = \frac{7}{9}$. Find the value of p if events A and B are independent.

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Section D: Conditional Probability

Sub-Section: Conditional Probability

Discussion: What is conditional probability?

Exploration: Conditional Probability

- Consider two probabilities below:

Probability that
a student James randomly selects is a male student wearing glasses.

- What is our sample space? [All students / Male students.]
- How about the second example?

Probability that
a male student James randomly selects is wearing glasses.

- What is our sample space? [All students / Male students.]
- Conditional probability is when we have a restricted sample space.
- We have [less / more] information about what has already occurred.
- So, which one of the two examples is a conditional probability? [First / Second]

Definition

Conditional Probability

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\text{"Likelihood of } A \text{ given that } B \text{ has happened"}$$

- **Conditional Probability**: Calculating the probability of an event given that another event has occurred.

Question 21 Walkthrough.

State whether each of the following probabilities would be conditional or not.

Event 1	Conditional / not
Probability of selecting a Y12 MHS student.	
Probability of selecting a MHS student out of Y12 students.	

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Question 22

James is selling blue T-shirts at Contour and is interested in the relationship between the size of the T-shirts that his mates purchase and their preferred sport. From his sales on Monday, James constructs the table below.

Size of T-shirts	Preferred Sport Soccer	Preferred Sport Cricket	Total
Large	12	4	16
Medium	20	12	32
Small	18	14	32
Total	50	30	80

What is the probability that a person chosen at random from this group:

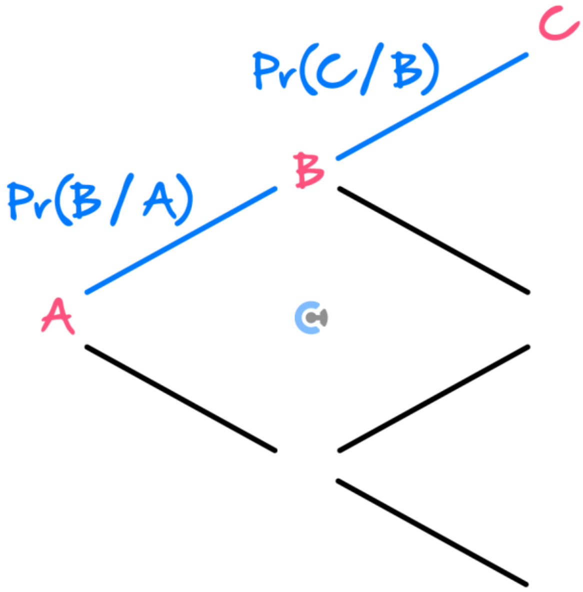
a. Wears small T-shirts?

b. Prefers soccer and wears small T-shirts?

c. Prefers soccer, given that they wear small T-shirts?

How do we solve conditional probabilities?

Tree Diagram for Condition Probability



- Tree diagram is perfect for conditional probability as each branch is ______________________________.

$$\textbf{Each branch} = \underline{\qquad\qquad\qquad\qquad}$$

Space for Personal Notes

Question 23 Walkthrough.

Lightbulbs are produced in factories A, B and C.

- 50% of all lightbulbs come from factory A, while 30% and 20% of lightbulbs are produced by factory B and C, respectively.
- The quality control of factory A is poor and the probability of lightbulbs produced from factory A being faulty is 20%, while only 15% and 10% are faulty from factory B and C, respectively.

a. Construct a tree diagram.

b. What is the probability of a faulty lightbulb being produced?

c. Given that a lightbulb is faulty, what is the likelihood that it came from Factory A?

TIP: Always draw a tree diagram!

<u>**Discussion:**</u> Why should our answer for **part c.** above be bigger than $\frac{1}{2}$?

TIP: Use logical reasoning to **check** your answer in the exam.

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Your turn!

Question 24

Batteries are produced in factories M and N. It is known that 40% of all batteries come from factory M. The quality control of factory M is poor and the probability of batteries produced in factory M being faulty is 25%, while factory N halves the chance.

a. Construct a tree diagram.

b. What is the probability of a faulty battery being produced?

c. Given that a battery is faulty, what is the likelihood that it came from Factory N?

Question 25 Extension.

Laptops are assembled in two facilities: Plant A and Plant B. It is known that 60% of all laptops come from Plant A, while the rest come from Plant B. The probability that a laptop from Plant A is defective is 20%, while the probability that a laptop from Plant B is defective is x.

It is observed that 18% of all laptops produced are defective.

a. Find the value of x, the probability that a laptop from Plant B is defective.

b. Given that a randomly selected laptop is defective, what is the probability that it was produced in Plant A?

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Sub-Section: Conditional Probability and Independence

Active Recall: Independent Events

- When A and B are independent events,

$$\Pr(A \cap B) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Exploration: Derivation of Independence Formula.

- Let's say A and B are independent events.
  - Does the given condition (B) affect the probability of A?
  - Hence, what does $\Pr(A|B)$ equal to?

$$\Pr(A|B) = \_\_\_\_\_\_\_\_\_\_$$

- Hence,

$$\frac{\Pr(A \cap B)}{\Pr(B)} = \_\_\_\_\_\_\_\_\_\_$$

- Rearrange, what do you get?

$$\Pr(A \cap B) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - We get the independent probability formula!

Conditional Probability with Independent Events

$$\Pr(A|B) = \_\_\_\_\_\_\_\_\_\_$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

Question 26

Let event A be "You draw a red card from a deck", and event B be "It rains today".

a. Are A and B independent?

b. Find an expression for $\Pr(A \mid B)$. Hence, find this probability if $\Pr(A) = 0.5$ and $\Pr(B) = 0.3$.

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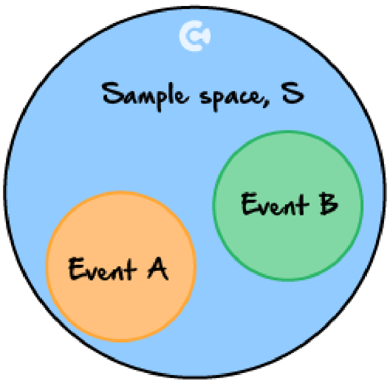
Section E: Cheat Sheets

Cheat Sheet

[5.1.1] -Basics of probability

- **Probability**
  - In probability, we are aiming to quantify uncertainty.



  - Sample space (ε) = Set of all possible outcomes.
  - Probability of an event = Proportion of the event that the sample space takes up.
  - The probability of an event must **always** take a value between 0 and 1, inclusive.
- **Sample Space (ε)**
  - The set of all possible outcomes in an experiment.
  - For tossing two coins in a row, the sample space is:

$$\varepsilon = \{HH, HT, TH, TT\}$$

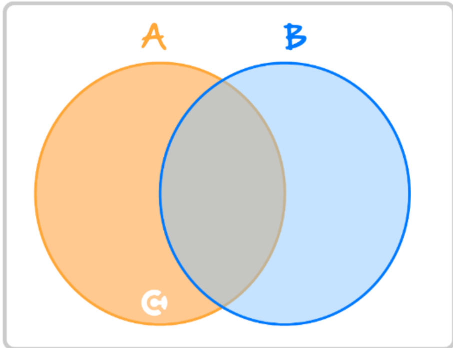
  - For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon = \{1,2,3,4,5,6\}$$

  - Total probability adds up to 1.
- **Calculating Probabilities for Equally Likely Outcomes**
  - When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(Success) = \frac{Number\ of\ successful\ outcomes}{Number\ of\ total\ outcomes}$$

- **Union, Intersection and Complement**



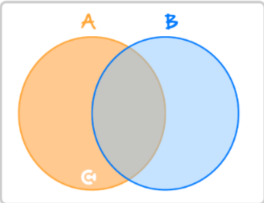
  - $A \cup B =$
    - Union of two events aka "or".
    - Equivalent to either event A **OR** event B **OR** BOTH occurring.
  - $A \cap B =$
    - Intersection of two events aka "and".
    - Equivalent to both event A **AND** event B occurring.
  - $A' =$
    - A complement aka "not".
    - e.g. if A = dice rolled a 6, then A' = dice rolled anything except a 6.

$$\Pr(A') = 1 - \Pr(A)$$

Cheat Sheet

[5.1.2] – Use Venn diagrams and/or Karnaugh maps and apply the addition rule

- **Venn Diagram**



Venn diagrams are useful to visualise the two events.

- **Karnaugh Tables**

We can also represent probability problems using a Karnaugh map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\mathbf{Pr}(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\mathbf{Pr}(A')$
	$\mathbf{Pr}(B)$	$\mathbf{Pr}(B')$	**1**

- The rows and columns add up to the last cell value.

Remember the **total** probability must always add to 1.

- **The Addition Rule**

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

[5.1.3] – Mutually exclusive and independent events

- **Mutually Exclusive Events**

$$\Pr(A \cap B) = 0$$

Mutually Exclusive Events: Events that cannot occur simultaneously.

- **Independent Events**

When A and B are independent:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Independent Events: Does not affect the likelihood of each other.

[5.1.4] – Conditional probability and tree diagrams.

- **Conditional Probability**

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

"Likelihood of A given that B has happened"

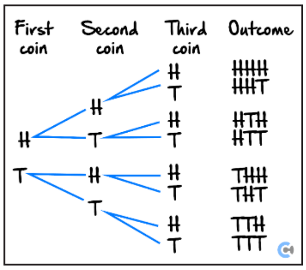
Conditional Probability: Calculating the probability of an event given that another event has occurred.

- **Tree Diagrams**

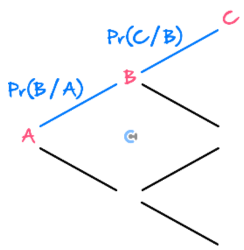
Useful for multiple sequence events.

To calculate the probability of a sequence, we multiply **the probabilities along the relevant branches.**

For instance, the following tree diagram shows the outcomes of three successive coin tosses.



- **Tree Diagram for Condition Probability**



Tree diagram is perfect for conditional probability as each branch is conditional probability.

$$\textit{Each branch} = \Pr(\text{Leaf}|\text{Root})$$

- **Conditional probability with independent events**

$$\Pr(A|B) = \Pr(A)$$

If A and B are independent, the given condition does **not** affect the probability of the event.

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