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VCE Mathematical Methods ¾
Discrete Random Variables I [5.1]
Test Solutions

31.5 Marks. 1 Minute Reading. 23 Minutes Writing.

Results:

Test Questions	________ / 14.5
Extension Test Questions	________ / 17

Section A: Test Questions (14.5 Marks)

Question 1 (3.5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. The sample space represents the set of all possible outcomes.	✓	
b. Mutually exclusive events can occur simultaneously.		✓
c. Independent events can occur simultaneously.	✓	
d. Independent events do not affect each other's probability and are given by $\Pr(A) \times \Pr(B) = \Pr(A \cup B)$.		✓
e. Conditional probability occurs when the sample space is restricted due to the given condition.	✓	
f. For conditional probability, a Venn diagram is best to use for complicated questions.		✓
g. The conditional probability for independent events is the same as the conditional probability for dependent events.		✓

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Question 2 (2 marks)

Two events A and B from a given event space are such that $\Pr(A) = \frac{1}{5}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A' \cap B)$ when $\Pr(A \cap B) = \frac{1}{8}$. (1 mark)

$\Pr(A' \cap B) = PR(B) - \Pr(A \cap B) = \frac{5}{24}$

Students who used a Venn diagram or Karnaugh map usually obtained the correct answer. Many students either just added or multiplied $\Pr(A')$ and $\Pr(B)$. Some students attempted unsuccessfully to develop a formula.

b. Calculate $\Pr(A' \cap B)$ when A and B are mutually exclusive events. (1 mark)

$\Pr(A' \cap B) = \Pr(B) = \frac{1}{3}$

A lot of incorrect attempts were seen for this question. A very popular incorrect response was $\frac{4}{15}$, obtained by confusing independent events with mutually exclusive events.

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Question 3 (5 marks)

For events A and B from a sample space, $\Pr(A \mid B) = \frac{1}{5}$ and $\Pr(B \mid A) = \frac{1}{4}$. Let $\Pr(A \cap B) = p$.

a. Find $\Pr(A)$ in terms of p. (1 mark)

$$\Pr(A) = \frac{\Pr(A \cap B)}{\Pr(B \mid A)}$$
$$= \frac{p}{\frac{1}{4}}$$
$$= 4p$$

This question was generally answered well. The most common errors included solving for Pr(*B*), and incorrectly transposing $\frac{p}{\Pr(A)}$ to yield $\frac{1}{4}$.

b. Find $\Pr(A' \cap B')$ in terms of p. (2 marks)

	A	A'	
B	p	$4p$	$5p$
B'	$3p$	$1-8p$	$1-5p$
	$4p$	$1-4p$	1

Or $\Pr(A \cup B) = 4p + 5p - p = 8p$
and $(A \cup B)' = A' \cap B'$

$\Pr(A' \cap B') = 1 - 8p$

Students who scored highly usually used a table or a Venn diagram to arrive at their answer. There were various misconceptions of the connection between conditional probabilities and $\Pr(A' \cap B')$. Many students assumed that events *A* and *B* were independent, hence incorrectly used $\Pr(A' \cap B') = \Pr(A') \times \Pr(B')$.

c. Given that $\Pr(A \cup B) \leq \frac{1}{5}$, state the largest possible interval for p. (2 marks)

$\Pr(A \cup B) = 8p$

If $8p \leq \frac{1}{5}$

$p \leq \frac{1}{40}$

Thus $0 < p \leq \frac{1}{40}$

Most students identified that $\Pr(A \cup B) = 8p$. Only a few students identified the correct interval because students did not consider that in this case $p \neq 0$. Common incorrect answers included $p = \frac{1}{40}$ or $p \leq \frac{1}{40}$ (allowing negative probabilities) and $0 \leq p \leq \frac{1}{40}$.

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Question 4 (4 marks)

Two boxes each contain four stones that differ only in colour.
Box 1 contains four black stones.
Box 2 contains two black stones and two white stones.
A box is chosen randomly, and one stone is drawn randomly from it.
Each box is equally likely to be chosen, as is each stone.

a. What is the probability that the randomly drawn stone is black? (2 marks)

Several approaches were possible using a tree diagram or a counting argument

- $\Pr(\text{Black}) = \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$
- $1 - \Pr(White) = 1 - \frac{1}{4} = \frac{3}{4}$
- Since choosing either box is equally likely and choosing any stone is equally likely and there are 8 stones, 6 of which are black, $\Pr(\text{Black}) = \frac{6}{8} = \frac{3}{4}$

This question was generally well answered. Many students showed their reasoning via a tree diagram or some written explanation. Some students overworked the problem by trying to use the binomial distribution.

b. It is known from which box the stone has been drawn.

Given that the stone that is drawn is black, what is the probability that it was drawn from Box 1? (2 marks)

$\Pr(\text{Box 1} | \text{Black}) = \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$

Students generally recognised the conditional probability (reduced sample space). Some students incorrectly worked $\Pr(\text{Black} | \text{Box 1})$, resulting in a probability greater than 1, which is not feasible.

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Section B: Extension Test Questions (17 Marks)

Question 5 (11 marks) **Tech-Active.**

A mobile phone technician is studying the performance of two brands of phones, Brand A and Brand B.

When a phone is sent in for repair, the technician classifies the phone as either" easily repairable" or "difficult to repair". Based on the past records, 60% of all phones are Brand A, and the remaining 40% are Brand B.

The technician believes:

- 90% of Brand A phones are easily repairable.
- 50% of Brand B phones are easily repairable.

Let R represent the event that a phone is easily repairable, A represent the event that the phone is Brand A, and B represent the event that the phone is Brand B.

a.

i. Find the probability that a phone is easily repairable. (2 marks)

$$\begin{aligned}\Pr(R) &= \Pr(R \mid A)\Pr(A) + \Pr(R \mid B)\Pr(B) \quad \textbf{[1M, or 1M for a diagram]}\\ &= 0.9 \times 0.6 + 0.5 \times 0.4\\ &= 0.54 + 0.2\\ &= \boxed{0.74} \quad \textbf{[1A]}\end{aligned}$$

ii. Given that a randomly chosen phone is easily repairable, find the probability that it is a Brand A phone. (1 mark)

$$\Pr(A \mid R) = \frac{\Pr(R \mid A)\Pr(A)}{\Pr(R)} = \frac{0.9 \times 0.6}{0.74} = \frac{0.54}{0.74} = \boxed{\frac{27}{37}} \quad \textbf{[1A]}$$

iii. Given that a randomly chosen phone is not easily repairable, find the probability that it is a Brand B phone. (2 marks)

$$\Pr(R') = 1 - 0.74 = 0.26 \quad \textbf{[1M]}$$
$$\Pr(R' \mid B) = 1 - 0.5 = 0.5$$
$$\Pr(B \mid R') = \frac{\Pr(R' \mid B)\Pr(B)}{\Pr(R')} = \frac{0.5 \times 0.4}{0.26} = \frac{0.2}{0.26} = \boxed{\frac{10}{13}} \quad \textbf{[1A]}$$

A third phone brand, Brand C, is introduced. The technician now records 67% of all phones sent in as easily repairable. Brand C makes up 20% of all phones sent in for repair.

Brand A and B keep the same statistics as above, and the ratio of Brand A phones to Brand B phones that are sent in for repair is $3:2$. Let c be the probability that a randomly chosen Brand C phone is easily repairable.

b.

i. Find the fraction of phones that are Brand A and the fraction of phones that are Brand B. (2 marks)

Since Brand C makes up 20% of phones, the remaining 80% are split between Brands A and B in a 3:2 ratio.

$$\Pr(A) = \frac{3}{3+2} \times 0.8 = \frac{3}{5} \times 0.8 = 0.48 = \frac{12}{25} \quad \textbf{[1M]}$$
$$\Pr(B) = \frac{2}{5} \times 0.8 = 0.32 = \frac{8}{25} \quad \textbf{[1M]}$$

ii. Hence, find the value of c. (2 marks)

Let c be the probability that a Brand C phone is easily repairable.
The total proportion of easily repairable phones is:

$$\Pr(R) = \Pr(A)\Pr(R \mid A) + \Pr(B)\Pr(R \mid B) + \Pr(C)\Pr(R \mid C)$$
$$0.67 = 0.48 \times 0.9 + 0.32 \times 0.5 + 0.2c$$
$$= 0.432 + 0.16 + 0.2c$$
$$0.67 = 0.592 + 0.2c$$
$$0.078 = 0.2c$$
$$c = \frac{0.078}{0.2} = \boxed{0.39} \quad \textbf{[1A]}$$

An external agency believes that the technician's assumption about repairability (that 67% of phones are easily repairable) is only valid for phones sent in during the morning. They believe that for phones sent in during the afternoon, the probability that a phone is easily repairable is x, where $0.8 \leq x \leq 0.95$.

c. Let the probability that a phone is sent in the afternoon be y. If the overall proportion of easily repairable phones observed is 0.78, find the minimum and maximum values of y, assuming the external agency's beliefs are correct. (2 marks)

Let the probability a phone is sent in the **morning** be $1-y$, and assume the technician's model gives a repairability rate of 0.67 for morning phones.
Then:

$$0.78 = (1-y)0.67 + yx$$
$$\implies y = \frac{0.11}{x - 0.67} \quad \text{[1M]}$$

Maximum value of y when $x = 0.8 \implies y = \dfrac{11}{13}$

Minimum value of y when $x = 0.95 \implies y = \dfrac{11}{28}$ **[1A]**

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Question 6 (3 marks)

There are 100 people in line to board a plane with 100 seats. The first person has lost his boarding pass, so he takes a random seat. Everyone that follows takes their assigned seat if it's available, but otherwise takes a random unoccupied seat. What is the probability that the last passenger ends up in his or her assigned seat?

Number the people 1, 2, 3, and so on, so that 1 boards first, 2 boards next, and 100 boards last.

First of all, observe that for $2 \leq n \leq 100$, * *n's seat is always taken after she finishes boarding.**. This is clear since if it isn't taken when she boards the plane, she will take it.

It follows that when 100 is getting on the plane, seats 2 through 99 are taken. Therefore, 100 will either get his own seat, or he will get 1's seat. That is,

$$P(100 \text{ gets his seat}) + P(100 \text{ gets 1's seat}) = 1 \qquad (*)$$

However, **1's seat and 100's seat are indistinguishable for the first 99 passengers**. If any of them are to take either 1's seat or 100's seat, they pick randomly between the two. Therefore, the chance that one of the first 99 passengers takes 1's seat is equal to the chance that one of them takes 100's seat. But one of the first 99 passengers takes a given seat *if and only if* 100 does *not* get that seat. So we have

$$P(100 \text{ doesn't get seat } 1) = P(100 \text{ doesn't get seat } 100)$$

which is the same as

$$P(100 \text{ gets his seat}) = P(100 \text{ gets 1's seat})$$

So from $(*)$,

$$P(100 \text{ gets his seat}) = \boxed{\frac{1}{2}}.$$

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Question 7 (3 marks)

A man is stranded on an island. A benevolent genie presents three boxes, 23 white marbles and 7 black marbles and instructs the man, "You may distribute the marbles into the boxes any way you see fit, but you must use all of the marbles. Once you finish, you will choose a box at random and then choose a marble from that box at random. If the marble is white, then I will help you escape from this place."

Assuming the man distributes the marbles in his best interest, what is the probability that he escapes the island? And what is the best way to distribute the marbles?

The man's best strategy is to place one white marble in the first two boxes and then the remaining marbles in the third box. This guarantees that he'll escape the island if one of the first two boxes are selected. If the man selects one of the boxes with only one white marble in it, then the probability that the man draws a white marble when selecting one of those boxes is 1. The remaining box will have 21 white marbles and 7 black marbles, so when that box is selected, the probability that the man draws a white marble from that box is $\frac{3}{4}$. Now we sum the probabilities that he draws a white marble over the boxes:

$$\left(\frac{1}{3}\right)(1) + \left(\frac{1}{3}\right)(1) + \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) = \frac{11}{12}$$

Therefore, the probability that the man escapes the island is $\frac{11}{12}$.

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