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VCE Mathematical Methods ¾
Integration Exam Skills [4.4]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 19
Supplementary Questions	Pg 20 - Pg 37

Section A: Compulsory Questions

Sub-Section: Exam 1 (Tech-Free)

Question 1

a. Find an anti-derivative of $3x^4 - \frac{2}{x^2}$ with respect to x.

b. Find $\int (4 - 2x)^{-5} dx$.

c. The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$. Given that $g(1) = \frac{1}{\pi}$, find $g(x)$.

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Question 2

The gradient of a curve is given by $2\sin(2x) - 4e^{-2x}$. The curve passes through the origin. What is the equation of the curve?

Question 3

a. Find the derivative of $x\sin(x)$.

b. Hence, find an antiderivative of $x\cos(x)$.

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Question 4

Find the average value of $y = -x^2 + 8x + 12$ over the interval $[1, 4]$.

Question 5

Find $3\int_0^{3k} \left(g\left(\frac{x}{3}\right) - 1\right) dx$, given $\int_0^k (g(x))dx = 3k$, where function g is continuous for $x \in R$ and given $g(x) \geq 0$ for $x \in [0, k]$.

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Question 6

Let $\boldsymbol{g}: \boldsymbol{R} \to \boldsymbol{R}, \boldsymbol{g}(\boldsymbol{x}) = (\boldsymbol{a} - \boldsymbol{x})^{\mathbf{2}}$, where a is a real constant.

The average value of a on the interval $[\mathbf{-1}, \mathbf{1}]$ is $\mathbf{\frac{31}{12}}$. Find the value(s) of $\boldsymbol{a}$.

Question 7

If $y = \frac{\tan(x)}{4}$, find $\frac{dy}{dt}$, given $\frac{dx}{dt} = \frac{2}{\sqrt{t}}$ and $x = 4$ when $t = 1$.

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Sub-Section: Exam 2 (Tech-Active)

Question 8

If $\int ae^{bx}dx = -2e^{2x} + c$, then:

A. $a = 4$ and $b = -2$.

B. $a = -2$ and $b = 2$.

C. $a = -1$ and $b = 2$.

D. $a = -4$ and $b = 2$.

Question 9

The gradient of a curve is given by $2\cos\left(\frac{x}{2}\right)$. If the x-intercept is $x = \frac{5\pi}{3}$ then, the y-intercept will be at $y =$

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. $\frac{\sqrt{3}}{2}$

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Question 10

Let $f(x) = px^r$ and $g(x) = qx^s$, where a, b, m and n are positive integers. The domain of f = domain of $g = R$. If $f'(x)$ is an anti-derivative of $g(x)$, then which one of the following must be true?

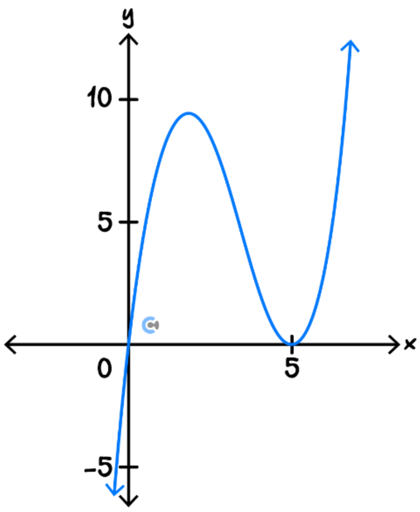
A. $\frac{r}{s}$ is an integer.

B. $\frac{s}{r}$ is an integer.

C. $\frac{p}{q}$ is an integer.

D. $\frac{q}{p}$ is an integer.

Question 11



The graph of $y = f'(x)$ is shown above. Which of the following statements is true for the graph of $y = f(x)$?

A. The graph has a local maximum at $x = 0$ and a stationary point of inflection at $x = 5$.

B. The graph has a local minimum at $x = 0$ and a stationary point of inflection at $x = 5$.

C. The graph has a local maximum at $x = 5$ and a stationary point of inflection at $x = 0$.

D. The graph has a local minimum at $x = 5$ and a stationary point of inflection at $x = 0$.

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Question 12

If $\int_2^6 f(x)dx = 8$, then $\int_0^2 f(2x+2)dx$ is equal to:

A. 4

B. 6

C. 8

D. 10

Question 13

If $\int_0^3 g(x)dx = 18$ and $\int_0^3 (2g(x) + ax)dx = 72$, then the value of a is:

A. 2

B. 4

C. 6

D. 8

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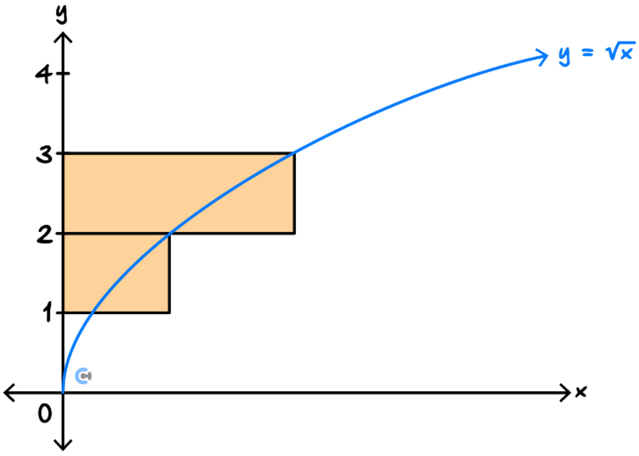
Question 14

Lily and Max are calculating the area between the graph of $y = \sqrt{x}$ and the y-axis between $y = 1$ and $y = 3$. Jake uses a partitioning, shown in the diagram below, while Anita uses a definite integral to find the exact area.



The difference between the results obtained by Jake and Anita is:

A. 0

B. $\frac{22}{3}$

C. $\frac{26}{3}$

D. $\frac{11}{3}$

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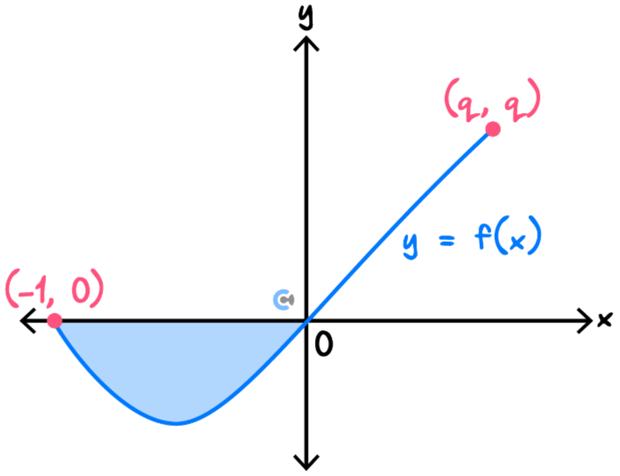
Question 15

The graph of a function $f: [-1, q] \to R$ is shown below.



The average value of f over the interval $[-1, q]$ is zero. The area of the shaded region is $\frac{9}{2}$.

If the graph is a straight line, for $0 \leq x \leq q$, then the value of q is:

A. 2

B. 5

C. $\frac{5}{2}$

D. 3

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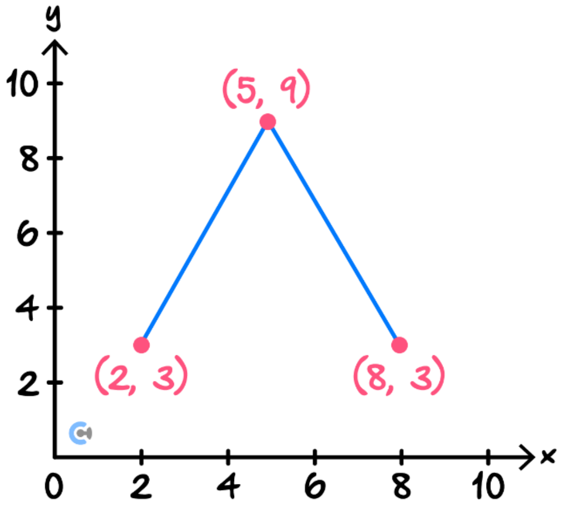
Question 16

The graph of a function, h, is shown below.



The average value of h is:

A. 3

B. 5

C. 6

D. 7

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Question 17

The algorithm below, described in pseudocode, estimates the value of a definite integral using the trapezium rule.

Inputs:

- $f(x)$, the function to integrate.
- a, the lower terminal of integration.
- b, the upper terminal of integration.
- n, the number of trapeziums to use.

Define trapezium $(f(x), a, b, n)$
$h \leftarrow (b - a) \div n$
$sum \leftarrow f(a) + f(b)$
$x \leftarrow a + h$
$i \leftarrow 1$
While $i < n$ Do
$sum \leftarrow sum + 2 \times f(x)$
$x \leftarrow x + h$
$i \leftarrow i + 1$
EndWhile
$area \leftarrow (h \div 2) \times sum$
Return area

Consider the algorithm implemented with the following inputs:

$$trapezium\,(\ln(x), 1, 5, 4)$$

What is the value of sum **after the 3rd iteration** of the loop?

A. $2\ln(1) + 2\ln(2) + 2\ln(3) + 2\ln(4) + 2\ln(5)$

B. $\ln(1) + 2\ln(2) + 2\ln(3) + \ln(5)$

C. $2\ln(2) + 2\ln(3) + 2\ln(4) + \ln(5)$

D. $2\ln(2) + 2\ln(3) + 2\ln(4)$

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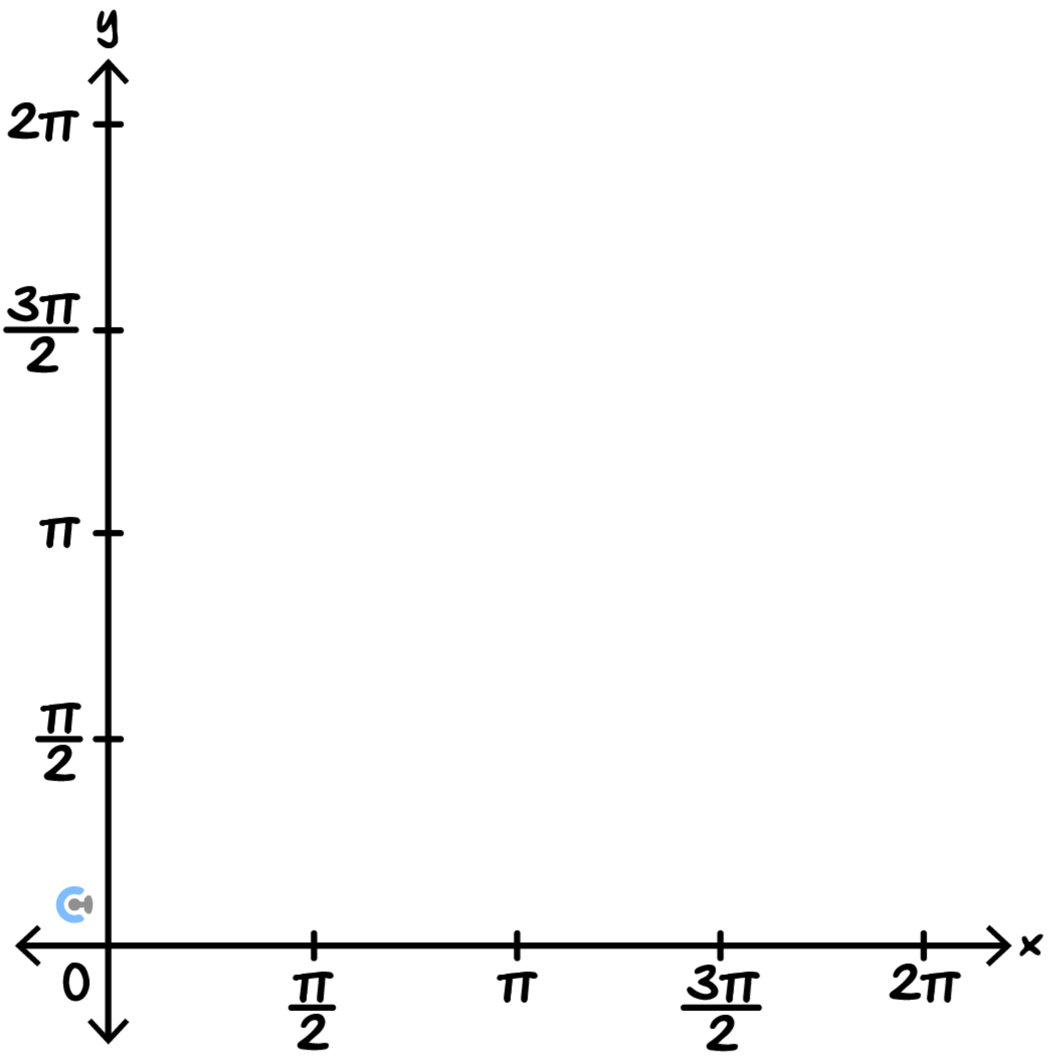
Question 18

A data science researcher is studying a nonlinear transformation model described by the function:
$f(x) = x + \sin(x)$, $x \in [0, 2\pi]$.

a. **Sketch the function** $f(x) = x + \sin(x)$ over the interval $[0, 2\pi]$. Label the endpoints clearly.



b. Given that $f(x)$ is strictly increasing on $[0, 2\pi]$, and thus invertible, **state the domain of** $f^{-1}(x)$.

c. **Sketch** $f^{-1}(x)$ on the same axis and **identify the coordinates of any intersections** between the graphs of $f(x)$ and $f^{-1}(x)$.

d. **Find the area between** $f(x)$ and $f^{-1}(x)$ on their domain of overlap.

e. **Calculate the area** under the curve $f^{-1}(x)$ from $x = 0$ to $x = 4$, bounded by the x-axis. Round your answer to **2 decimal places**.

f. **Verify** that the point $A\left(\frac{5\pi}{6} + \frac{1}{2}, \frac{5\pi}{6}\right)$ lies on the graph of $f^{-1}(x)$.

g. **Find the area bounded** by the tangent to $f^{-1}(x)$ at point A and the curve $f^{-1}(x)$. Give your answer correct to **2 decimal places**.

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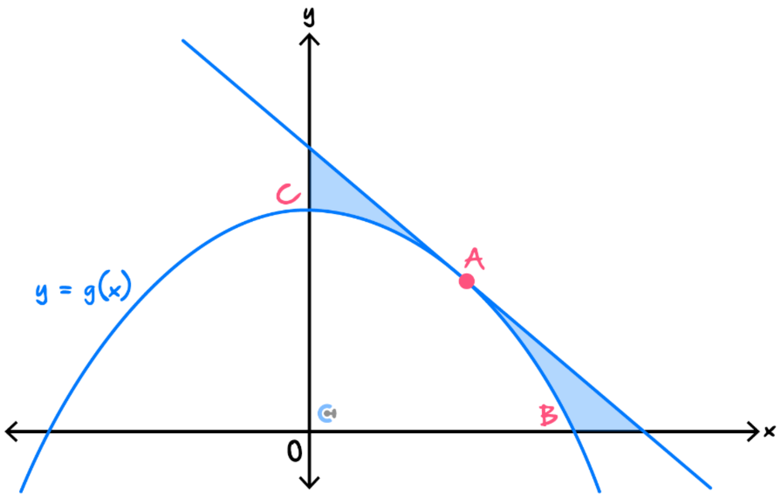
Question 19

Part of the graph of a function $R \to R, g(x) = 12 - 2x^2$ is shown below.



a. Points B and C are the positive x-intercept and y-intercept of the graph of g, respectively, as shown in the diagram above. The tangent to the graph of g at the point A is parallel to the line segment BC.

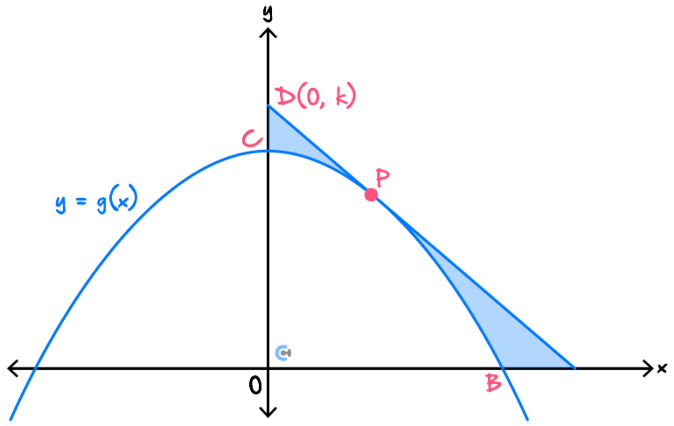
i. Find the equation of the line perpendicular to the graph of g at the point A.

ii. Find the average rate of change of $f(x)$ between $x = 0$ and the x-coordinate of point A.

iii. The shaded region shown in the diagram above is bounded by the graph of g, the tangent at the point A, and the x-axis and y-axis.

Calculate the area of this shaded region.

b. The tangent to the graph of g at a point p has a negative gradient and intersects the y-axis at point $D(0, k)$, where $14 \leq k \leq 20$.



i. Find the equation of the tangent line at point p in terms of k.

ii. Find the rule $A(k)$ for the function of k that gives the area of the shaded region.

iii. Find the maximum area of the shaded region and the value of k for which this occurs, give to 2 decimal places.

iv. If $12 \leq k \leq 24$, find the minimum area of the shaded region and the value of k for which this occurs, give to 2 decimal places.

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Section B: Supplementary Questions

Sub-Section: Exam 1 (Tech-Free)

Question 20

a. Evaluate $\int_1^5 \left(\frac{1}{\sqrt{x}}\right) dx$.

b. If $f'(x) = 2\cos(x) - \sin(2x)$ and $f\left(\frac{\pi}{2}\right) = \frac{1}{2}$, find $f(x)$.

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Question 21

a. Find $\int_1^2 3x^2 - 4x + \frac{5}{x} + \sin(x)$.

b. If $f(x) = x\log_e(x)$,

i. Find $f'(x)$.

ii. Hence, find $\int \log_e(x)dx$.

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Question 22

Let f be a differentiable function defined for all real x, where $f(x) \geq 0$ for all $x \in [0, a]$.

If $\int_0^a f(x)dx = a$, find $2\int_0^{7a} \left(f\left(\frac{x}{7}\right) + 2\right) dx$.

Question 23

Find the value(s) of $\boldsymbol{k}$ for which the average value of $\boldsymbol{y = \sin(kx)}$ over the interval $[\boldsymbol{0, \pi}]$ is equal to the average value of $\boldsymbol{y = \cos(x)}$ over the same interval.

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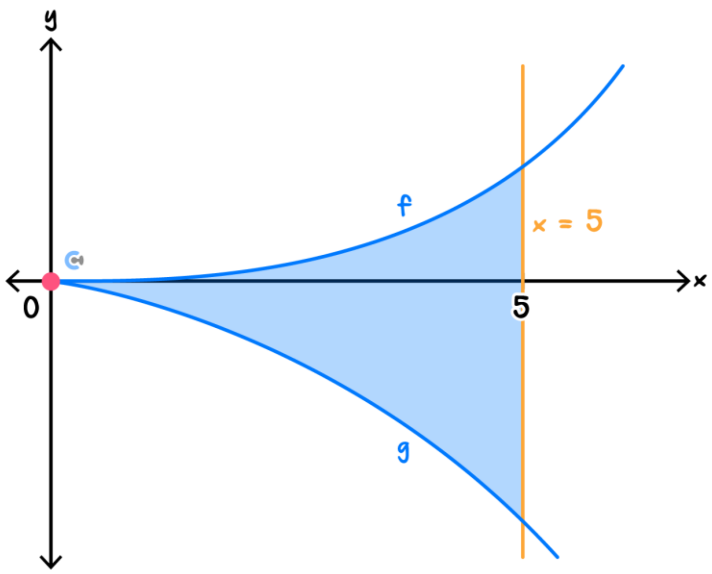
Question 24

Let $f: R \to R, f(x) = x^2 e^{-kx}$, where k is a negative real constant.

a. Show that $f'(x) = xe^{-kx}(-kx + 2)$.

b. Find the value(s) of k for which the graphs of $y = f(x)$ and $y = f'(x)$ have exactly one point of intersection.

Let $g(x) = \frac{2xe^{-kx}}{k}$. The diagram below shows sections of the graphs of f and g for $x \geq 0$.



Let A be the area of the region bounded by the curves $y = f(x), y = g(x)$ and the line $x = 5$.

c. Write down a definite integral that gives the value of A.

d. Using your result from **part a.**, or otherwise, find the value of k such that $A = \frac{10}{-k}$.

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Sub-Section: Exam 2 (Tech-Active)

Question 25

If $f(x) = \int_0^x \left(\sqrt{t^2+4}\right)dt$, then $f'(-2)$ is equal to:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. $2\sqrt{2}$

D. $-2\sqrt{2}$

Question 26

Which one of the following options is an anti-derivative of $\frac{1}{x^2} - \frac{1}{\cos^2\left(\frac{x}{2}\right)}$?

A. $-\frac{1}{x} - 2\tan\left(\frac{x}{2}\right)$

B. $-\frac{2}{x^3} - \frac{(2)}{\cos^3\left(\frac{x}{2}\right)}$

C. $\frac{1}{x} - \frac{1}{2}\tan\left(\frac{x}{2}\right)$

D. $\log_e(x^2) - \tan\left(\frac{x}{2}\right)$

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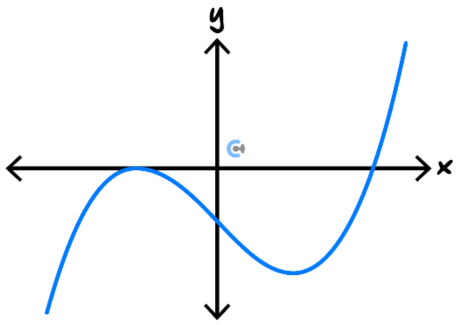
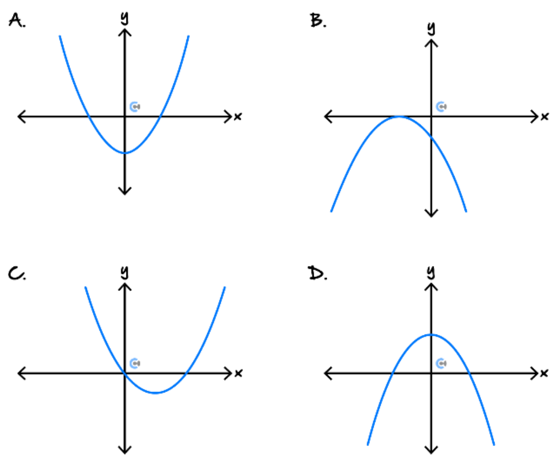
The following information applies to the two questions that follow.

For Questions 27 and 28, refer to the graph of $y = f(x)$ shown below.



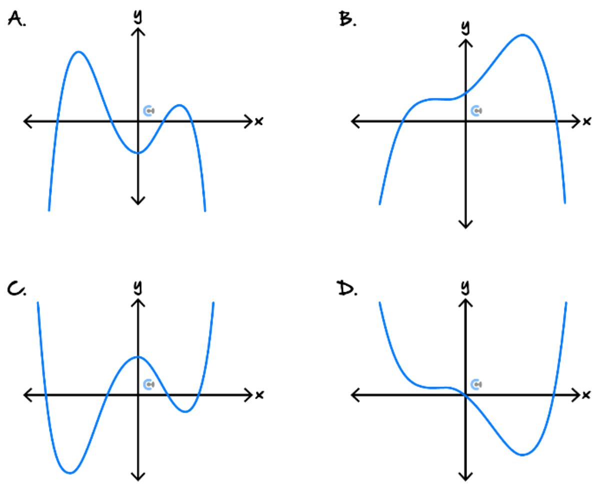
Question 27

The corresponding part of the derivative graph of $y = f(x)$ is best represented by:



Question 28

The corresponding part of the antiderivative graph of $y = f(x)$ is best represented by:



Question 29

Given that $\int_1^5 (f(x))dx = 4$, $\int_5^1 (f(x) - 2)dx$ is equal to:

A. 0

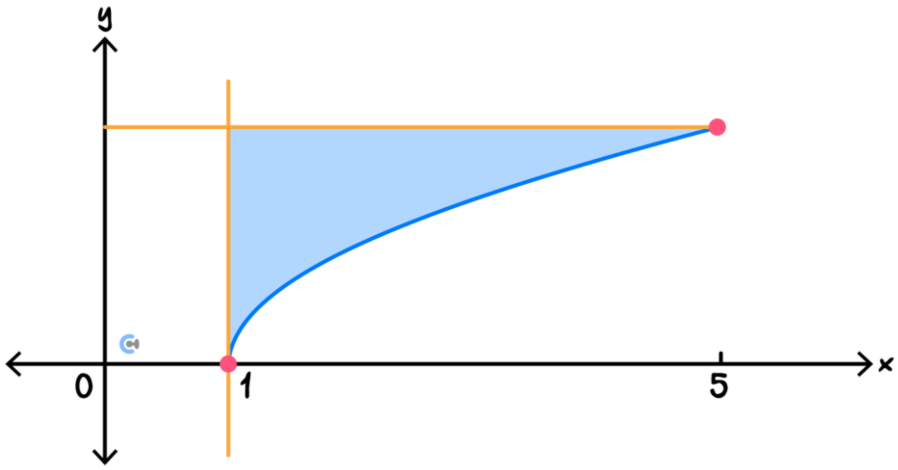
B. 1

C. 4

D. 7

Question 30

The graph of $\boldsymbol{g}: [\mathbf{1}, \mathbf{5}] \to \boldsymbol{R}, \boldsymbol{g}(\boldsymbol{x}) = 2\sqrt{\boldsymbol{x} - \mathbf{1}}$ is shown below.



Which one of the following definite integrals could be used to find the area of the shaded region?

A. $\int_1^5 (2\sqrt{x - 1})dx$

B. $\int_0^4 \left(\frac{x^2}{4}\right) dx$

C. $\int_0^4 (4 - 2\sqrt{x - 1})dx$

D. $\int_0^4 \left(\frac{x^2}{4} + 1\right) dx$

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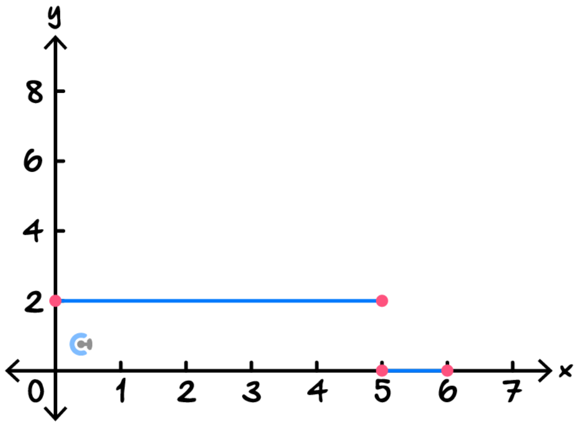
Question 31

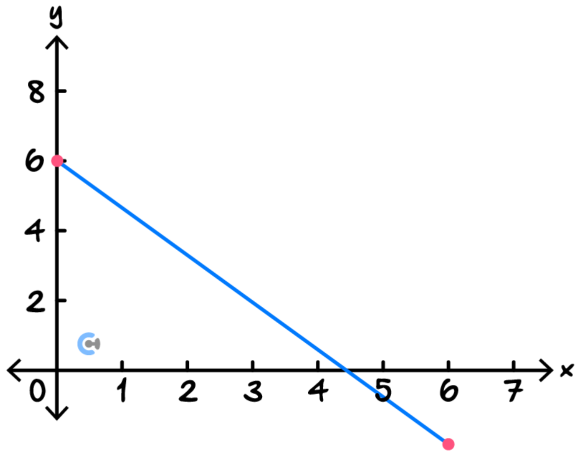
Let g be a function with an average value of 2 over the interval $[0, 6]$.

The graph of g over this interval could be:

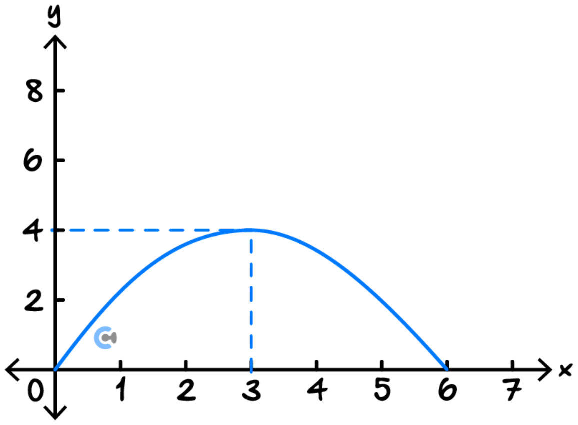
A.



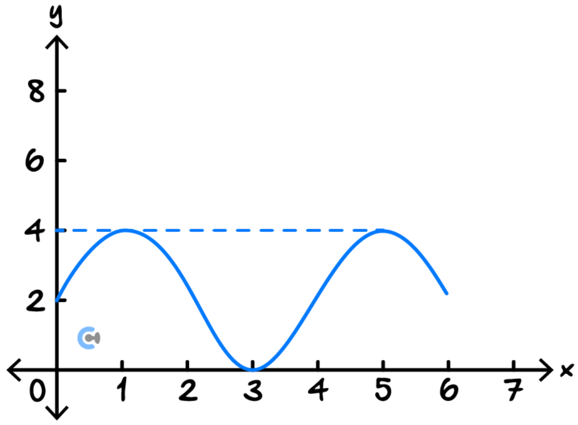
B.



C.



D.



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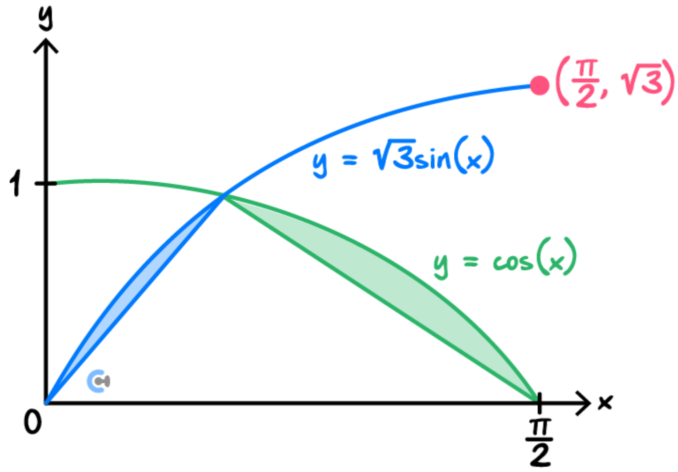
Question 32

The area of the shaded region would be:



A. $\sqrt{3} - 1 - \frac{\sqrt{3}\pi}{8}$

B. $\sqrt{3} - 1 - \frac{\sqrt{3}\pi}{4}$

C. $\frac{\sqrt{3}\pi}{8}$

D. $\frac{\pi}{2}(\sqrt{3} - 1)$

E. $\frac{3\pi}{8} - \sqrt{3}$

Question 33

The temperature $T°$ over a time period of a day is given by the function $T(t) = 17 - 6\sin\left(\frac{\pi t}{12}\right)$, where t is the time in hours. Using the given function, the average temperature over the first 12 hours is equal to:

A. 17

B. $204 - \frac{12}{\pi}$

C. $17 + \frac{12}{\pi}$

D. $17 - \frac{12}{\pi}$

E. $\frac{12}{\pi}$

Question 34

The following pseudocode is intended to estimate the value of a definite integral using the trapezium rule. However, one line in the loop is missing.

Inputs:

- $f(x)$, the function to integrate.
- a, the lower terminal of integration.
- b, the upper terminal of integration.
- n, the number of trapeziums to use.

Define trapezium $(f(x), a, b, n)$
    $h \leftarrow (b - a) \div n$
    $sum \leftarrow f(a) + f(b)$
    $x \leftarrow a + h$
    $i \leftarrow 1$
    While $i < n$ Do
        ____
        $x \leftarrow x + h$
        $i \leftarrow i + 1$
    EndWhile
    $area \leftarrow (h \div 2) \times sum$
    Return area

A. $sum \leftarrow f(x) + 2$

B. $sum \leftarrow sum + f(x)$

C. $sum \leftarrow sum + 2 \times f(x)$

D. $sum \leftarrow sum \times 2 \times f(x)$

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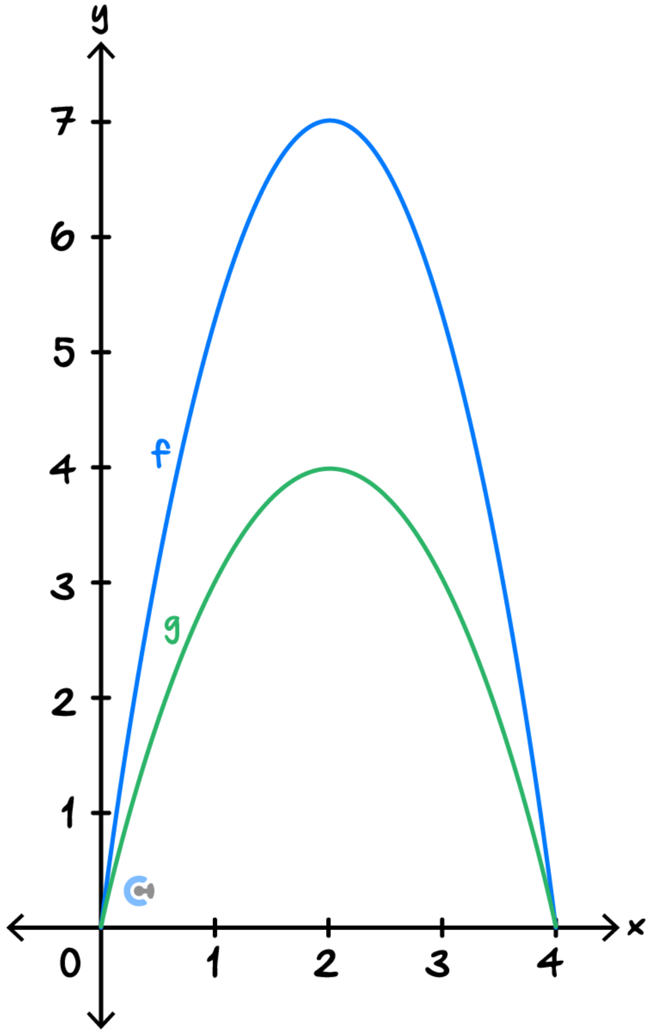
Question 35

Assume that $f(x) = \frac{7x}{4}(4 - x)$ and $g(x) = 4x - x^2$, $0 \le x \le 4$.



a. Find the angle between the tangents drawn to f and g when $x = 0$, in degrees, correct to 2 decimal places.

b. Find the average value of the function $y = f(x) - g(x)$ on the interval $[0, 4]$.

c. Let $(k, g(k))$ be a random point on graph of g, find the value of area bounded by the tangent of $g(x)$ at $x = k$, $g(x)$, and $f(x)$.

d. Find the value of k such that the bounded area is minimum, and state the minimum value.

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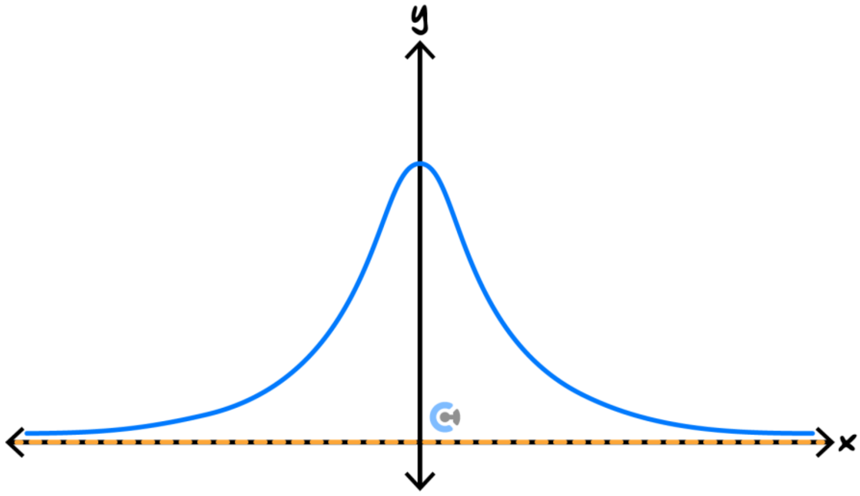
Question 36

An engineer is exploring the safety of the jumps that have been built along the track. A typical jump follows the rule $h(x) = \frac{1}{x^2+1}$ as shown in the diagram.



a. Find an approximation, for the area under the curve from $x = -3$ to $x = 3$ using intervals of width one unit and right endpoint rectangles.

b. Find the exact area under $h(x)$ from $x = -3$ to $x = 3$.

c. Hence, show your working to find $\int_{-1}^{1} h(3x)dx - \frac{\pi}{6}$.

Let $g: [a, \infty] \to R, g(x) = \frac{1}{x^2+1}$ where a is the least possible value such that the inverse function $g^{-1}(x)$ exists.

d. State the value of a.

e. Find the integral(s) required to find the area defined by the regions bounded by the graphs of $g(x), g^{-1}(x)$ and the lines $x = \frac{7}{10}$ and $x = 2$. You do not need to evaluate the integrals.

The graph of g undergoes the listed transformations below to become the graph of p:

- Dilated by a factor of 3 from x-axis.
- Dilated by a factor of $\frac{1}{2}$ from y-axis.
- Reflected in the y-axis.

f. Find the rule for p^{-1} and state the domain.

g. Find the average value of p^{-1} in the interval $[1, 2]$, correct to 2 decimal places.

h. The area between $y = kp^{-1}(x)$, the lines $x = 1$, $x = 2$ and the x-axis is found to be at least 6 square units. Find the possible values of k correct to 1 decimal place.

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Question 37

Let $g: [0, \infty) \to R, g(x) = \frac{3}{4x^2+1}$.

a. Using the fact that $g\left(g^{-1}(x)\right) = x$ or otherwise, find the rule for g^{-1} and its domain.

b. Find the area bounded between $g(x)$ and $g^{-1}(x)$, correct to 2 decimal places.

c. Find the value of a so that $\int_0^{0.5} g(x)dx = 1.5 - \int_a^3 0.5 - g^{-1}(x)dx$.

Now, consider $f: [0, \infty) \to R, f(x) = \frac{3}{kx^2+1}, k > 0$.

d. State possible number of intersection points between $f(x)$ and $f^{-1}(x)$.

e. Consider the set of k that there is only one intersection point between $f(x)$ and $f^{-1}(x)$, find the largest possible area that is bounded by $f(x)$, $f^{-1}(x)$, x-axis and y-axis in terms of k.

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