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VCE Mathematical Methods ¾
Integration II [4.3]
Test Solutions

38 Marks. 1 Minute Reading. 30 Minutes Writing.

Results:

Test Questions	____________ / 19
Extension Questions	____________ / 19

Section A: Test Questions (19 Marks)

Question 1 (3 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. Area between two inverse functions can be found by finding the area between the function and $y = x$. False. You gotta double.		✓
b. We can cut the area up into horizontal strips where each strip has an area of xdy.	✓	
c. Average value of the function is simply the average height of the function.	✓	
d. Area bounded by a, b, c is the reciprocal of area bounded by a^{-1}, b^{-1} and c^{-1}. False, it's the same area.		✓
e. When finding the average value of the function, we divide the **total area** by the width $(b - a)$. False. We divide the signed area.		✓
f. For integration by recognition, the question will always give a function to derive first.	✓	

Space for Personal Notes

Question 2 (4 marks)

Consider a function $f(x) = \log_e(x) + 3$.

Find the area bounded by the function $f(x)$, x-axis, y-axis and $y = 4$.

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$y - 3 = \log_e(x)$

$e^{y-3} = x$

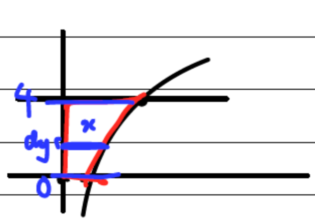
$$\int_0^4 x \cdot dy$$

$$= \int_0^4 e^{y-3}\, dy$$

$$= \left[e^{y-3}\right]_0^4$$

$$= e^{4-3} - e^{-3}$$

$$= e - e^{-3}$$



Question 3 (3 marks)

Find the average value of the function given by $f(x) = 3\cos(3x) - 2$ for $x \in \left[0, \frac{\pi}{12}\right]$.

Find the average value of the function given by $f(x) = 3\cos(3x) - 2$ for $x \in \left[\overset{a}{0}, \overset{b}{\frac{\pi}{12}}\right]$.

$$\frac{1}{\frac{\pi}{12}-0}\int_0^{\frac{\pi}{12}} 3\cos(3x)-2\,dx$$

R. Chain

$$=\frac{12}{\pi}\cdot\left[3\sin(3x)\frac{1}{3}-2x\right]_0^{\frac{\pi}{12}}$$

$$=\frac{12}{\pi}\left[\sin(3x)-2x\right]_0^{\frac{\pi}{12}}$$

$$=\frac{12}{\pi}\left(\sin\left(\frac{\pi}{4}\right)-\frac{\pi}{6}\right)-\frac{12}{\pi}\left(\underbrace{\sin(0)-0}_{=0}\right)$$

$$=\frac{12}{\pi}\cdot\left(\frac{\sqrt{2}}{2}-\frac{\pi}{6}\right)$$

$$=\frac{6\sqrt{2}}{\pi}-2$$

Space for Personal Notes

Question 4 (6 marks)

a. Find the derivative of xe^{2x}. (2 marks)

$$1\cdot e^{2x} + xe^{2x}\cdot 2$$

$$\frac{d}{dx}\left(xe^{2x}\right) = e^{2x} + 2xe^{2x}$$

b. Hence, find the area under xe^{2x} for $x = 0$ to $x = 1$. (4 marks)

$$\int_0^1 xe^{2x}\,dx$$

$$= \int_0^1 \frac{1}{2}\frac{d}{dx}\left(xe^{2x}\right) - \frac{1}{2}e^{2x}\,dx$$

R.Chn

$$= \left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^1$$

$$= \left(\frac{1}{2}e^2 - \frac{1}{4}e^2\right) - \left(0 - \frac{1}{4}e^0\right)$$

$$= \frac{1}{4}e^2 + \frac{1}{4}$$

$$\frac{d}{dx}\left(xe^{2x}\right) - e^{2x} = 2xe^{2x}$$

$$\frac{1}{2}\frac{d}{dx}\left(xe^{2x}\right) - \frac{1}{2}e^{2x} = xe^{2x}$$

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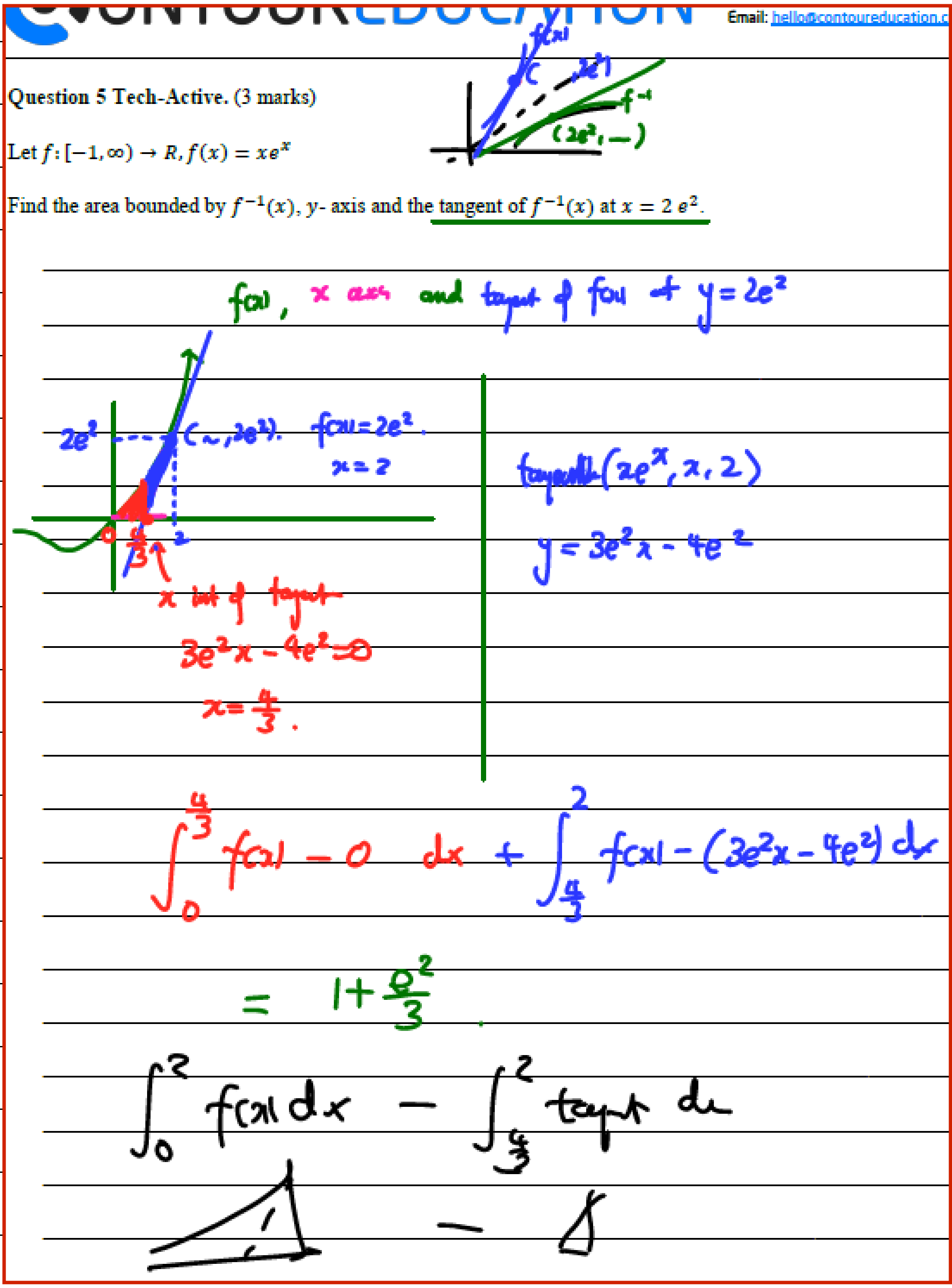
Question 5 (3 marks) **Tech-Active.**

Let $f: [-1, \infty) \to R, f(x) = xe^x$.

Find the area bounded by $f^{-1}(x)$, y-axis and the tangent of $f^{-1}(x)$ at $x = 2e^2$.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Question 5 Tech-Active. (3 marks)

Let $f: [-1, \infty) \to R, f(x) = xe^x$

Find the area bounded by $f^{-1}(x)$, y- axis and the tangent of $f^{-1}(x)$ at $x = 2e^2$.

$f(x)$, x axis and tangent of $f(x)$ at $y = 2e^2$

$2e^2$ $(\sim, 2e^2)$. $f(x) = 2e^2$, $x = 2$

tangentLine$(xe^x, x, 2)$

$y = 3e^2x - 4e^2$

x int of tangent

$3e^2x - 4e^2 = 0$

$x = \frac{4}{3}$.

$$\int_0^{\frac{4}{3}} f(x) - 0 \, dx + \int_{\frac{4}{3}}^{2} f(x) - (3e^2x - 4e^2) \, dx$$

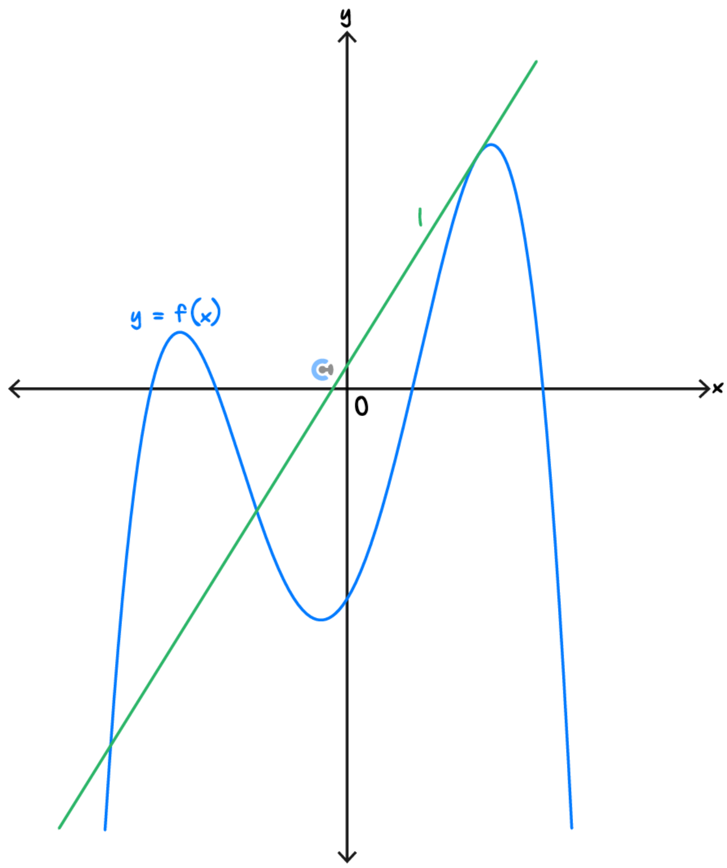
$$= 1 + \frac{e^2}{3}$$

$$\int_0^2 f(x)\,dx - \int_{\frac{4}{3}}^{2} \text{tangent}\, dx$$

Section B: Extension Questions - Tech Active (19 Marks)

Question 6 (9 marks)

Consider the quartic $f : \mathbb{R} \to \mathbb{R}, f(x) = -x^4 - x^3 + 11x^2 + 9x - 18$. Part of the graph $y = f(x)$ and a line l that is tangent to f is shown below.



a. The line l is tangent to f at $x = 2$. Find the equation for the line l. (2 marks)

$f'(2) = 9$. Want line with gradient 9 and through the point $(2, 20)$. **[1M]**
$y = 9x + 2$. **[1A]**

b. The tangent l intersects $y = f(x)$ at $x = 2$ and two other points. State the x values of the two other points of intersection. Express your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b, and c are integers. (2 marks)

Let $t(x) = 9x + 2$. We solve $t(x) = f(x)$. **[1M]**

$x = \dfrac{-5 \pm \sqrt{5}}{2}$ are the other two solutions. **[1A]**

c. Find the total area of the region bounded by the tangent l and $y = f(x)$. Express your answer in the form $\frac{a+b\sqrt{c}}{d}$. (3 marks)

The area is given by:

$$A = \int_{\frac{-5-\sqrt{5}}{2}}^{\frac{-5+\sqrt{5}}{2}} (f(x) - t(x))dx + \int_{\frac{-5+\sqrt{5}}{2}}^{2} (t(x) - f(x))dx \quad \textbf{[1M for each integral]}$$

$$= \frac{801 + 1025\sqrt{5}}{40} \quad \textbf{[1A]}$$

d. The average value of the function f on the interval $[1, b]$, where $b > 1$, is 10. Find the possible value(s) of b correct to three decimal places. (2 marks)

We must solve $\dfrac{1}{b-1}\displaystyle\int_1^b f(x)dx = 10$ **[1M]**

We get $b = 1.869, 3.322$. **[1A]**

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Question 7 (10 marks)

Consider functions of the form:

$$f : \mathbb{R} \to \mathbb{R}, f(x) = \frac{324x^2(a - 2x)}{a^4}$$

and

$$h : \mathbb{R} \to \mathbb{R}, h(x) = \frac{36x}{a^2}$$

where a is a positive real number.

a. Find the coordinates of the local maximum of f in terms of a. (2 marks)

Solve $f'(x) = 0 \implies x = 0, \frac{a}{3}$. **[1M]**

The local maximum is at $\left(\frac{a}{3}, \frac{12}{a}\right)$ **[1A]**

b. Find the x-values of all of the points of intersection between the graphs of f and h, in terms of a where appropriate. (1 mark)

Solve $f(x) = h(x)$

$x = 0, \frac{a}{6}, \frac{a}{3}$. **[1A]**

c. Determine the total area of the regions bounded by the graphs of $y = f(x)$ and $y = h(x)$. (2 marks)

The area is given by

$$A = \int_0^{a/6} (h(x) - f(x))\, dx + \int_{a/6}^{a/3} ((f(x) - h(x)))\, dx \quad \textbf{[1M]}$$

$$= \frac{1}{4} \quad \textbf{[1A]}$$

Consider the function:

$$g : \left[0, \frac{a}{3}\right] \to \mathbb{R}, g(x) = \frac{324x^2(a-2x)}{a^4}$$

where a is a positive real number.

d. Evaluate $\frac{a}{3} \times g\left(\frac{a}{3}\right)$. (1 mark)

$$\frac{a}{3} \times g\left(\frac{a}{3}\right) = 4 \quad \textbf{[1A]}$$

e. Find the area bounded by the graph of g^{-1}, the x-axis and the line $x = g\left(\frac{a}{3}\right)$. (2 marks)

Area $= \frac{a}{3} \times \frac{12}{a} - \int_0^{a/3} g(x)\,dx = 2.$ **[1M set up a correct expression, 1A]**

Or by symmetry could do $\frac{1}{2} \times \frac{a}{3} \times g\left(\frac{a}{3}\right) = 2.$

f. Find the value of a for which the graphs of g and g^{-1} have the same endpoints. (1 mark)

We must have that $\frac{a}{3} = \frac{12}{a} \implies a = 6.$ **[1A]**

g. Find the area enclosed by the graphs of g and g^{-1} when they have the same endpoints. (1 mark)

When $a = 6, h(x) = x$. So then using result from part c.

Area $= 2 \times \frac{1}{4} = \frac{1}{2}$ **[1A]** .

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