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VCE Mathematical Methods ¾

Integration II [4.3]

Test

38 Marks. 1 Minute Reading. 30 Minutes Writing.

Results:

Test Questions	_____ / 19
Extension Questions	_____ / 19

Section A: Test Questions (19 Marks)

Question 1 (3 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. Area between two inverse functions can be found by finding the area between the function and $y = x$.		
b. We can cut the area up into horizontal strips where each strip has an area of xdy.		
c. Average value of the function is simply the average height of the function.		
d. Area bounded by a, b, c is the reciprocal of area bounded by a^{-1}, b^{-1} and c^{-1}.		
e. When finding the average value of the function, we divide the **total area** by the width $(b - a)$.		
f. For integration by recognition, the question will always give a function to derive first.		

Space for Personal Notes

Question 2 (4 marks)

Consider a function $f(x) = \log_e(x) + 3$.

Find the area bounded by the function $f(x)$, x-axis, y-axis and $y = 4$.

Space for Personal Notes

Question 3 (3 marks)

Find the average value of the function given by $f(x) = 3\cos(3x) - 2$ for $x \in \left[0, \frac{\pi}{12}\right]$.

Space for Personal Notes

Question 4 (6 marks)

a. Find the derivative of xe^{2x}. (2 marks)

b. Hence, find the area under xe^{2x} for $x = 0$ to $x = 1$. (4 marks)

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Question 5 (3 marks) **Tech-Active.**

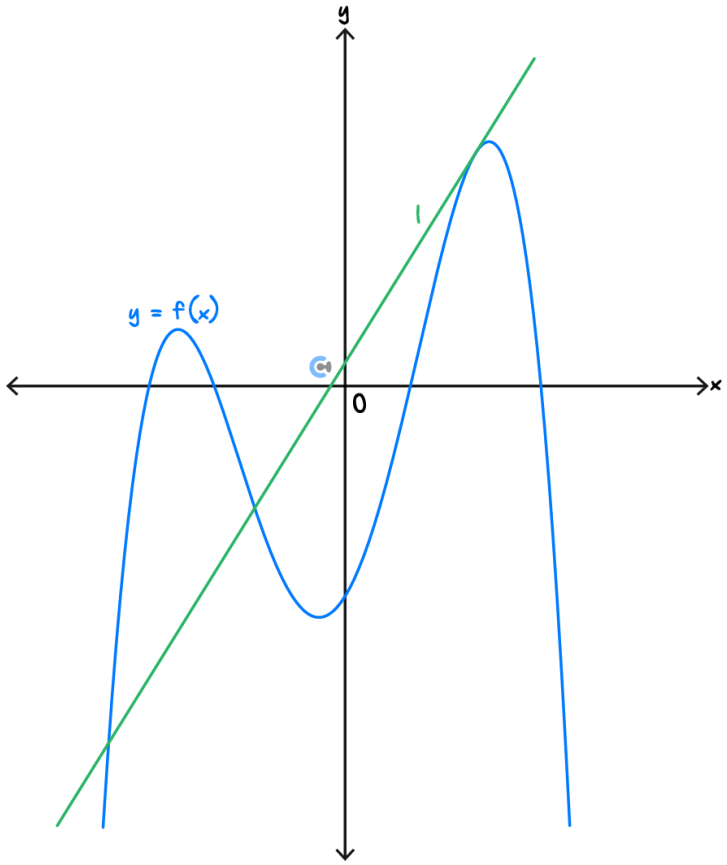
Let $f: [-1, \infty) \to R, f(x) = xe^x$.

Find the area bounded by $f^{-1}(x)$, y-axis and the tangent of $f^{-1}(x)$ at $x = 2e^2$.

Section B: Extension Questions - Tech Active (19 Marks)

Question 6 (9 marks)

Consider the quartic $f : \mathbb{R} \to \mathbb{R}, f(x) = -x^4 - x^3 + 11x^2 + 9x - 18$. Part of the graph $y = f(x)$ and a line l that is tangent to f is shown below.



a. The line l is tangent to f at $x = 2$. Find the equation for the line l. (2 marks)

b. The tangent l intersects $y = f(x)$ at $x = 2$ and two other points. State the x values of the two other points of intersection. Express your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b, and c are integers. (2 marks)

c. Find the total area of the region bounded by the tangent l and $y = f(x)$. Express your answer in the form $\frac{a+b\sqrt{c}}{d}$. (3 marks)

d. The average value of the function f on the interval $[1, b]$, where $b > 1$, is 10. Find the possible value(s) of b correct to three decimal places. (2 marks)

Space for Personal Notes

Question 7 (10 marks)

Consider functions of the form:

$$f : \mathbb{R} \to \mathbb{R}, f(x) = \frac{324x^2(a - 2x)}{a^4}$$

and

$$h : \mathbb{R} \to \mathbb{R}, h(x) = \frac{36x}{a^2}$$

where a is a positive real number.

a. Find the coordinates of the local maximum of f in terms of a. (2 marks)

b. Find the x-values of all of the points of intersection between the graphs of f and h, in terms of a where appropriate. (1 mark)

c. Determine the total area of the regions bounded by the graphs of $y = f(x)$ and $y = h(x)$. (2 marks)

Consider the function:

$$g : \left[0, \frac{a}{3}\right] \to \mathbb{R}, g(x) = \frac{324x^2(a-2x)}{a^4}$$

where a is a positive real number.

d. Evaluate $\frac{a}{3} \times g\left(\frac{a}{3}\right)$. (1 mark)

e. Find the area bounded by the graph of g^{-1}, the x-axis and the line $x = g\left(\frac{a}{3}\right)$. (2 marks)

f. Find the value of a for which the graphs of g and g^{-1} have the same endpoints. (1 mark)

g. Find the area enclosed by the graphs of g and g^{-1} when they have the same endpoints. (1 mark)

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