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VCE Mathematical Methods ¾
Integration II [4.3]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2-Pg 10
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Section A: Compulsory Questions

Sub-Section: Fundamentals

Question 1

a. Let $f(x) = \ln(3x + 1)$.

Find the rule for the inverse function $f^{-1}(x)$. **[4.3.1]**

$$y = \ln(3x + 1)$$

Swap x and y for inverse $\Rightarrow x = \ln(3y + 1)$ **1M**

$$\Rightarrow y = \frac{e^x - 1}{3}$$

$$f^{-1}(x) = \frac{e^x - 1}{3} \quad \textbf{1A}$$

b. Hence, find the area bounded by the graph of $y = f(x)$, the y-axis, the x-axis and the line $y = 2$. **[4.3.1]**

$$A = \int_0^2 \frac{e^y - 1}{3}\, dy \quad \textbf{1M}$$

$$= \frac{1}{3}[e^y - y]_0^2 \quad \textbf{1M}$$

$$= \frac{1}{3}(e^2 - 2 - (1 - 0))$$

$$= \frac{1}{3}(e^2 - 3) \quad \textbf{1A}$$

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Question 2 [4.3.2]

Let $f(x) = x^2 - 2x + 3$. Find the average value of the function over the interval $[1,4]$.

5

Question 3

a. Show that $\frac{d}{dx}(x^2 \ln x) = 2x\ln\ x + x$. **[4.3.3]**

$$\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x^2 \times \frac{1}{x} \quad \textbf{1M}$$

$$\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x \text{ as required.}$$

b. Hence, evaluate $\int x\ln\ x\, dx$. **[4.3.3]**

$$\frac{d}{dx}(x^2 \ln x) = 2x\ln\ x + x$$

$$2x\ln\ x = \frac{d}{dx}(x^2 \ln\ x) - x$$

$$x\ln\ x = \frac{1}{2}\left(\frac{d}{dx}(x^2\ln\ \ x) - x\right)$$

$$\int x\ln\ x\, dx = \frac{1}{2}\int \left(\frac{d}{dx}(x^2\ln\ \ x) - x\right) dx \quad \textbf{1M}$$

$$= \frac{1}{2}\left(x^2 \ln x - \frac{x^2}{2}\right) + C$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \quad \textbf{1A}$$

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Question 4

Let $f(x) = x^3$.

a. Find the points where $f(x) = f^{-1}(x)$. **[4.3.1]**

$$f(x) = x$$
$$\Rightarrow x^3 = x \textbf{ 1M}$$
$$\Rightarrow x^3 - x = 0$$
$$\Rightarrow x(x^2 - 1) = 0$$
$$\Rightarrow x = 0, \pm 1 \textbf{ 1M}$$

$(-1,-1), (0,0), (1,1)$ **1A**

b. Hence, find the exact area between the graphs of $f(x)$ and $f^{-1}(x)$. **[4.3.1]**

$$\text{Area} = 2\int_{-1}^{0} (x^3 - x)\,dx + 2\int_{0}^{1} (x - x^3)\,dx \textbf{ 1M}$$

$$\int_{-1}^{0} (x^3 - x)\,dx = \left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]_{-1}^{0} = (0-0) - \left(\tfrac{1}{4} - \tfrac{1}{2}\right) = -\left(-\tfrac{1}{4}\right) = \frac{1}{4}$$

$$\int_{0}^{1} (x - x^3)\,dx = \left[\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right]_{0}^{1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

1M for antiderivatives

$$2 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} = \frac{1}{2} + \frac{1}{2} = \boxed{1} \textbf{ 1A}$$

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Sub-Section: Problem Solving

Question 5

Let $f(x) = e^{-3x}\sin(2x)$ and $g(x) = e^{-3x}\cos(2x)$.

a. Differentiate $e^{-3x}\sin(2x)$ and $e^{-3x}\cos(2x)$ with respect to x. **[4.3.3]**

$$\frac{d}{dx}(e^{-3x}\sin(2x)) = \frac{d}{dx}(e^{-3x}) \cdot \sin(2x) + e^{-3x} \cdot \frac{d}{dx}(\sin(2x))$$
$$= -3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x) \quad \mathbf{1A}$$
$$\frac{d}{dx}(e^{-3x}\cos(2x)) = -3e^{-3x}\cos(2x) - 2e^{-3x}\sin(2x) \quad \mathbf{1A}$$

b. Hence, show that: **[4.3.3]**

$$e^{-3x}\sin(2x) + c_1 = -3\int e^{-3x}\sin(2x)\,dx + 2\int e^{-3x}\cos(2x)\,dx$$

and

$$e^{-3x}\cos(2x) + c_2 = -3\int e^{-3x}\cos(2x)\,dx - 2\int e^{-3x}\sin(2x)\,dx$$

$$\frac{d}{dx}(e^{-3x}\sin(2x)) = -3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x)$$
$$e^{-3x}\sin(2x) + c_1 = \int(-3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x))\,dx \quad \mathbf{1M}$$
$$\Rightarrow e^{-3x}\sin(2x) + c_1 = -3\int e^{-3x}\sin(2x)\,dx + 2\int e^{-3x}\cos(2x)\,dx$$
$$\frac{d}{dx}(e^{-3x}\cos(2x)) = -3e^{-3x}\cos(2x) - 2e^{-3x}\sin(2x)$$
$$e^{-3x}\cos(2x) + c_2 = \int(-3e^{-3x}\cos(2x) - 2e^{-3x}\sin(2x))\,dx \quad \mathbf{1M}$$
$$e^{-3x}\cos(2x) + c_2 = -3\int e^{-3x}\cos(2x)\,dx - 2\int e^{-3x}\sin(2x)\,dx$$

c. Use the two equations from **part b.** to determine $\int e^{-3x}\sin(2x)\,dx$. **[4.3.3]**

Let $I = \int e^{-3x}\sin(2x)\,dx, J = \int e^{-3x}\cos(2x)\,dx$

$$e^{-3x}\sin(2x) = -3I + 2J \quad (1)$$
$$e^{-3x}\cos(2x) = -3J - 2I \quad (2)$$
$$3e^{-3x}\sin(2x) = -9I + 6J \quad (3)$$
$$2e^{-3x}\cos(2x) = -6J - 4I \quad (4)$$

1M for equations

Add eq. 3 and 4:

$$3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x) = -13I \quad \mathbf{1M}$$

$$3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x) = -13I \Rightarrow I = \frac{-1}{13}(3e^{-3x}\sin(2x)+2e^{-3x}\cos(2x))$$

$$\boxed{\int e^{-3x}\sin(2x)\,dx = \frac{e^{-3x}}{13}(-3\sin(2x)-2\cos(2x)) + C} \quad \mathbf{1A}$$

Question 6

For this question, only consider quadrant 1 of the cartesian x-y plane.

a. Find the area enclosed between the parabolas $y = x^2$ and $y^2 = x$. **[4.3.1]**

$$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx \quad \mathbf{1M}$$
$$= \int_0^1 x^{\frac{1}{2}} - x^2\,dx$$
$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^1 \quad \mathbf{1M}$$
$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}} \quad \mathbf{1A}$$

Or find area with $y = x$ then double.

b. Show that the curves $y = x^n$ and $y^n = x$ intersect at the point $(1,1)$ for all positive integers n. **[4.3.1]**

$(x^n)^n = x^{n^2} = x \Rightarrow x^{n^2-1} = 1$ **1M**

So $x = 1$ then $y = x^n = 1^n = 1$

Intersect $(1,1)$

c. Show that the area of the region contained between the curves $y = x^n$ and $y^n = x$ is $\frac{n-1}{n+1}$. **[4.3.1]**

Area between $y = x$ and x^n

$$A = \int_0^1 (x - x^n)\,dx \textbf{ 1M}$$

$$= \left[\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right]_0^1 \textbf{ 1M}$$

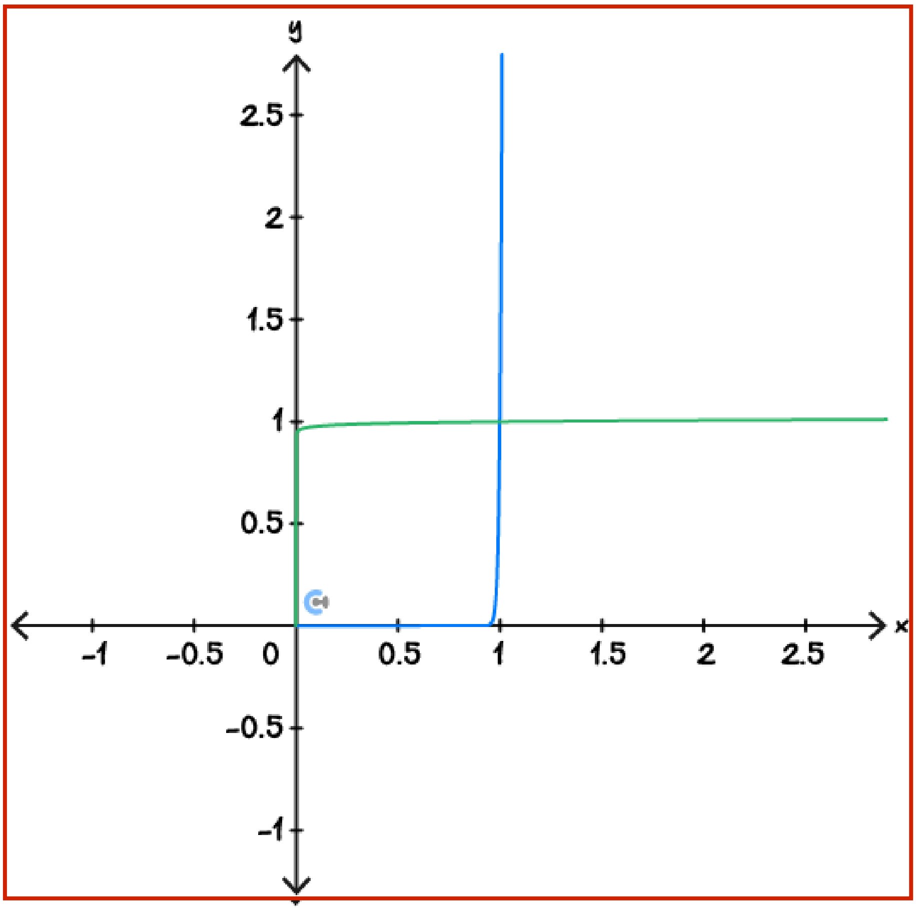
$$= \frac{1}{2} - \frac{1}{n+1}$$

$$\text{Total Area} = 2A = 2\left(\frac{1}{2} - \frac{1}{n+1}\right) = 1 - \frac{2}{n+1} \textbf{ 1M}$$

$$= \frac{n+1-2}{n+1}$$

$$= \frac{n-1}{n+1}$$

d. Describe the area between the curves for very large values of n. You use a CAS calculator to help you visualise. **[4.3.1]**



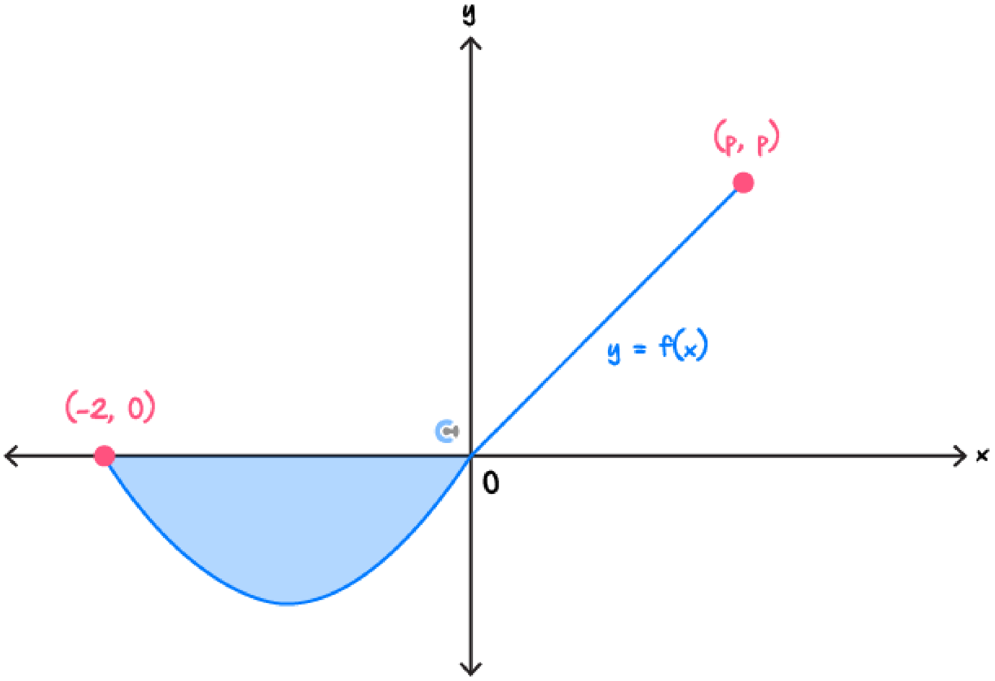
The area approaches 1 as $n \to \infty$ **1M**

Either visualise behaviour or determine the horizontal asymptote of answer in **part c.**

https://www.desmos.com/calculator/7gxcco74lz

Question 7 [4.3.2]

The graph of a function $f: [-2, p] \to \mathbb{R}$ is shown. It consists of a curved segment from $x = -2$ to $x = 0$ and and a straight line from the origin O to the point (p, p), where $p > 0$. The area of the shaded region under the curve from $x = -2$ to $x = 0$ is $\frac{25}{8}$. The average value of f over the interval $[-2, p]$ is zero.



Find the value of p.

$$\text{Area of triangle from 0 to } p = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot p \cdot p = \frac{p^2}{2}$$

$$\frac{p^2}{2} = \frac{25}{8} \quad \textbf{1M}$$

$$p = \sqrt{\frac{25}{4}} = \frac{5}{2} \text{ as } p > 0 \quad \textbf{1A}$$

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Question 8

a. If $y = x\log_e x$, find $\frac{dy}{dx}$. **[4.3.3]**

$$\frac{dy}{dx} = 1 \cdot \log_e x + x \cdot \frac{1}{x} \ \mathbf{1M}$$
$$= \log_e x + 1. \mathbf{1A}$$

b. Hence, evaluate $\int_1^e \log_e x \, dx$. **[4.3.3]**

$$\int_1^e \log_e x \, dx = \int_1^e \left(\frac{dy}{dx} - 1\right) dx \ \mathbf{1M}$$
$$= [x\log_e x]_1^e - \int_1^e 1 \, dx. \ \mathbf{1M}$$
$$= (e - 0) - (e - 1)$$
$$= 1. \mathbf{1A}$$

c. If $y = x(\log_e x)^n$, where n is a natural number, find $\frac{dy}{dx}$. **[4.3.3]**

$$\frac{dy}{dx} = (\log_e x)^n + x \cdot n(\log_e x)^{n-1} \cdot \frac{1}{x} \ \mathbf{1M}$$
$$= (\log_e x)^n + n(\log_e x)^{n-1}. \ \mathbf{1A}$$

d. Let $I_n = \int_1^e (\log_e x)^n \, dx$.

Show that, for $n > 1$, $I_n + nI_{n-1} = e$. **[4.3.3]**

$$\int_1^e \frac{dy}{dx} \, dx = \int_1^e (\log_e x)^n \, dx + n \int_1^e (\log_e x)^{n-1} \, dx \ \mathbf{1M}$$
$$y(e) - y(1) = I_n + nI_{n-1} \ \mathbf{1M}$$
$$I_n + nI_{n-1} = e - 0 \ \ \mathbf{1M}$$
$$I_n + nI_{n-1} = \mathrm{e}$$

e. Find $\int_1^e (\log_e x)^3 \, dx$. **[4.3.3]**

$$I_n + nI_{n-1} = e.$$
$$I_3 + 3I_2 = e \quad (1)$$
$$I_2 + 2I_1 = e \quad (2)$$
$$I_1 = \int_1^e \log_e x \, dx = 1$$
$$I_2 + 2(1) = e \Rightarrow I_2 = e - 2. \ \mathbf{1M}$$
$$I_3 + 3(e-2) = e \ \mathbf{1M}$$
$$\Rightarrow I_3 + 3e - 6 = e$$
$$\Rightarrow I_3 = e - (3e - 6)$$
$$= -2e + 6. \mathbf{1A}$$

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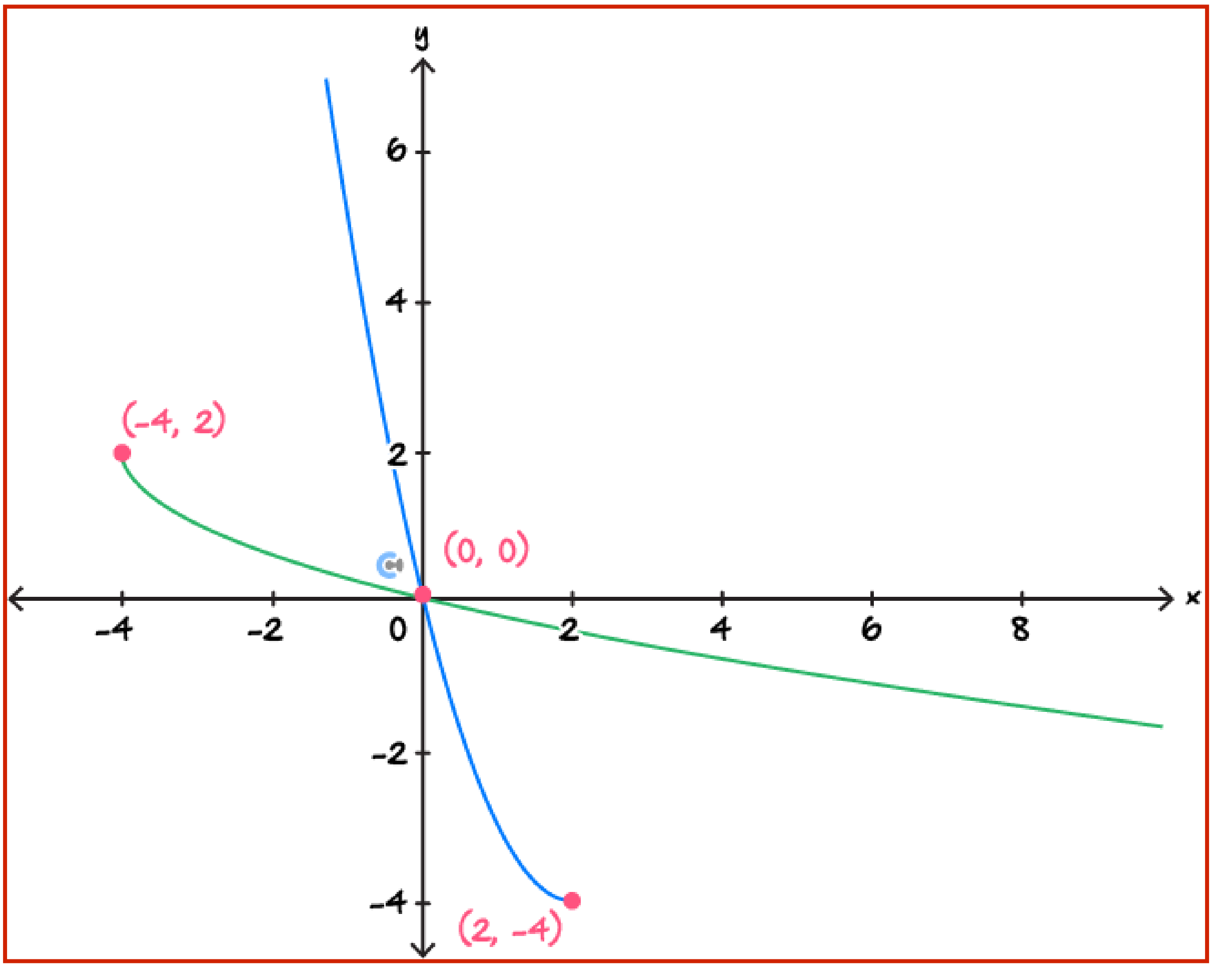
Section B: Supplementary Questions

Sub-Section: Exam 1

Question 9

Let $f:(-\infty, 2] \to \mathbb{R}, f(x) = x^2 - 4x$.

A portion of the graph of $y = f(x)$ is shown below.



a. State the range of f. (1 mark) **[4.3.1]**

$[-4, \infty)$. **1A**

b. On the same set of axes, sketch the graph of the inverse function $y = f^{-1}(x)$. Label any endpoints and any intercepts with their coordinates. **[4.3.1]**

1A shape
1A coordinates

c. Find the inverse function f^{-1}. **[4.3.1]**

$$f(x) = x^2 - 4x = (x-2)^2 - 4.$$

Swap x and y: $x = (y-2)^2 - 4$ **1M**

$$y = 2 \pm \sqrt{4+x}$$

As range of inverse is $(-\infty, 2]$,

$$f^{-1}(x) = 2 - \sqrt{x+4} \text{ } \mathbf{1A}$$

f^{-1} has domain $[-4, \infty)$. **1A**

d. Calculate the total area of the region(s) enclosed by the curves $y = f(x)$, $y = f^{-1}(x)$ and the line $y = -x + 2$. **[4.3.1]**

$$\text{Area} = 2\int_0^1 x^2 - 4x - (-x-2)\ dx + 2\int_1^2 -x - 2 - (x^2 - 4x)\ dx + \frac{1}{2} * 2 * 2 \text{ } \mathbf{2M}$$

1M for integrals, 1M triangle – using symmetry see graph.

Integral part:

$$2[\int_0^1 (x^2 - 3x + 2)\,dx + \int_1^2 (-x^2 + 3x - 2)\,dx]$$

$$= 2\left[\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)]_0^1 + \left(-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right)]_1^2\right] \text{ } \mathbf{1M}$$

$$= 2[((\frac{1}{3} - \frac{3}{2} + 2) - 0) + ((-\frac{8}{3} + 6 - 4) - (-\frac{1}{3} + \frac{3}{2} - 2))]$$

$$= 2[\frac{5}{6} + (-\frac{8}{3} + 2 - (-\frac{1}{3} + \frac{3}{2} - 2))]$$

$$= 2[\frac{5}{6} - \frac{8}{3} + 2 + \frac{1}{3} - \frac{3}{2} + 2]$$

$$= 2[\frac{5}{6} + (-\frac{8}{3} + \frac{6}{3}) + \frac{1}{3} - \frac{3}{2} + 2]$$

$$= 2[\frac{5}{6} - \frac{2}{3} + \frac{1}{3} - \frac{3}{2} + 2]$$

$$= 2[\frac{5}{6} + (-\frac{4}{6} + \frac{2}{6}) - \frac{9}{6} + \frac{12}{6}]$$

$$= 2[\frac{5}{6} - \frac{2}{6} - \frac{9}{6} + \frac{12}{6}]$$

$$= 2[\frac{5-2-9+12}{6}]$$

$$= 2[\frac{6}{6}]$$

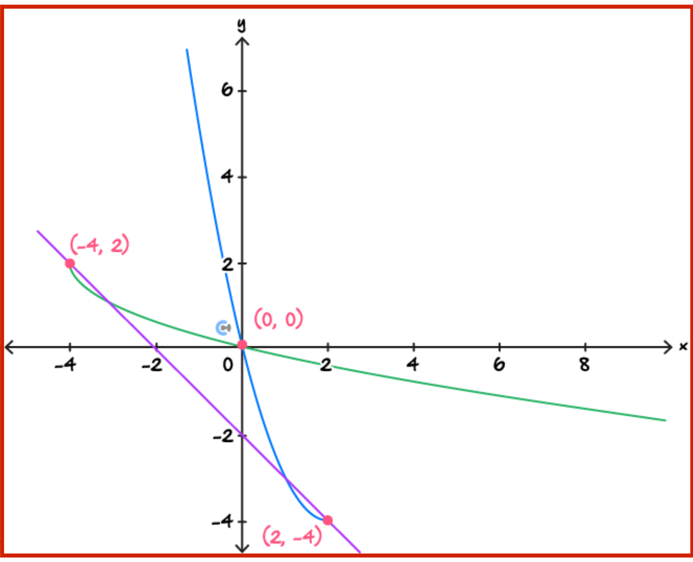
$$= 2 \cdot 1 = 2 \text{ } \mathbf{1M}$$

Area = 2 + triangle area

= 2 + 2

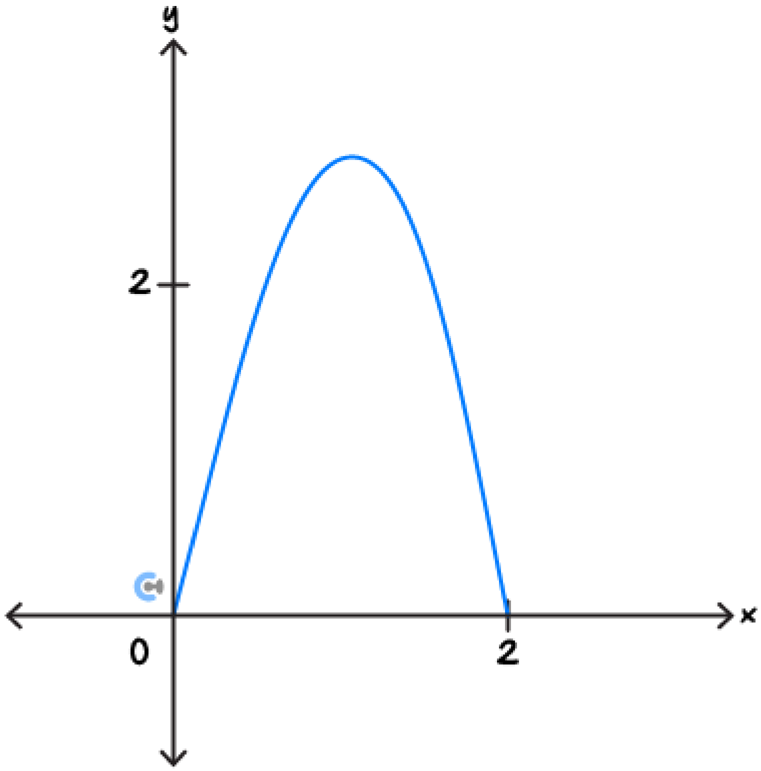
= 4

1A



Question 10

Part of the graph of $y = f(x)$ is shown below. The rule $A(k) = 2k\sin(k)$ gives the area bounded by the graph of f, the horizontal axis, and the vertical line $x = k$.



a. State the value of $A\left(\frac{\pi}{4}\right)$. **[4.3.2]**

$$A\left(\frac{\pi}{4}\right) = 2\cdot\frac{\pi}{4}\sin\left(\frac{\pi}{4}\right) = \frac{\pi}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\pi\sqrt{2}}{4}.\ \mathbf{1A}$$

b. Evaluate $f\left(\frac{\pi}{4}\right)$. **[4.3.2]**

$$\frac{d}{dk}[A(k)] = f(k).$$

$$f(k) = \frac{d}{dk}[2k\sin(k)] = 2\sin(k) + 2k\cos(k)\ \mathbf{1M}$$

$$f\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right).$$

$$f\left(\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} + 2\cdot\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} = \sqrt{2} + \frac{\pi\sqrt{2}}{4}.\ \mathbf{1A}$$

c. Consider the average value of the function f over the interval $[0, k]$ where k lies in the interval $[0, 2]$. Find the value of k that gives the maximum average value and state this maximum average value. **[4.3.2]**

$$\frac{1}{k}\int_0^k f(x)\,dx = \frac{A(k)}{k}.$$

$$\frac{A(k)}{k} = \frac{2\,k\sin(k)}{k} = 2\sin(k).\ \mathbf{1M}$$

$$\boxed{k = \frac{\pi}{2}.}\ \mathbf{1A}$$

Max average value $= 2$ **1A**

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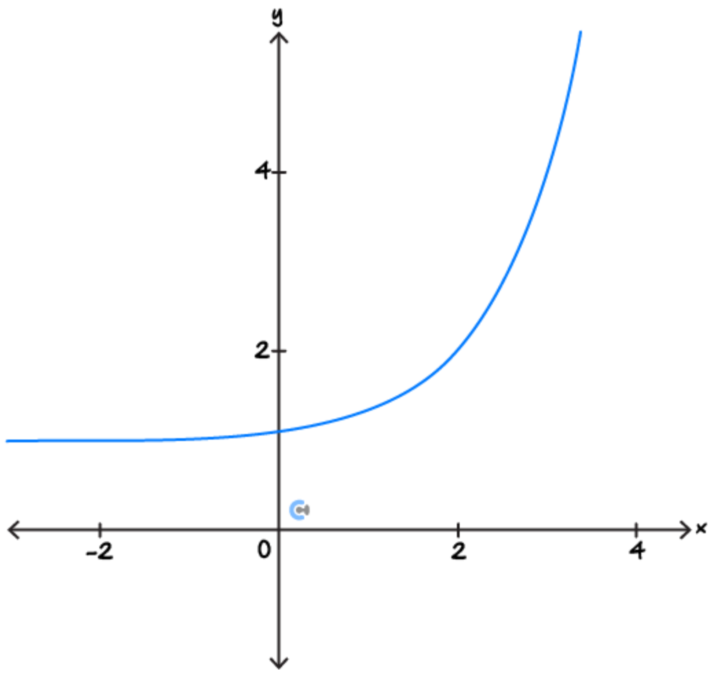
Sub-Section: Exam 2

Question 11

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^{x-2} + 1$. Part of the graph of f is shown below.



a. State f^{-1}, the inverse function of f. **[4.3.1]**

$f^{-1}(x) = \log_3(x-1) + 2$ **1A**, Domain: $x > 1$ **1M**

Define $f(x)=3^{x-2}+1$ *Done*

solve$(f(y)=x, y)$ $y=\dfrac{\ln(9\cdot(x-1))}{\ln(3)}$ and $x>1$

b. Find the area bounded by the graphs of f and f^{-1}. Give your answer correct to 5 decimal places. **[4.3.1]**

solve$(f(x)=x, x)$ $x=1.826018$ or $x=2.$

1M for intersections

$$2\cdot\int_{1.826018}^{2}(x-f(x))\,dx \quad 0.0009637$$

1A for 0.00096

1M

Exact same for CASIO

```
In[16]:= f[x_] := 3^(x-2) + 1
In[12]:= NSolve[f[x] == x && Element[x, Reals], x]
         {{x → 1.8260175600961008`}, {x → 2.`}}
Out[13]= {{x → 1.82602}, {x → 2.}}
In[15]:= NIntegrate[x - f[x], {x, 1.826018, 2}]
Out[15]= 0.000481854
```

c. Find the gradient of f and the gradient of f^{-1} at $x = 2$. **[4.3.1]**

$\frac{d}{dx}(f(x))|x=2$ 1.098612

1A each

$\frac{1}{1.0986122886681}$ 0.9102392

The functions g, where $k \in \mathbb{R}^+$, are defined with domain $\mathbb{R}$ such that, $g(x) = \frac{1}{9}e^{kx} + 1$.

d.

i. Find the value of k such that $g(x) = f(x)$. **[4.3.1]**

Define $g(x)=\frac{1}{9}\cdot e^{k\cdot x}+1$ *Done*

solve$(f(x)=g(x),k)$ 1A $k=\ln(3)$

ii. Find the rule and domain for the inverse function $g^{-1}(x)$ in terms of k. **[4.3.1]**

$g^{-1}(x) = \frac{1}{k}\ln[9(x-1)]$ **1A** *where* $x > 1$ **1A**

solve$(g(y)=x,y)$ $y=\frac{\ln(9\cdot(x-1))}{k}$ and $x>1$

e. The lines L_1 and L_2 are the tangents at the origin to the graphs of g and g^{-1}, respectively. Find the value(s) of k for which the angle between L_1 and L_2 is 30°. **[4.3.1]**

$\frac{d}{dx}(g(x))|x=0$ 1M $\frac{k}{9}$

$$\text{solve}\left(\tan(30°)=\left|\frac{\frac{k}{9}-\frac{1}{\frac{k}{9}}}{1+\frac{k}{9}\cdot\frac{1}{\frac{k}{9}}}\right|,k\right)|k>0$$

$k=3\cdot\sqrt{3}$ or $k=9\cdot\sqrt{3}$

1A

1M or split into two cases + and -

f. Let p be the value of k for which $g(x) = g^{-1}(x)$ has only one solution.

i. Find p correct to three decimal places. **[4.3.1]**

$$\text{solve}\left(g(x)=x \text{ and } \frac{d}{dx}(g(x))=1, x, k\right)$$

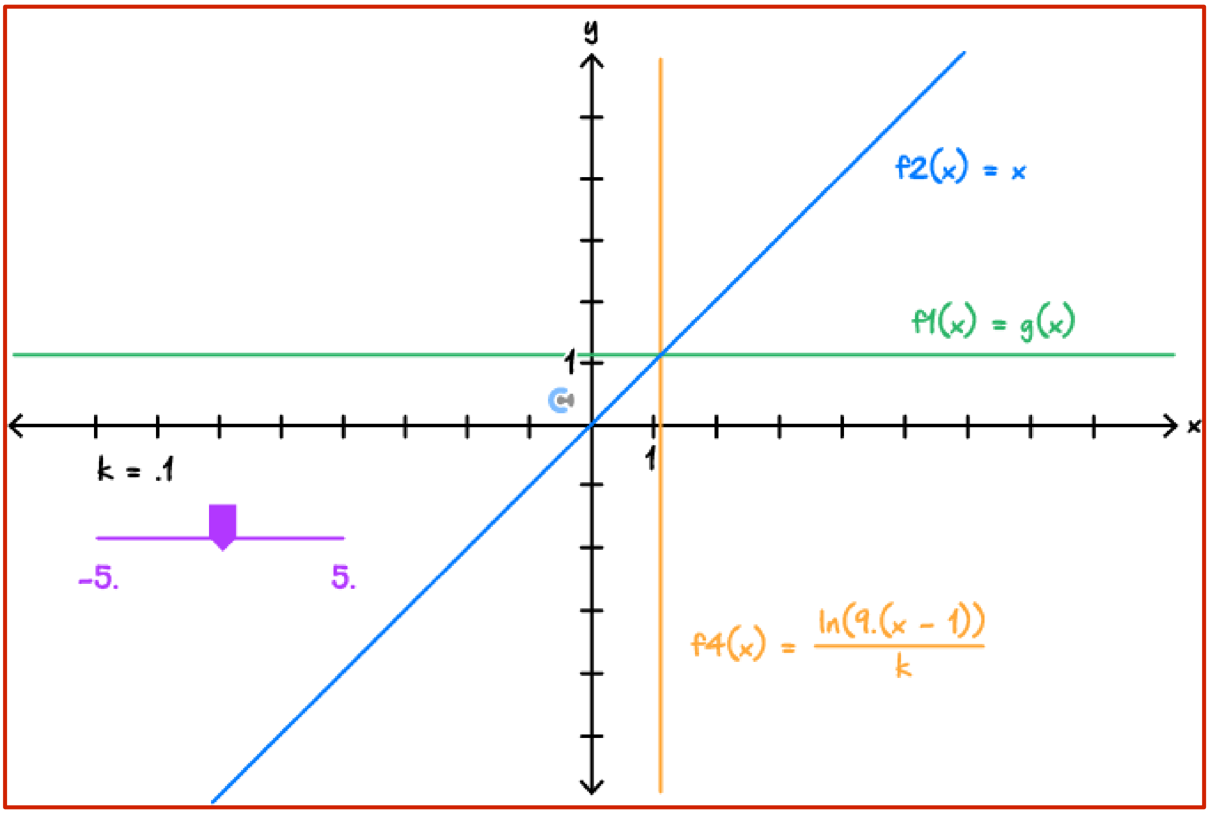
1M 1M

$$x=1.908263 \text{ and } k=1.101003$$

1A for p=1.101

ii. Let $A(k)$ be the area bounded by the graphs of $g,\ g^{-1}$ and both horizontal and vertical axes for all $k < p$.

State the largest value of b such that $A(k) > b$. **[4.3.1]**



As k approaches 0 we see the area becomes a square of length 1. Hence, $b = 1 \times 1 = 1$.

Question 12

a. Show that $\frac{d}{dx}\left(\frac{1}{1+e^x}\right) = -\frac{e^x}{(1+e^x)^2}$. **[4.3.3]**

$$\frac{d}{dx}((1+e^x)^{-1}) = -(1+e^x)^{-2} \cdot e^x \ \mathbf{1M} = -\frac{e^x}{(1+e^x)^2}$$

b. Hence, or otherwise, find the exact value of $\int_0^{\ln 3} \frac{e^x}{(1+e^x)^2}\, dx$ using **integration by recognition. [4.3.3]**

$$\int_0^{\ln(3)} \frac{e^x}{(1+e^x)^2}\, dx \qquad \frac{1}{4}$$

1A

$$\frac{d}{dx}\left(\frac{1}{1+e^x}\right) = -\frac{e^x}{(1+e^x)^2} \Rightarrow \frac{e^x}{(1+e^x)^2} = -\frac{d}{dx}\left(\frac{1}{1+e^x}\right) \ \mathbf{1M}$$

$$\int_0^{\ln 3} \frac{e^x}{(1+e^x)^2}\, dx = \left[-\frac{1}{1+e^x}\right]_0^{\ln 3} \ \mathbf{1M} = -\frac{1}{1+3} + \frac{1}{1+1} = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$$

c. Let $A(k) = \int_0^{\ln k} \frac{e^x}{(1+e^x)^2}\, dx$ where $k > 0$.

i. Show that $A(k) = \frac{k-1}{k+1} \cdot \frac{1}{2}$. **[4.3.3]**

$$A(k) = \int_0^{\ln k} \frac{e^x}{(1+e^x)^2}\, dx = \left[-\frac{1}{1+e^x}\right]_0^{\ln k} \textbf{1M} = -\frac{1}{1+k} + \frac{1}{2}$$

$$= \frac{1}{2} - \frac{1}{1+k} = \frac{(1+k)-2}{2(1+k)} \textbf{1M} = \frac{k-1}{2(k+1)}$$

ii. It is known that $A(k)$ represents the area of a function $f(x)$ from $x = 0$ to $x = \ln k$. Find the smallest b such that $b > A(k)$ for all $k > 0$. Indicate what this means in terms of area. **[4.3.3]**

$$\lim_{k \to \infty} \left(\int_0^{\ln(k)} \frac{e^x}{(1+e^x)^2}\, dx \right) \quad \frac{1}{2}$$

1M using limits or finding horizontal asymptote of A(k)

1A for $b = \frac{1}{2}$

This means the graph has a finite area **1A**

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VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make the Most of These (Free) Services!

- Experienced Contour tutors ($45+$ raw scores, $99+$ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
- Book via bit.ly/contour-methods-consult-2025 (or QR code below). - One active booking at a time (must attend before booking the next).	- Message +61 440 138 726 with questions. - Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726