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VCE Mathematical Methods ¾
Integration II [4.3]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2-Pg 10
Supplementary Questions	Pg 11-Pg 19

Section A: Compulsory Questions

Sub-Section: Fundamentals

Question 1

a. Let $f(x) = \ln(3x + 1)$.

Find the rule for the inverse function $f^{-1}(x)$. **[4.3.1]**

b. Hence, find the area bounded by the graph of $y = f(x)$, the y-axis, the x-axis and the line $y = 2$. **[4.3.1]**

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Question 2 [4.3.2]

Let $f(x) = x^2 - 2x + 3$. Find the average value of the function over the interval $[1,4]$.

Question 3

a. Show that $\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x$. **[4.3.3]**

b. Hence, evaluate $\int x \ln x \, dx$. **[4.3.3]**

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Question 4

Let $f(x) = x^3$.

a. Find the points where $f(x) = f^{-1}(x)$. **[4.3.1]**

b. Hence, find the exact area between the graphs of $f(x)$ and $f^{-1}(x)$. **[4.3.1]**

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Sub-Section: Problem Solving

Question 5

Let $f(x) = e^{-3x}\sin(2x)$ and $g(x) = e^{-3x}\cos(2x)$.

a. Differentiate $e^{-3x}\sin(2x)$ and $e^{-3x}\cos(2x)$ with respect to x. **[4.3.3]**

b. Hence, show that: **[4.3.3]**

$$e^{-3x}\sin(2x) + c_1 = -3\int e^{-3x}\sin(2x)\,dx + 2\int e^{-3x}\cos(2x)\,dx$$

and

$$e^{-3x}\cos(2x) + c_2 = -3\int e^{-3x}\cos(2x)\,dx - 2\int e^{-3x}\sin(2x)\,dx$$

c. Use the two equations from **part b.** to determine $\int e^{-3x}\sin(2x)\,dx$. **[4.3.3]**

Question 6

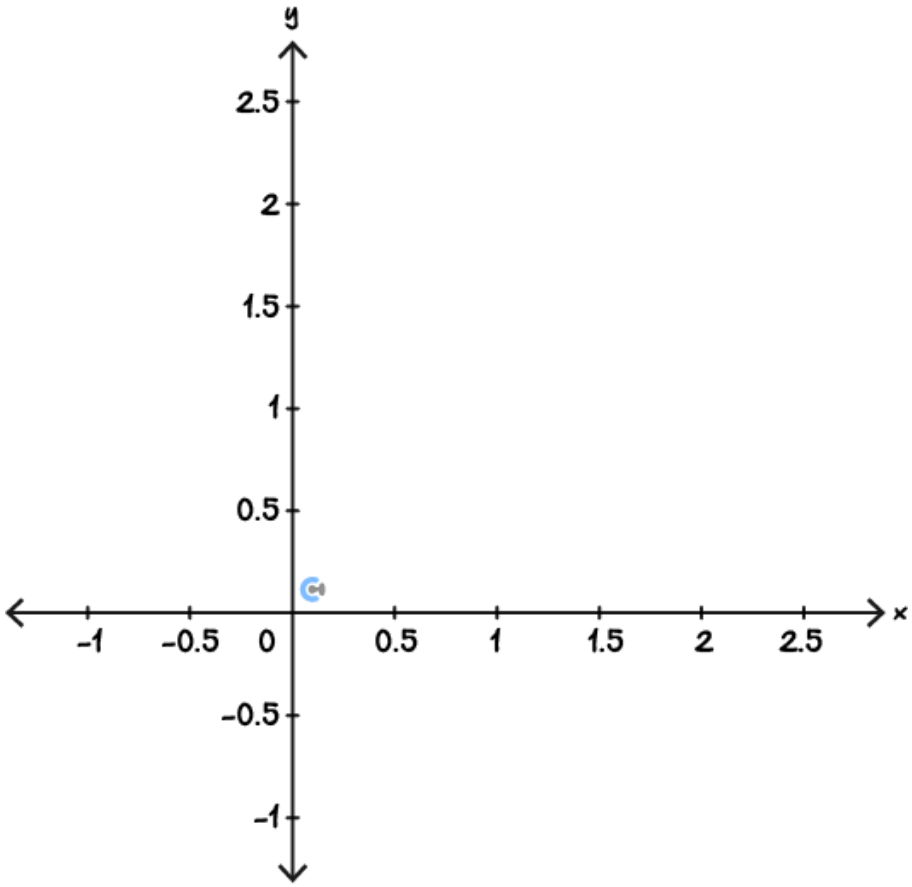
For this question, only consider quadrant 1 of the cartesian x-y plane.

a. Find the area enclosed between the parabolas $y = x^2$ and $y^2 = x$. **[4.3.1]**

b. Show that the curves $y = x^n$ and $y^n = x$ intersect at the point $(1,1)$ for all positive integers n. **[4.3.1]**

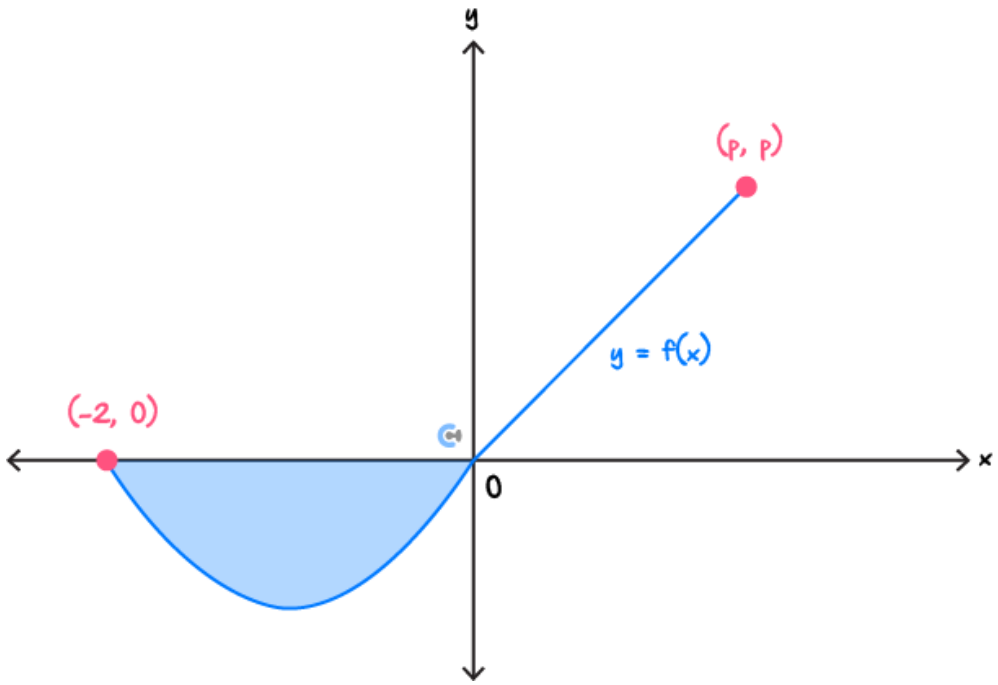
c. Show that the area of the region contained between the curves $y = x^n$ and $y^n = x$ is $\frac{n-1}{n+1}$. **[4.3.1]**

d. Describe the area between the curves for very large values of n. You use a CAS calculator to help you visualise. **[4.3.1]**



Question 7 [4.3.2]

The graph of a function $f: [-2, p] \to \mathbb{R}$ is shown. It consists of a curved segment from $x = -2$ to $x = 0$ and and a straight line from the origin O to the point (p, p), where $p > 0$. The area of the shaded region under the curve from $x = -2$ to $x = 0$ is $\frac{25}{8}$. The average value of f over the interval $[-2, p]$ is zero.



Find the value of p.

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Question 8

a. If $y = x\log_e x$, find $\frac{dy}{dx}$. **[4.3.3]**

b. Hence, evaluate $\int_1^e \log_e x \, dx$. **[4.3.3]**

c. If $y = x(\log_e x)^n$, where n is a natural number, find $\frac{dy}{dx}$. **[4.3.3]**

d. Let $I_n = \int_1^e (\log_e x)^n \, dx$.

Show that, for $n > 1$, $I_n + nI_{n-1} = e$. **[4.3.3]**

e. Find $\int_1^e (\log_e x)^3 \, dx$. **[4.3.3]**

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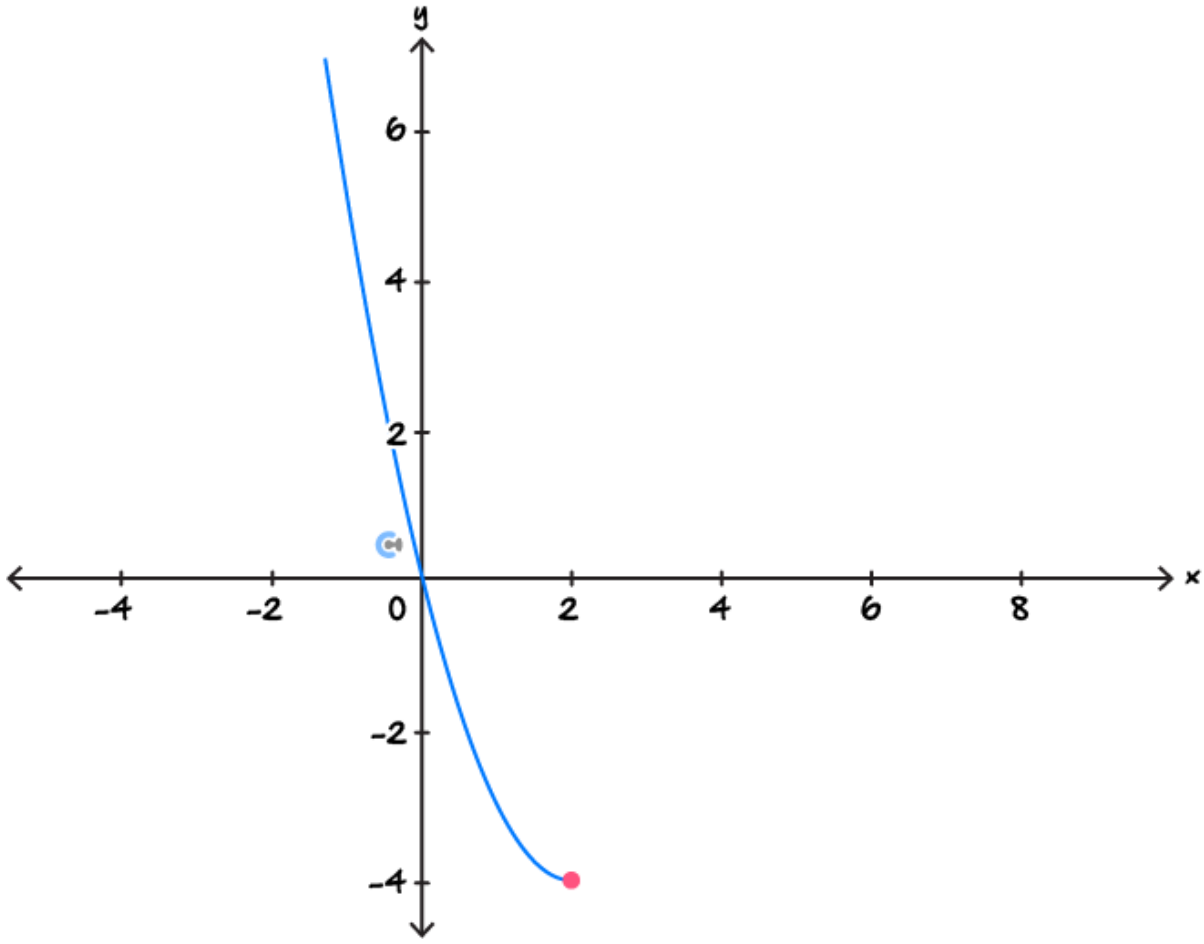
Section B: Supplementary Questions

Sub-Section: Exam 1

Question 9

Let $f : (-\infty, 2] \to \mathbb{R}, f(x) = x^2 - 4x$.

A portion of the graph of $y = f(x)$ is shown below.



a. State the range of f. (1 mark) **[4.3.1]**

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b. On the same set of axes, sketch the graph of the inverse function $y = f^{-1}(x)$. Label any endpoints and any intercepts with their coordinates. **[4.3.1]**

c. Find the inverse function f^{-1}. **[4.3.1]**

d. Calculate the total area of the region(s) enclosed by the curves $y = f(x)$, $y = f^{-1}(x)$ and the line $y = -x + 2$. **[4.3.1]**

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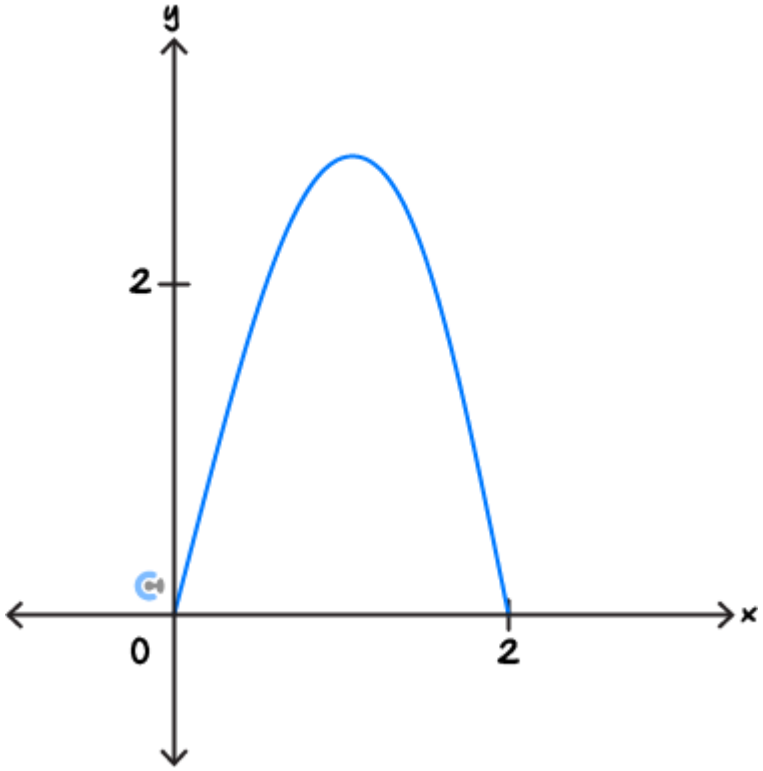
Question 10

Part of the graph of $y = f(x)$ is shown below. The rule $A(k) = 2k \sin(k)$ gives the area bounded by the graph of f, the horizontal axis, and the vertical line $x = k$.



a. State the value of $A\left(\frac{\pi}{4}\right)$. **[4.3.2]**

b. Evaluate $f\left(\frac{\pi}{4}\right)$. **[4.3.2]**

c. Consider the average value of the function f over the interval $[0, k]$ where k lies in the interval $[0, 2]$. Find the value of k that gives the maximum average value and state this maximum average value. **[4.3.2]**

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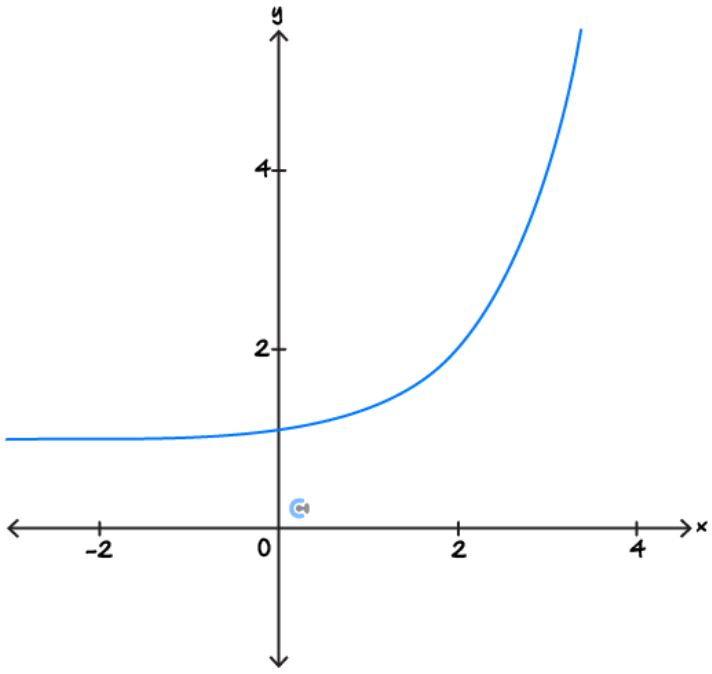
Sub-Section: Exam 2

Question 11

Let $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 3^{x-2} + 1$. Part of the graph of f is shown below.



a. State f^{-1}, the inverse function of f. **[4.3.1]**

b. Find the area bounded by the graphs of f and f^{-1}. Give your answer correct to 5 decimal places. **[4.3.1]**

c. Find the gradient of f and the gradient of f^{-1} at $x = 2$. **[4.3.1]**

The functions g, where $k \in \mathbb{R}^+$, are defined with domain $\mathbb{R}$ such that, $g(x) = \frac{1}{9}e^{kx} + 1$.

d.

i. Find the value of k such that $g(x) = f(x)$. **[4.3.1]**

ii. Find the rule and domain for the inverse function $g^{-1}(x)$ in terms of k. **[4.3.1]**

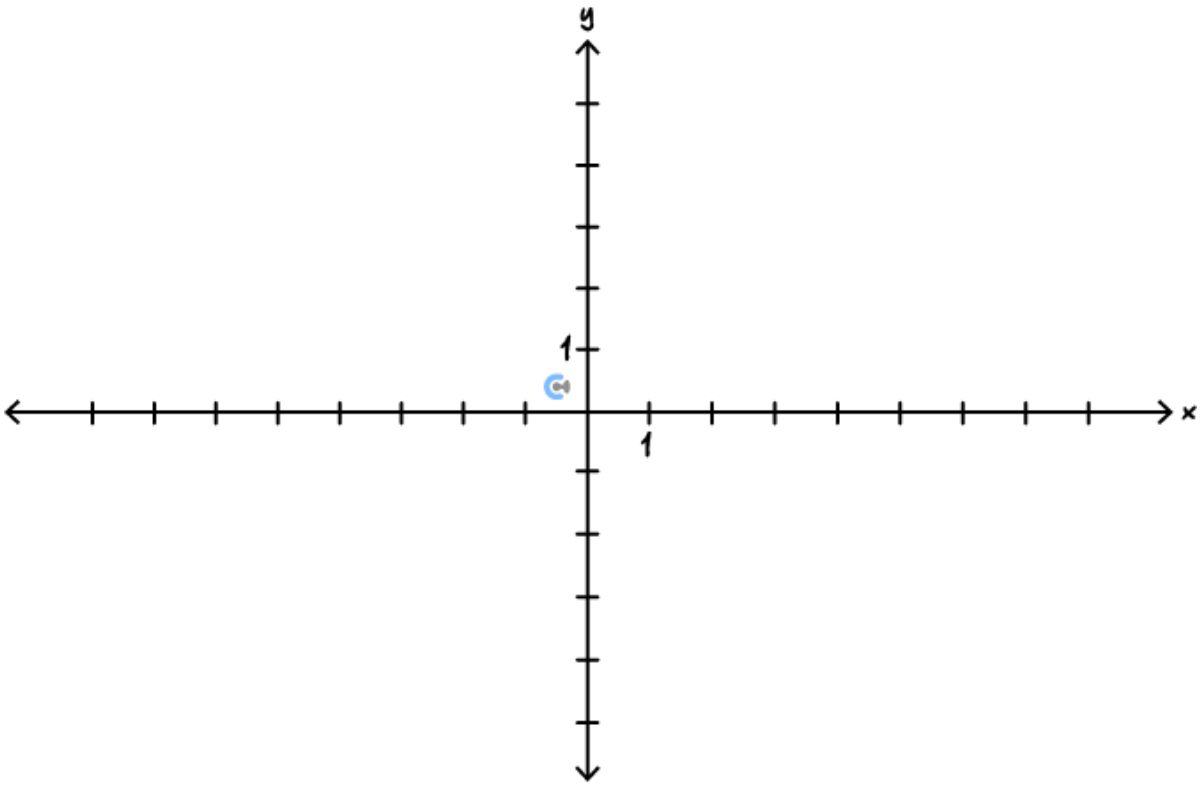
e. The lines L_1 and L_2 are the tangents at the origin to the graphs of g and g^{-1}, respectively. Find the value(s) of k for which the angle between L_1 and L_2 is 30°. **[4.3.1]**

f. Let p be the value of k for which $g(x) = g^{-1}(x)$ has only one solution.

i. Find p correct to three decimal places. **[4.3.1]**

ii. Let $A(k)$ be the area bounded by the graphs of g, g^{-1} and both horizontal and vertical axes for all $k < p$.

State the largest value of b such that $A(k) > b$. **[4.3.1]**



Question 12

a. Show that $\frac{d}{dx}\left(\frac{1}{1+e^x}\right) = -\frac{e^x}{(1+e^x)^2}$. **[4.3.3]**

b. Hence, or otherwise, find the exact value of $\int_0^{\ln 3} \frac{e^x}{(1+e^x)^2}\,dx$ using **integration by recognition. [4.3.3]**

c. Let $A(k) = \int_0^{\ln k} \frac{e^x}{(1+e^x)^2}\, dx$ where $k > 0$.

i. Show that $A(k) = \frac{k-1}{k+1} \cdot \frac{1}{2}$. **[4.3.3]**

ii. It is known that $A(k)$ represents the area of a function $f(x)$ from $x = 0$ to $x = \ln k$. Find the smallest b such that $b > A(k)$ for all $k > 0$. Indicate what this means in terms of area. **[4.3.3]**

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