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VCE Mathematical Methods ¾

Integration I [4.2]

Test Solutions

24 Marks. 1 Minute Reading. 19 Minutes Writing.

Results:

Test Questions	_____ / 18
Extension Test Questions	_____ / 6

Section A: Test Questions (18 Marks)

Question 1 (3 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. When the function is increasing, the right-endpoint approximation will always be an underestimation.		✓
b. Trapezoidal approximation is always the average of left and right-endpoint approximation.	✓	
c. When the question says "find the area", it can be signed or total area.		✓
d. Signed area is always positive and represents the change of the antidiff. function.		✓
e. When finding the total area (without using modulus), we solve for areas above and below the x-axis separately.	✓	
f. If the two functions are both below the x-axis, we do **Bottom–Top** to find the area between them.		✓

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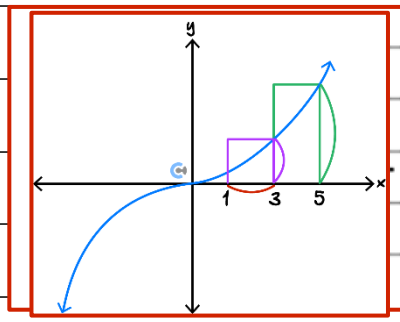
Question 2 (6 marks)

a. Approximate the area under $y = x^3$ between $x = 1$ to $x = 5$ using the right-endpoint method.

Use step size of 2. (2 marks)



$2 \times f(3) + 2 \times f(5)$

$= 2 \cdot 27 + 2 \cdot 125$

$= 304$

b. Approximate the area under $y = x^3$ between $x = 1$ to $x = 5$ using the trapezoidal method.

Use a step size of 2. (2 marks)

```
f[x_] := x^3

1/2 * (f[1] + f[3]) * 2 + 1/2 * (f[3] + f[5]) * 2

180
```

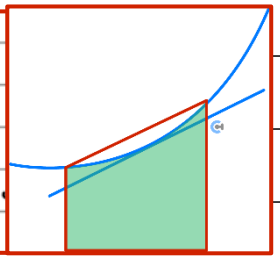
c. Determine which answers out of **part a.** and **b.** were more accurate overall.

State whether the approximation was underestimation or overestimation. (2 marks)

b. Trapezoidal is better.

∴ Overestimation (Concave up)

⇒ "Above tangent"



Question 3 (2 marks)

Let $f(x) = (2x-1)^3$. Find the signed area under the function $f(x)$ from $x = 0$ to $x = 2$.

Tutor: Emphasise that we only need one integral here.

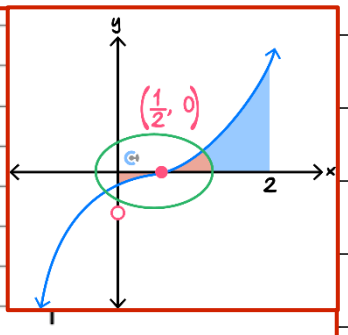
$$\int_0^2 (2x-1)^3\,dx$$

$$= \left[\frac{1}{4}(2x-1)^4 \cdot \frac{1}{2}\right]_0^2$$

$$= \frac{1}{8}(4-1)^4 - \frac{1}{8}(0-1)^4$$

$$= \frac{1}{8}\cdot 3^4 - \frac{1}{8}\cdot 1^4 = \frac{81}{8} - \frac{1}{8}$$

$$= \frac{80}{8} = 10.$$



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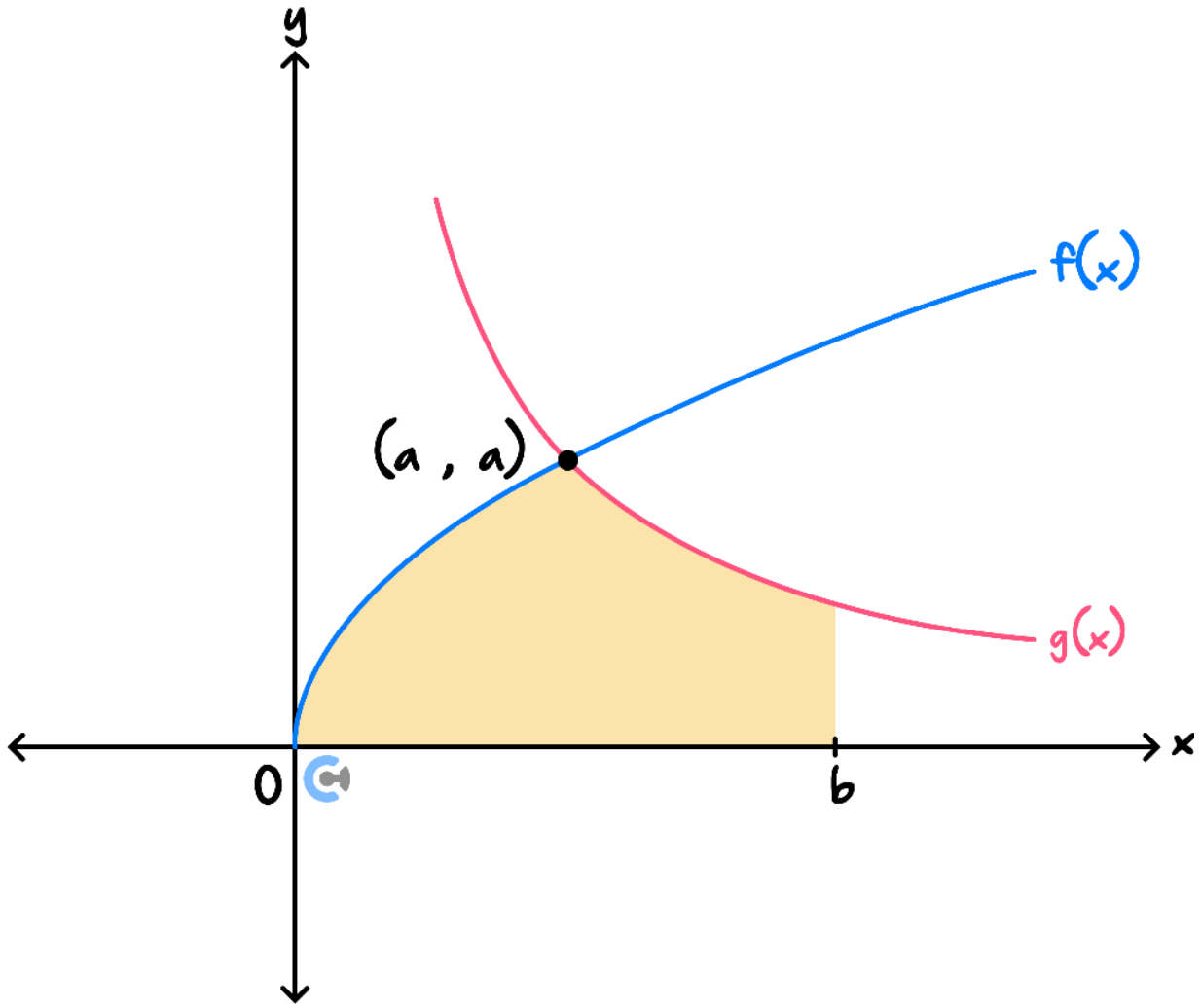
Question 4

Construct the integral for the shaded region given in the diagram.



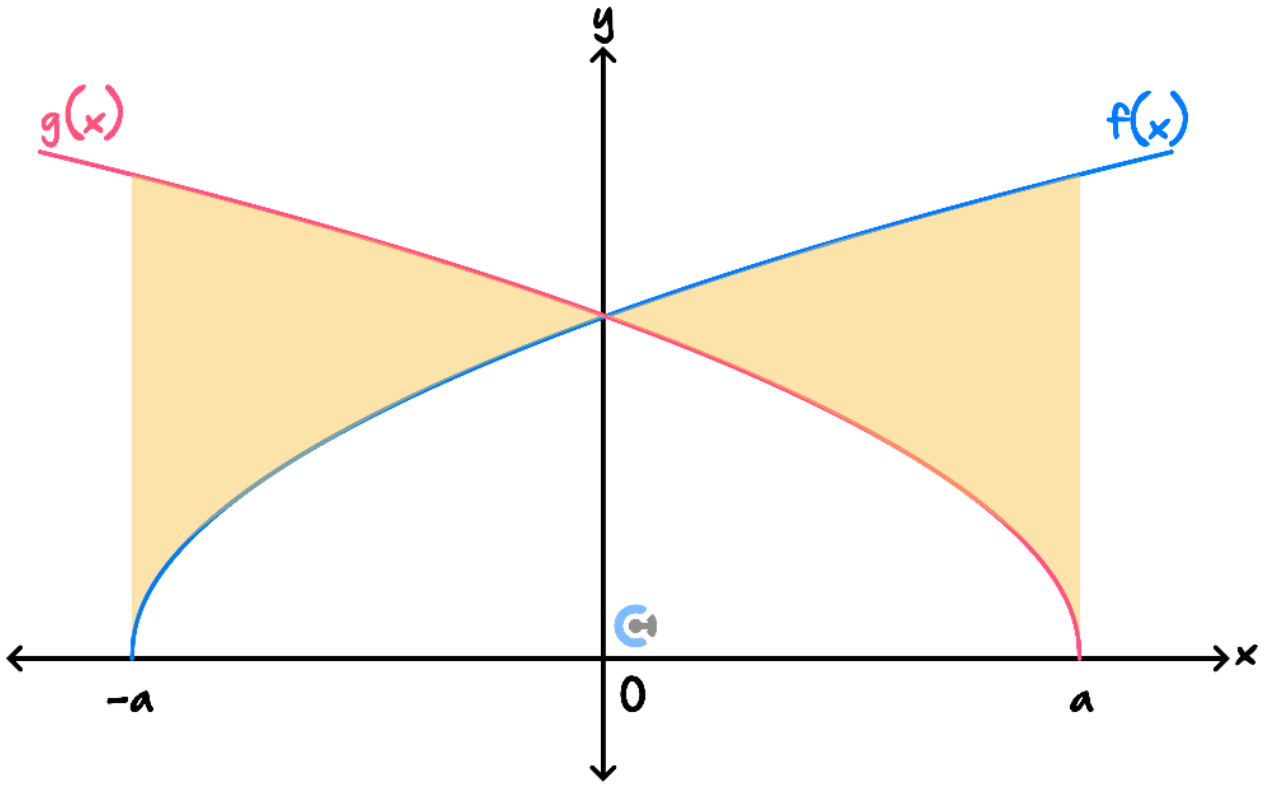
$$\int_0^a f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_0^b f(x)\,dx$$

$$-\int_a^b f(x) - g(x)\,dx$$

Question 5

Construct the integral for the shaded region given in the diagram.



$$2\int_0^a f(x) - g(x)\,dx$$

$$\int_0^a f(x) - g(x)\,dx + \int_{-a}^0 g(x) - f(x)\,dx$$

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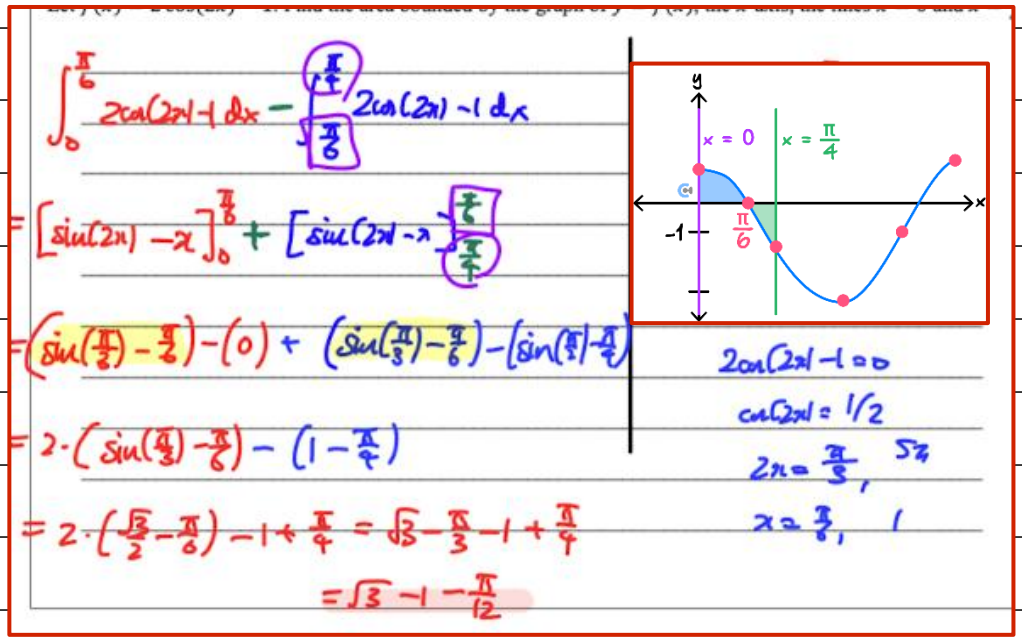
Question 6 (4 marks)

Let $f(x) = 2\cos(2x) - 1$. Find the area bounded by the graph of $y = f(x)$, the x-axis, the lines $x = 0$ and $x = \frac{\pi}{4}$.



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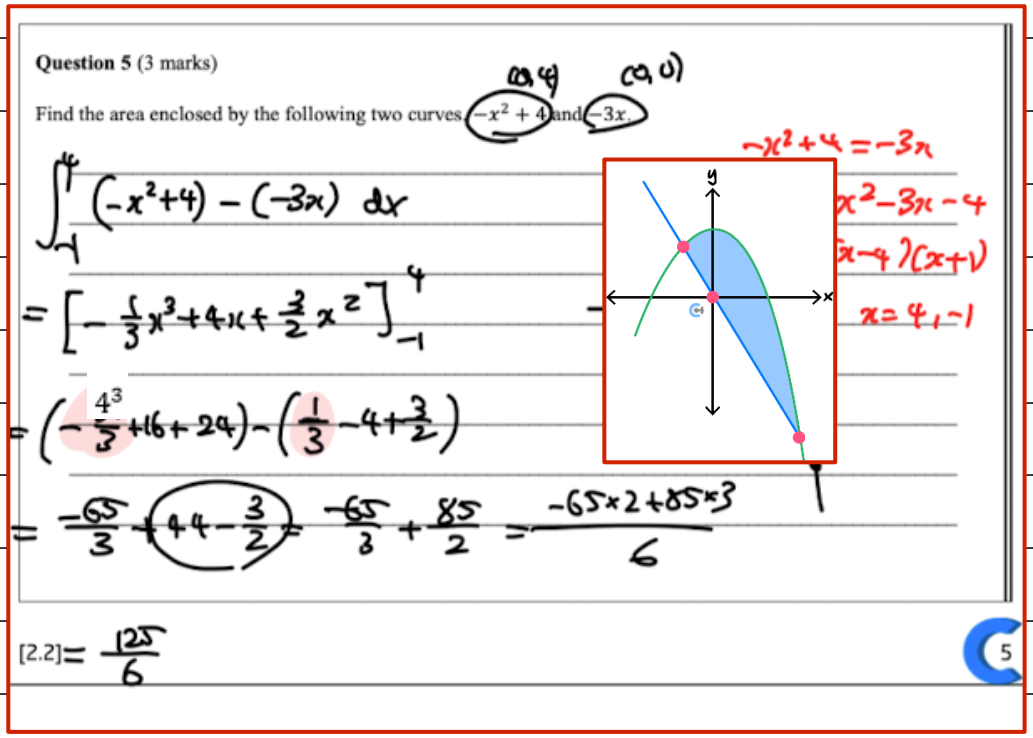
Question 7 (3 marks)

Find the area enclosed by the following two curves $-x^2 + 4$ and $-3x$.



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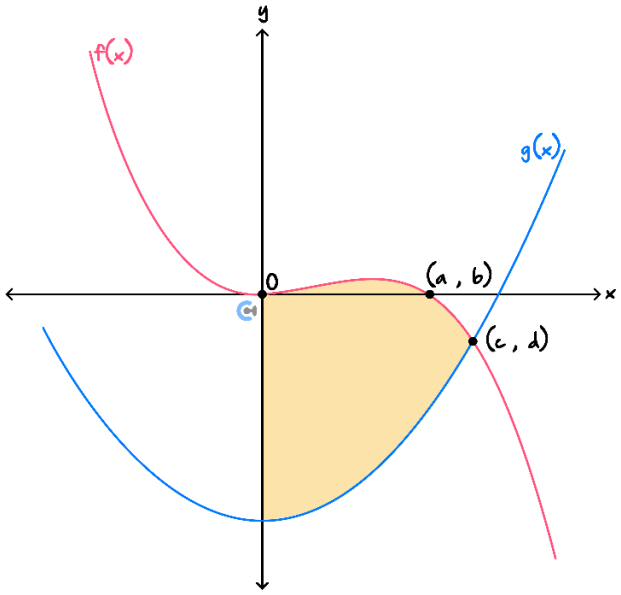
Question 8

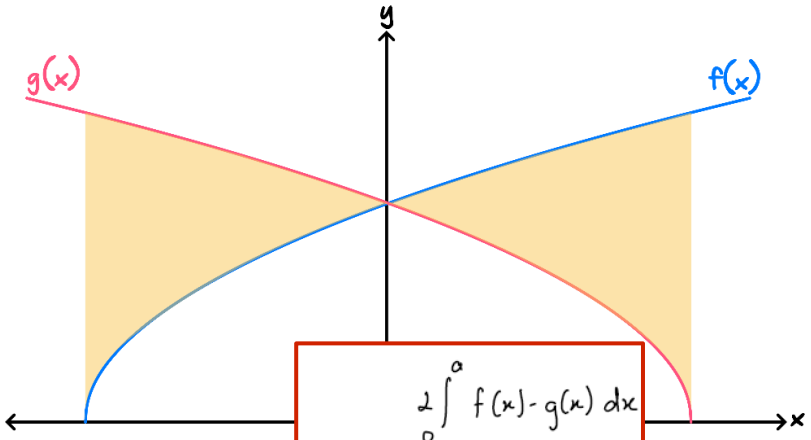
Construct the integral in the most simplified way possible for the shaded region given in the diagram.

a.



$$\int_0^c f(x) - g(x)\,dx$$

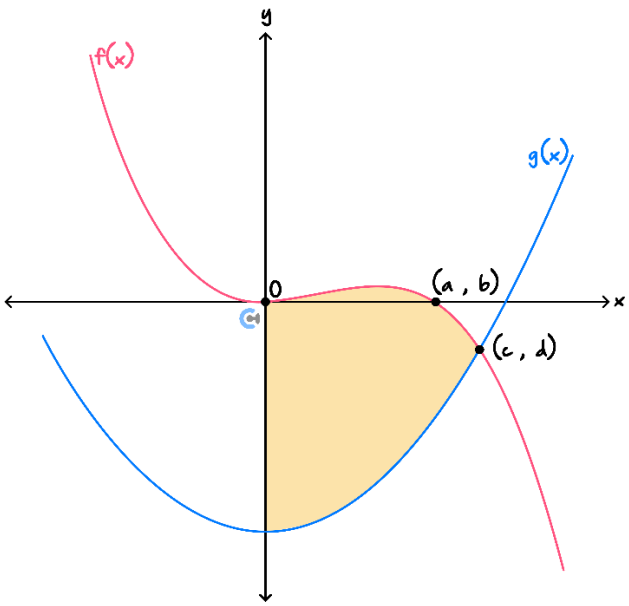
b.



$$2\int_0^a f(x) - g(x)\,dx$$

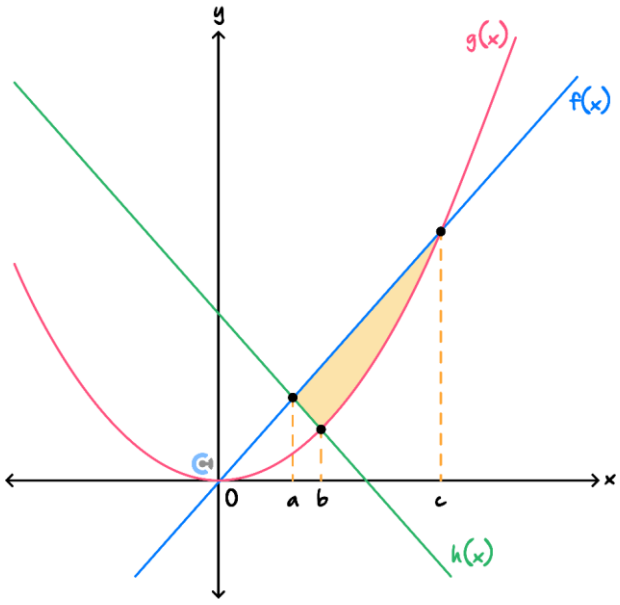
$$\int_0^a f(x) - g(x)\,dx + \int_{-a}^0 g(x) - f(x)\,dx$$

c.



$$\int_0^c f(x) - g(x)\,dx$$

d.



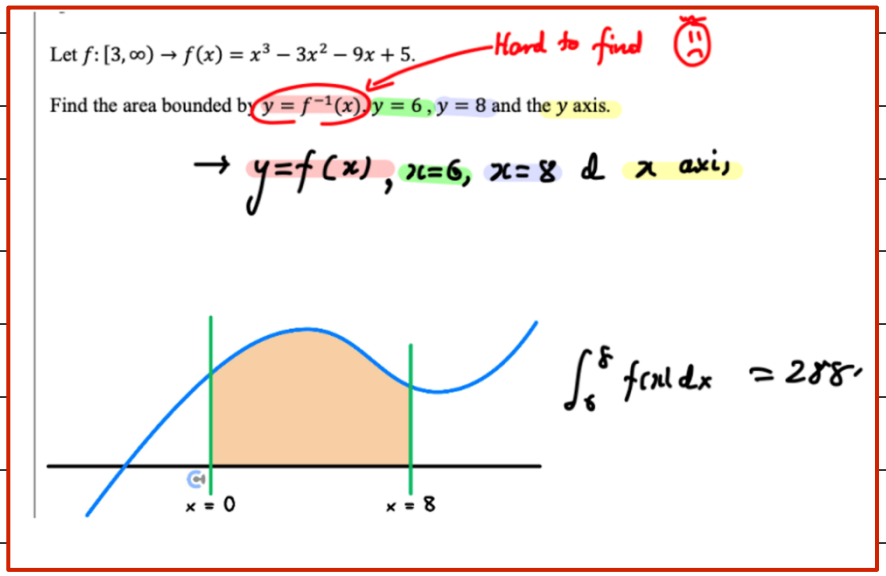
$$\int_a^b f(x) - h(x)\,dx + \int_b^c f(x) - g(x)\,dx$$

Section B: Extension Test Questions (6 Marks)

Question 9 (3 marks)

Let $f: [3, \infty) \to f(x) = x^3 - 3x^2 - 9x + 5$.

Find the area bounded by $y = f^{-1}(x), y = 6, y = 8$ and the y-axis.



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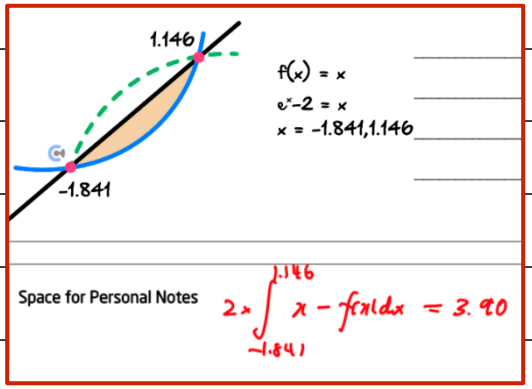
Question 10 (3 marks) **Tech-Active.**

Let $f(x) = e^x - 2$.

Find the area bounded by $y = f(x)$ and $y = f^{-1}(x)$.



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