CONTOUR EDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Integration I [4.2]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 - Pg 22
Supplementary Questions	Pg 23 - Pg 36

Section A: Compulsory Questions

Sub-Section: Basics

Question 1

Consider the function $f(x) = x^2 + 1$.

a. Approximate the area under the curve $y = f(x)$ from $x = 0$ to $x = 2$ using two rectangles of equal width with left endpoints.

$$\Delta x = \frac{2-0}{2} = 1$$

$f(0) = 0^2 + 1 = 1$ and $f(1) = 1^2 + 1 = 2$

$A = 1 \times (1 + 2)$ **1M**

$= 3$ **1A**

b. Is this approximation an overestimate or an underestimate of the true area? Explain briefly.

Since $f(x) = x^2 + 1$ is increasing on $[0,2]$ **1M**

the left-endpoint method underestimates the area. **1A**

Space for Personal Notes

Question 2

Find the area bounded by the curve $y = 1 - x^2$ and the x-axis.

$$1 - x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = -1,\ 1.$$

$$\int_{-1}^{1} (1 - x^2)dx \quad \mathbf{1M} = \left[x - \frac{x^3}{3}\right]_{-1}^{1} \quad \mathbf{1M}$$

$$= \left(1 - \frac{1}{3}\right) - \left((-1) - \left(-\frac{1}{3}\right)\right)$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

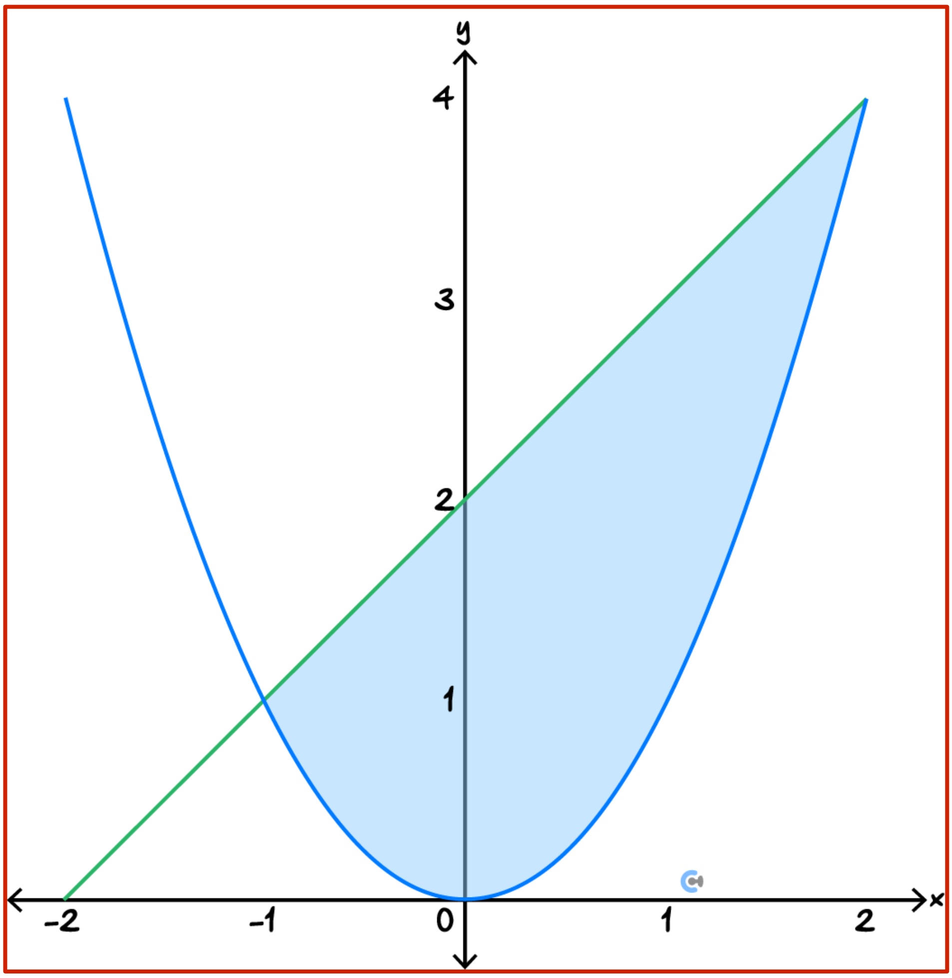
$$= \frac{2}{3} - \left(-\frac{2}{3}\right)$$

$$= \frac{4}{3}. \quad \mathbf{1A}$$

Space for Personal Notes

Question 3

Find the area enclosed between the curve $y = x + 2$ and $y = x^2$.



$$x + 2 = x^2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0, \quad x = -1, 2. \quad \textbf{1M}$$

$$\int_{-1}^{2} [(x+2) - x^2]dx \quad \textbf{1M} = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} \textbf{1M}$$

At $x = 2$: $\frac{2^2}{2} + 2(2) - \frac{2^3}{3} = 2 + 4 - \frac{8}{3} = \frac{10}{3}$,

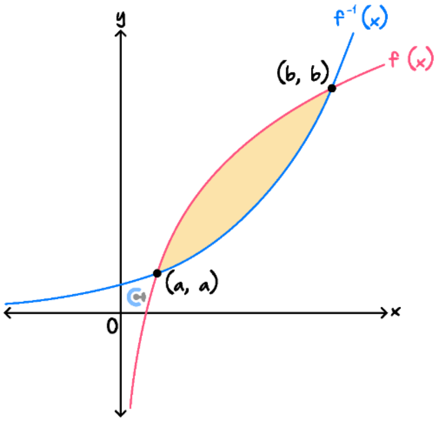
At $x = -1$: $\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3} = \frac{1}{2} - 2 + \frac{1}{3} = -\frac{7}{6}$,

Difference: $\frac{10}{3} - (-\frac{7}{6}) = \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$. **1A**

Sub-Section: Problem Solving

Question 4

Construct the integral for the shaded region given in the diagram.



$$2\int_a^b x - f^{-1}(x)\,dx$$

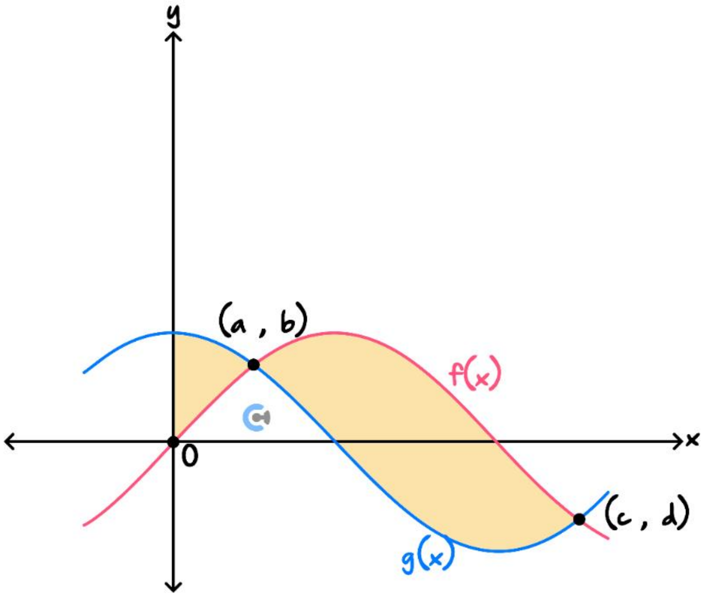
$$2\int_a^b f(x) - x\,dx$$

$$\int_a^b f(x) - f^{-1}(x)\,dx$$

Space for Personal Notes

Question 5

Construct the integral for the shaded region given in the diagram.



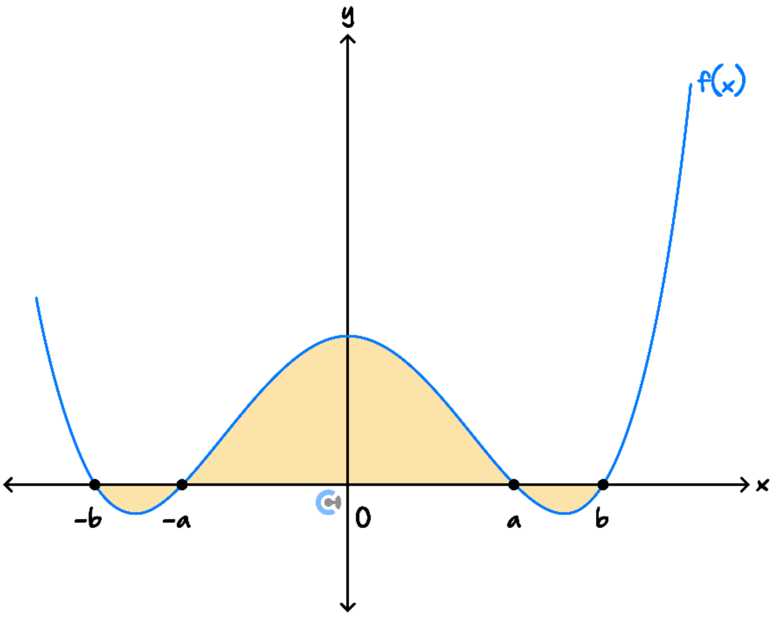
$$\int_0^a g(x)-f(x)\,dx + \int_a^c f(x)-g(x)\,dx$$

$$\int_0^c |f(x)-g(x)|\,dx$$

Space for Personal Notes

Question 6

Construct the integral for the shaded region given in the diagram.



$$2\left(\int_0^a f(x)\,dx - \int_a^b f(x)\,dx\right)$$

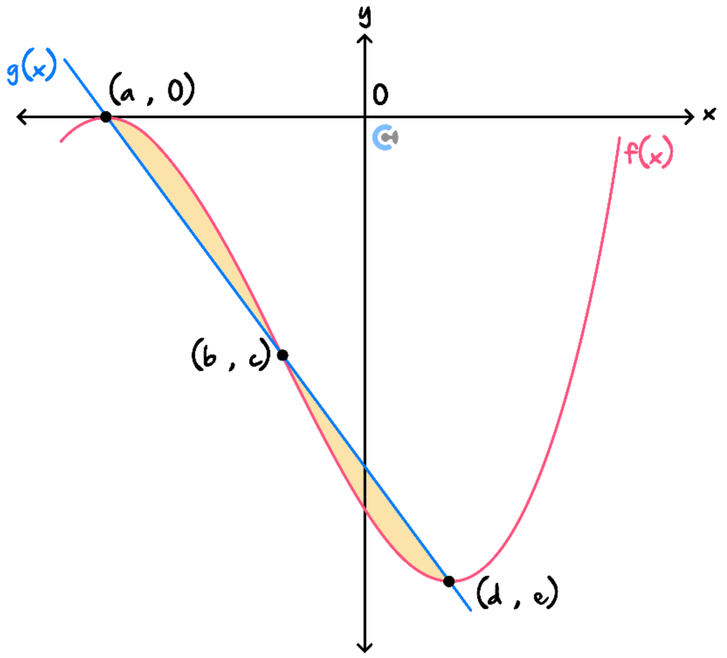
$$\int_{-a}^{a} f(x)\,dx - \int_a^b f(x)\,dx$$

$$-\int_{-b}^{-a} f(x)\,dx$$

Space for Personal Notes

Question 7

Construct the integral for the shaded region given in the diagram.



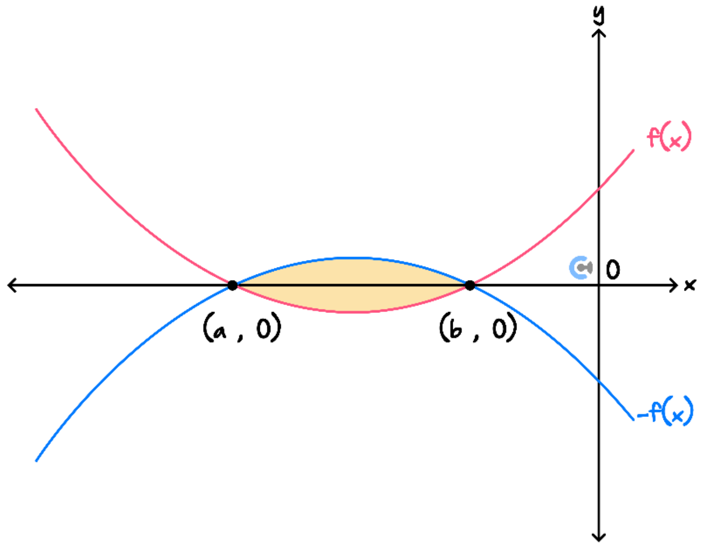
$$2\int_a^b f(x)-g(x)\,dx$$

$$\int_a^b f(x)-g(x)\,dx+\int_b^d g(x)-f(x)\,dx$$

Space for Personal Notes

Question 8

Construct the integral for the shaded region given in the diagram.

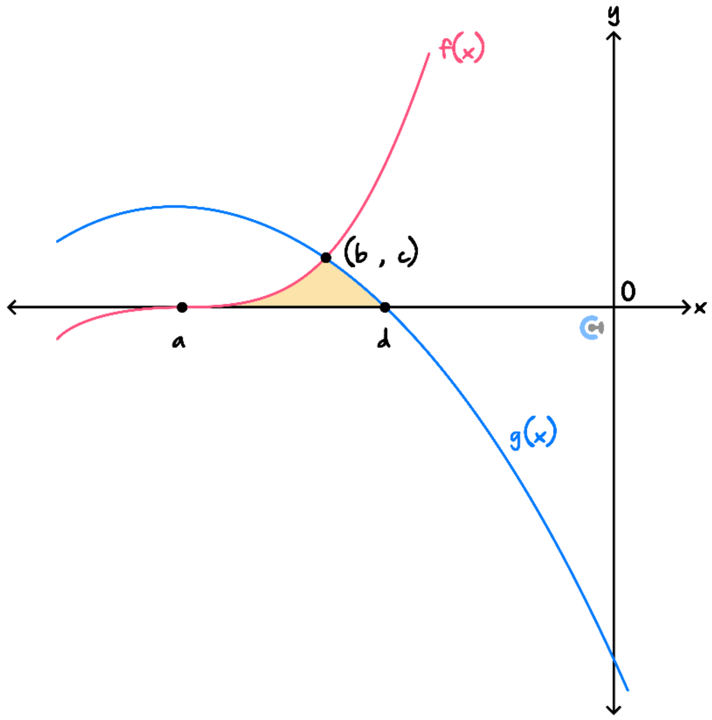


$$2\int_a^b f(x)\,dx$$

Space for Personal Notes

Question 9

Construct the integral for the shaded region given in the diagram.

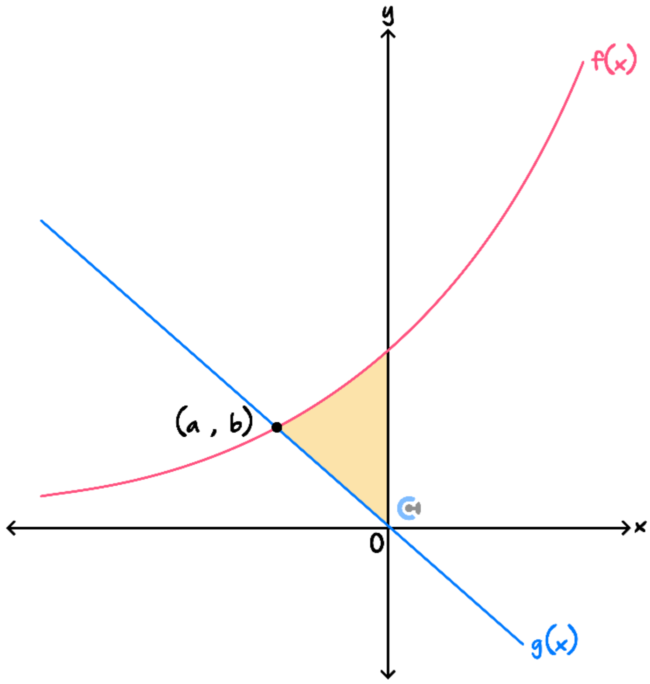


$$\int_a^b f(x)\,dx + \int_b^d g(x)\,dx$$

Space for Personal Notes

Question 10

Construct the integral for the shaded region given in the diagram.

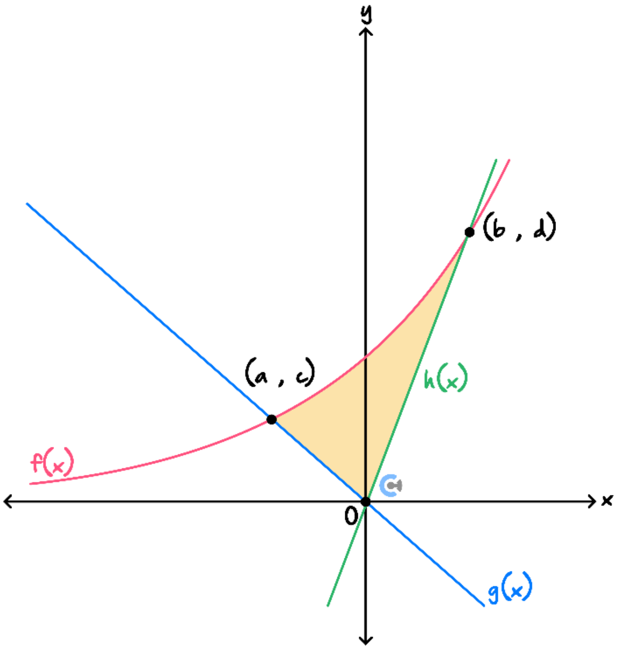


$$\int_a^0 f(x) - g(x)\,dx$$

Space for Personal Notes

Question 11

Construct the integral for the shaded region given in the diagram.

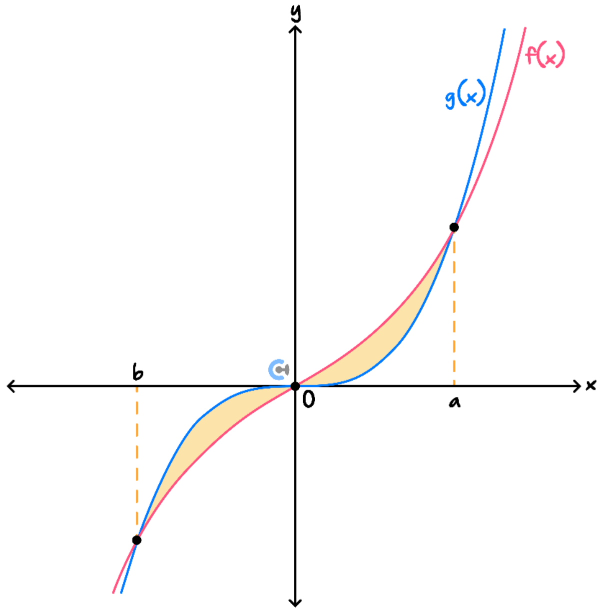


$$\int_{a}^{0} (f(x) - g(x))\ dx + \int_{0}^{b} (f(x) - h(x))\ dx$$

Space for Personal Notes

Question 12

Construct the integral for the shaded region given in the diagram.



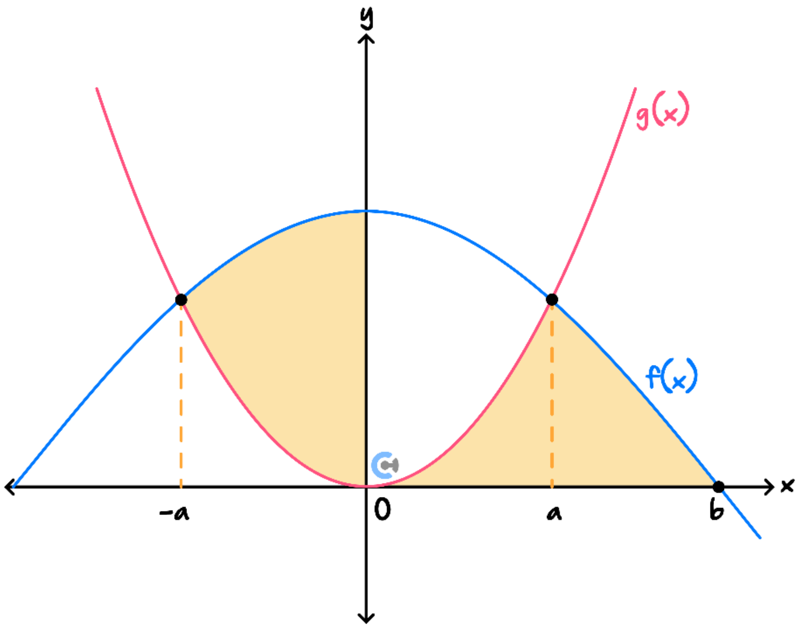
$$2\int_0^a f(x)-g(x)\,dx$$

$$\int_b^0 g(x)-f(x)\,dx + \int_0^a f(x)-g(x)\,dx$$

Space for Personal Notes

Question 13

Construct the integral for the shaded region given in the diagram.

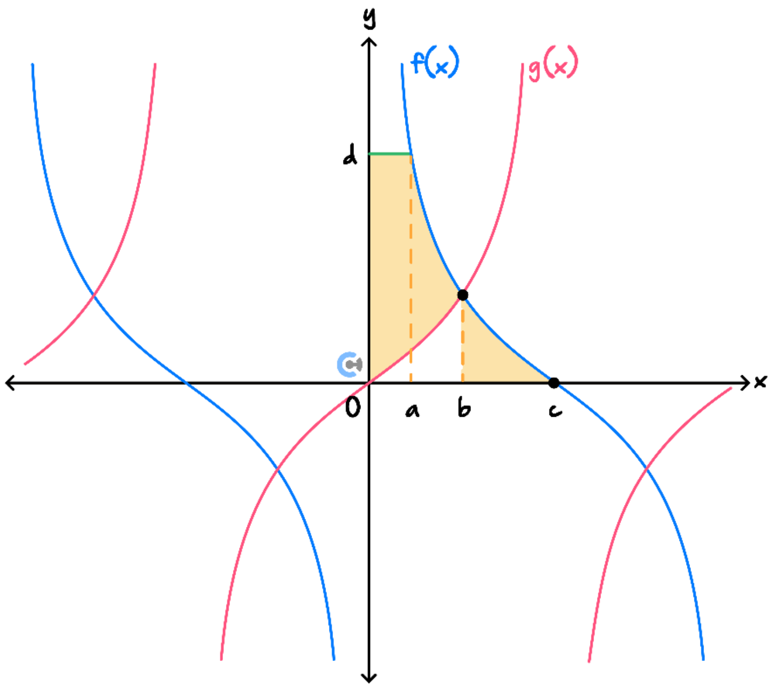


$$\int_{-a}^{0} f(x)-g(x)\,dx + \int_{0}^{a} g(x)\,dx + \int_{a}^{b} f(x)\,dx$$

Space for Personal Notes

Question 14

Construct the integral for the shaded region given in the diagram.



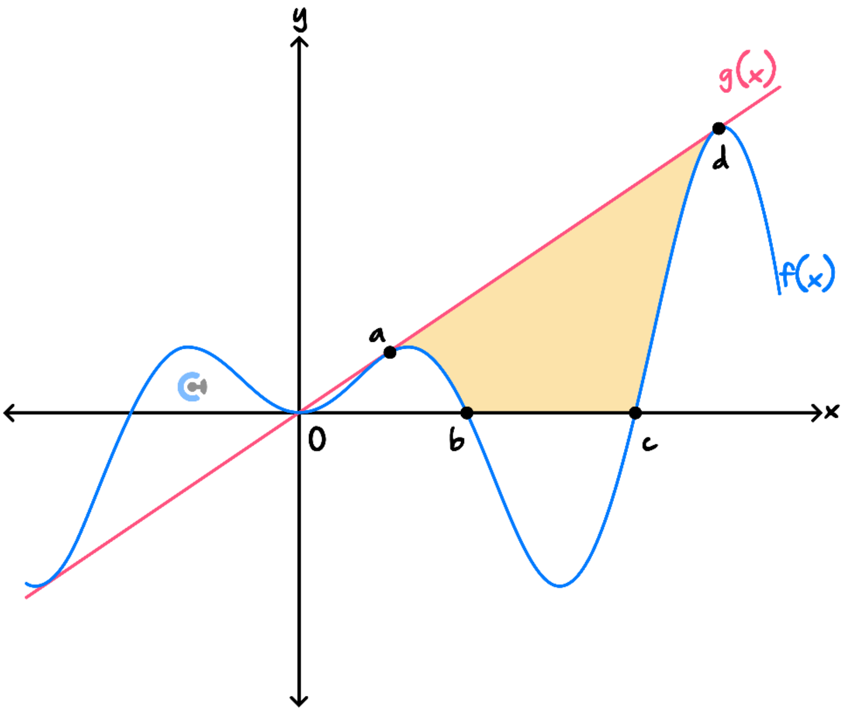
$$\int_0^a d - g(x)\,dx + \int_a^b f(x) - g(x)\,dx + \int_b^c f(x)\,dx$$

$$ad + \int_a^c f(x)\,dx - \int_0^b g(x)\,dx$$

Space for Personal Notes

Question 15

Construct the integral for the shaded region given in the diagram.



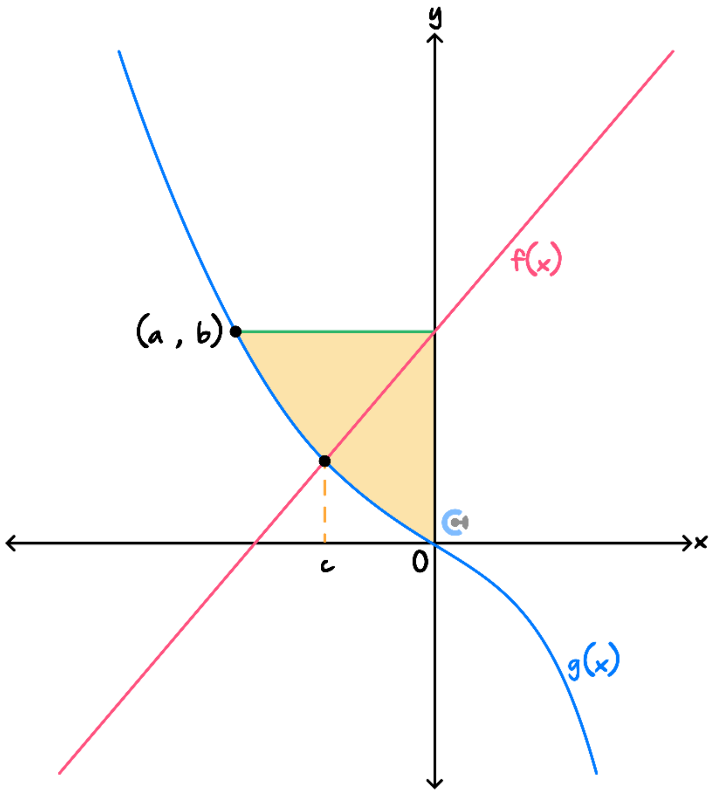
$$\int_a^d g(x)\,dx - \int_a^b f(x)\,dx - \int_c^d f(x)\,dx$$

$$\int_a^b g(x) - f(x)\,dx + \int_c^d g(x) - f(x)\,dx + \int_b^c g(x)\,dx$$

Space for Personal Notes

Question 16

Construct the integral for the shaded region given in the diagram.



$$\int_a^0 b - g(x)\, dx$$

$$\int_a^c b - g(x)\, dx + \int_c^0 b - f(x)\, dx + \int_c^0 f(x) - g(x)\, dx$$

Space for Personal Notes

Question 17

Given that $\int_a^c f(x)\,dx = 10$ and the total area between $x = a$ and $x = b$ is 16 with $f(x) \geq 0$ on $[a, b]$ and $a < b < c$, find $\int_b^c f(x)\,dx$.

$$\int_a^c f(x)dx = 10, \int_a^b f(x)dx = 16,$$
$$\int_b^c f(x)dx = 10 - 16 \textbf{ 1M} = -6 \textbf{ 1A}$$

Question 18

Let $f(x)$ be a function such that $\int_0^4 f(x)\,dx = 5$ and the total area bounded by the graph of $y = f(x)$ and the x-axis on $[0, 4]$ is 13. The function $f(x)$ crosses the x-axis exactly once in $(0, 4)$ at $x = k$, with $f(x) \leq 0$ on $[0, k]$ and $f(x) \geq 0$ on $[k, 4]$.

a. Write two equations that relate $A = \int_0^k f(x)\,dx$ and $B = \int_k^4 f(x)\,dx$.

$$A + B = 5,$$
$$-A + B = 13.$$

1A each equation.

b. Hence, find the value of $\int_0^k f(x)\,dx$.

$$(-A + B) - (A + B) = 13 - 5, \quad \textbf{1M}$$
$$-2A = 8,$$
$$A = -4. \quad \textbf{1A}$$

Space for Personal Notes

Question 19

The graph of $y = f(x)$ consists of a straight-line segment from $(0, 1)$ to $(1, 2)$ and a quarter circle of radius 1 centred at $(1, 1)$ connecting $(1, 2)$ to $(2, 1)$. Evaluate $\int_0^2 f(x)\,dx$.

$$0 \le x \le 1: f(x) = x + 1,$$
$$1 \le x \le 2: f(x) = 1 + \sqrt{1 - (x-1)^2}$$

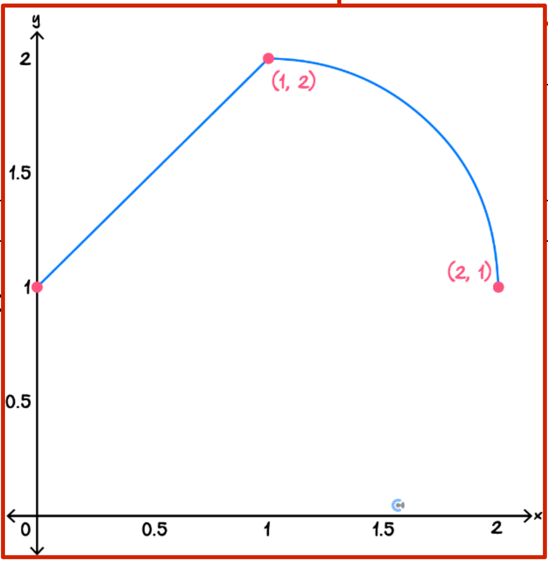
$$\int_0^2 f(x)dx = \int_0^1 (x+1)dx + \int_1^2 \left[1 + \sqrt{1-(x-1)^2}\right] dx \ \textbf{1M},$$

$$\int_0^1 (x+1)dx = \left[\frac{x^2}{2} + x\right]_0^1 = \frac{3}{2}, \ \textbf{1M}$$

$$\int_1^2 1\,dx = [\,x\,]_1^2 = 1,$$

$$\text{Quarter-circle area} = \frac{\pi}{4}, \ \textbf{1M}$$

$$\text{Total area} = \frac{3}{2} + 1 + \frac{\pi}{4}. \ \textbf{1A}$$



Sp

Question 20

Consider the function $f(x) = x + 1$ on $[0, 2]$ using 2 equal subintervals.

a. Find the left endpoint approximation L.

$$\begin{aligned} \Delta x &= \frac{2-0}{2} = 1, \\ x_0 &= 0, x_1 = 1, \\ L &= 1\left[f(0) + f(1)\right] = 1\left[(0+1) + (1+1)\right] \mathbf{1M} = 1\left[1+2\right] = 3 \ \mathbf{1A}. \end{aligned}$$

b. Find the trapezoidal rule approximation T.

$$\begin{aligned} T &= \frac{1}{2}\left[f(0) + 2f(1) + f(2)\right], \\ f(0) &= 0 + 1 = 1, \\ f(1) &= 1 + 1 = 2, \\ f(2) &= 2 + 1 = 3, \\ T &= \frac{1}{2}\left[1 + 2(2) + 3\right]\mathbf{1M} = \frac{1}{2}\left[1 + 4 + 3\right] = \frac{8}{2} = 4 \ \mathbf{1A}. \end{aligned}$$

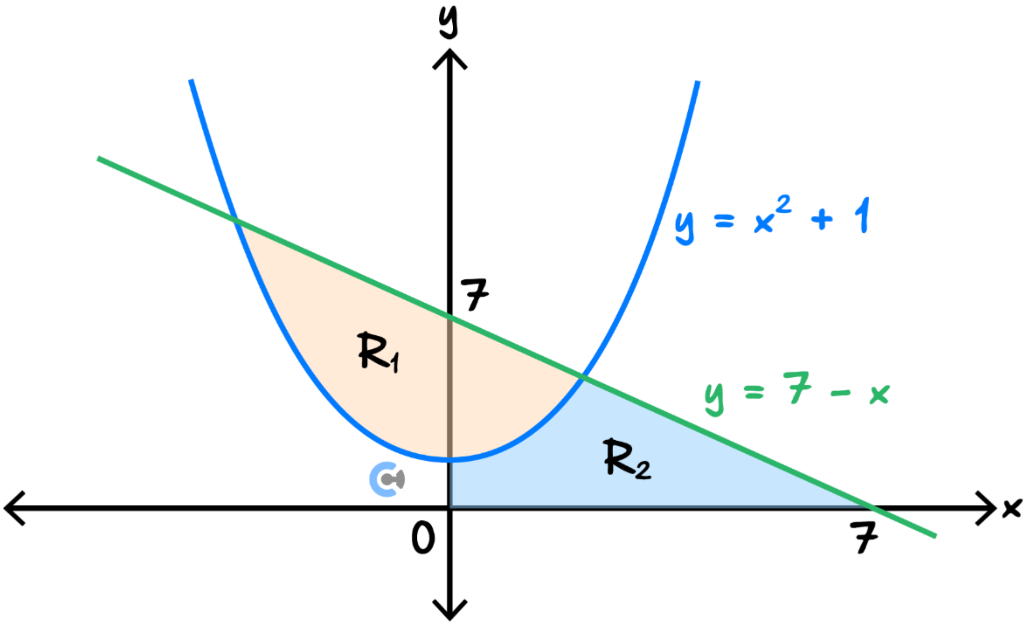
c. **Hence,** determine the right endpoint approximation R.

$$\begin{aligned} T &= \frac{L+R}{2}, \\ R &= 2T - L = 2(4) - 3 \ \mathbf{1M} = 8 - 3 = 5 \ \mathbf{1A}. \end{aligned}$$

Space for Personal Notes

Question 21

The diagram shows the curve $y = x^2 + 1$ and the line $y = 7 - x$. These graphs enclose two finite regions. Region R_1 is bounded by the curve and the line. Region R_2 lies below both graphs and is also bounded by the positive x and y-axes.



a. Find the area of R_1.

$x^2 + 1 = 7 - x$

$x^2 + x - 6 = 0$

$x = -3, 2$ **1M**

$\int_{-3}^{2} [(7 - x) - (x^2 + 1)]\, dx$ **1M**

$= \int_{-3}^{2} (6 - x - x^2)\, dx$

$= [\, 6x - \frac{x^2}{2} - \frac{x^3}{3}]_{-3}^{2}$ **1M**

$= (6(2) - \frac{(2)^2}{2} - \frac{(2)^3}{3}) - (6(-3) - \frac{(-3)^2}{2} - \frac{(-3)^3}{3})$

$= \left(12 - 2 - \frac{8}{3}\right) - \left(-18 - \frac{9}{2} + \frac{-27}{-3}\right)$

$= \frac{22}{3} - (-\frac{27}{2})$

$\boxed{\frac{125}{6}}$ **1A**

b. Find the area of R_2.

$$\int_0^2 (x^2+1)\,dx + \int_2^7 (7-x)\,dx \ \textbf{1M}$$

$$\int_0^2 (x^2+1)\,dx = [\frac{x^3}{3}+x]_0^2 = (\frac{8}{3}+2) - 0 = \frac{14}{3} \ \textbf{1M}$$

$$\int_2^7 (7-x)\,dx = [7x - \frac{x^2}{2}]_2^7$$

$= (7\times 7 - \frac{49}{2}) - (7\times 2 - \frac{4}{2}) = (49 - \frac{49}{2}) - (14-2) = \frac{49}{2} - 12 = \frac{25}{2}$ **1M (or use area of triangle)**

$$\frac{14}{3} + \frac{25}{2} = \frac{28}{6} + \frac{75}{6} = \frac{103}{6} \ \textbf{1A}$$

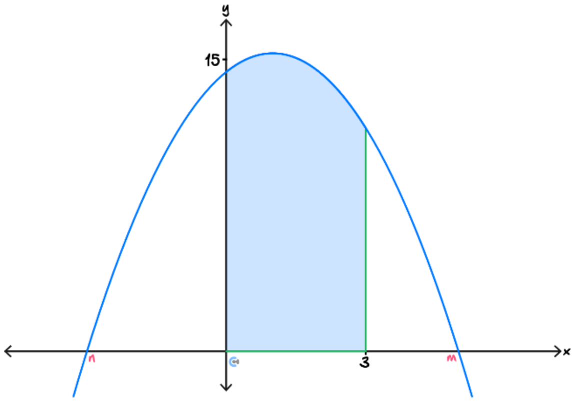
Space for Personal Notes

Section B: Supplementary Questions (47 Marks)

Sub-Section: Exam 1

Question 22 (5 marks)

Part of the graph of $f : \mathbb{R} \to \mathbb{R}, f(x) = -x^2 + ax + 15$ is shown below. If the shaded area is 45 square units, find the values of a, m and n where m and n are the x-axis intercepts of the graph of $y = f(x)$.



We have that $\int_0^3 (-x^2 + ax + 15)dx = 45$. (1M).

$$\left[\left(-\frac{x^3}{3}\right) + \left(\frac{1}{2}\right)x^2 + 15x\right]_0^3 = 45 \text{ (1M)}$$

$$\left(-9 + \frac{9}{2}a + 45\right) - 0 = 45$$

$$a = 2 \text{ (1A)}$$

So $f(x) = -x^2 + 2x + 15$.

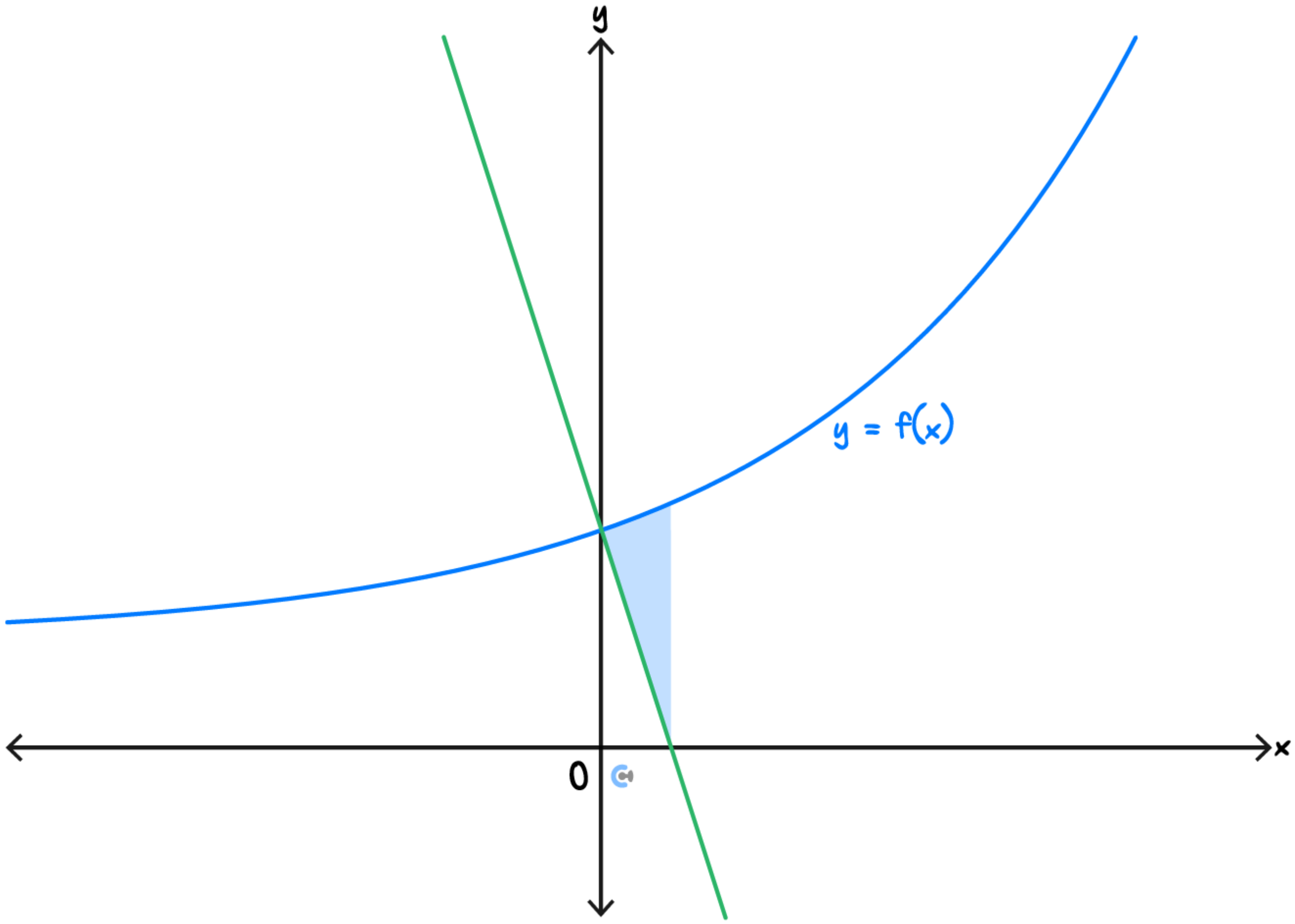
Solve $f(x) = 0 \Rightarrow x^2 - 2x - 15 = 0 \Rightarrow (x - 5)(x + 3) = 0$.

$$x = -3, 5.$$

Thus $n = -3$ and $m = 5$. (1A each)

Question 23 (5 marks)

The graph of $f: \mathbb{R} \to \mathbb{R}, f(x) = e^{\frac{x}{3}} + 1$ is shown. The normal to the graph of f where it crosses the y-axis is also shown.



a. Find the equation of the **normal** to the graph of f where it crosses the y-axis. (2 marks)

$f'(x) = \left(\frac{1}{3}\right) e^{\frac{x}{3}}$. So $f'(0) = \frac{1}{3}$. (1M)
Then gradient of normal must be -3. Line with gradient -3 that passes through $(0, 2)$.
Normal is $y = 2 - 3x$. (1A)

b. Find the exact area of the shaded region. (3 marks)

Normal has x-intercept when $x = \frac{2}{3}$. We have:

$$\int_0^{\frac{2}{3}} f(x)dx = \left[3e^{\frac{x}{3}} + x^{\frac{2}{3}}\right]_0^{\frac{2}{3}}$$

$$= 3e^{\frac{2}{9}} + \frac{2}{3} - (3)$$

$$= 3e^{\frac{2}{9}} - \frac{7}{3}. \text{ (1M)}$$

Area unshaded triangle $= \frac{1}{2} \times \frac{2}{3} \times 2 = \frac{2}{3}$. (1M)

Therefore, shaded area is $3e^{\frac{2}{9}} - \frac{7}{3} - \frac{2}{3} = 3e^{\frac{2}{9}} - 3$. (1A)

Question 24 (3 marks)

The area of the region bounded by the curve with equation $y = k\sqrt{x}$, where k is a positive constant, the x-axis and the line with equation $x = 9$ is 54. Find k.

We require that $\int_0^9 k\sqrt{x}\, dx = 54$ (1M).

$$k\left[\left(\frac{2}{3}\right)x^{\frac{3}{2}}\right]_0^9 = 54 \text{ (1M)}$$

$$k \times \frac{2}{3} \times 27 = 54$$

$$\frac{2}{3} \times k = 2$$

$$k = 3 \text{ (1A)}$$

Question 25 (3 marks)

The area of the region bounded by the y-axis, the x-axis, the curve $y = e^{3x}$ and the line $x = c$, where c is a positive real constant is $\frac{8}{3}$. Find c.

We have that $\int_0^c e^{3x}\,dx = \frac{8}{3}$. (1M)

$\frac{1}{3}\left[e^{3x}\right]_0^3 = \frac{8}{3}$ (1M)

$e^{3c} - 1 = 8$

$3c = \log_e(9)$

$c = \left(\frac{1}{3}\right)\log_e(9)$. (1A)

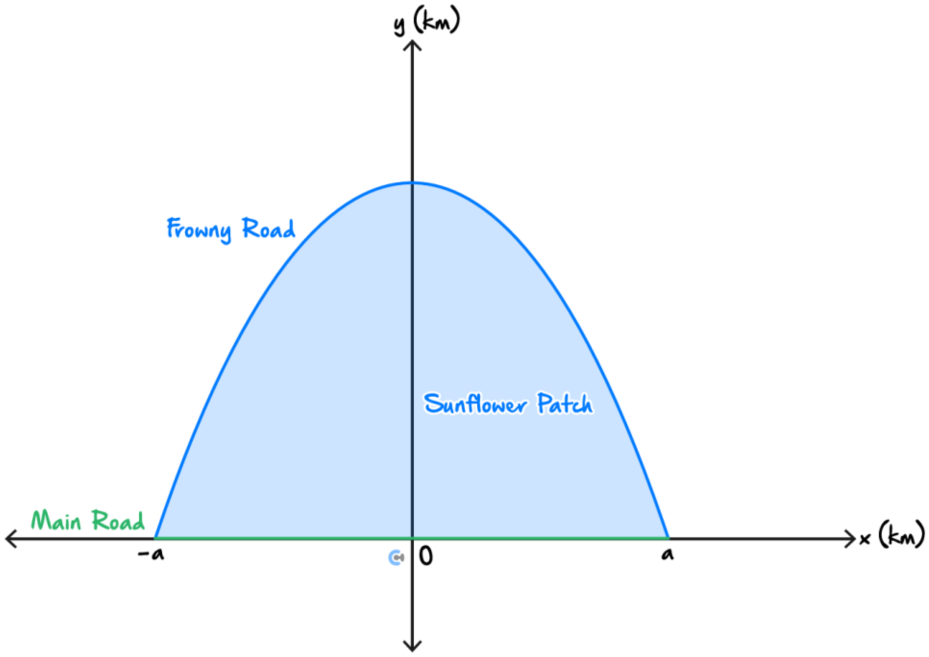
Space for Personal Notes

Sub-Section: Exam 2

Question 26 (8 marks)

Juliette's sunflower patch lies between two roads. Main Road lies along the x-axis and Frowny Road lies along the graph of the function:

$$y = 4 - \frac{4}{3}\left(e^{x} + e^{-x}\right)$$



a. Show that the y-axis intercept of the curve representing Frowny Road is $\left(0, \frac{4}{3}\right)$. (1 mark)

When $x = 0$ we have $y = 4 - \frac{4}{3}(1 + 1) = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$. (1M)

Therefore y-axis intercept is $\left(0, \frac{4}{3}\right)$

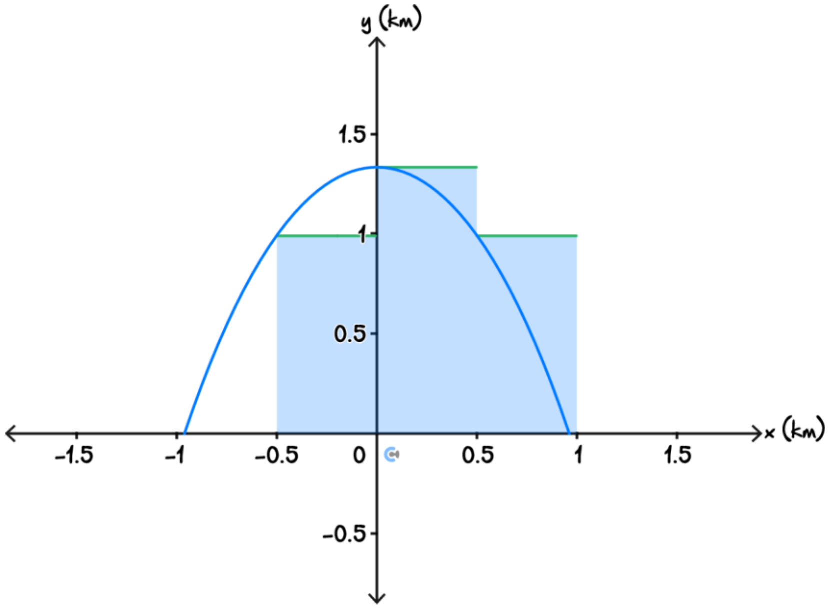
b. Find the exact value of a such that the sunflower patch lies between $x = -a$ and $x = a$. (1 mark)

We solve $f(x) = 4 - \frac{4}{3}(e^{x} + e^{-x}) = 0$.

We get $x = \log_e\left(\frac{(3 \pm \sqrt{5})}{2}\right)$.

Since $a > 0$, we have $a = \log_e\left(\frac{(3 + \sqrt{5})}{2}\right)$.

c. Juliette wants to estimate the area of the sunflower patch. She uses a rectangle method with a width of 0.5 km and three rectangles between $x = -1$ and $x = 1$.



i. Complete the table of values for $y = 4 - \frac{4}{3}(e^{x} - e^{-x})$, correct to two decimal places. (1 mark)

x	-0.5	0	0.5
y	0.99	1.33	0.99

ii. Use the table to estimate the area of the patch in square kilometres, correct to one decimal place. (1 mark)

$0.5 \times (0.99 + 1.33 + 0.99) = 1.66 \approx 1.7$ (1A)

iii. If she expects to harvest w kg of sunflowers per square kilometre and she can sell them for \$$m$ per kilogram, write a formula for her expected revenue R. (Use your answer from the previous part.) (1 mark)

$R = 1.7 \times w \times m$ (1A)

d. Juliette decides to try another approximation method by fitting a parabola through three known points: $(-1,0)$, $\left(0,\frac{4}{3}\right)$ and $(1,0)$. She then estimates the area under this parabola.

i. Find the equation of the parabola that fits these three points. (1 mark)

$y = a(x - 1)(x + 1)$. Then we get $y = -\frac{4}{3}(x - 1)(x + 1)$. (1A)

ii. Find the area enclosed by the parabola and the x-axis, giving your answer correct to two decimal places. (2 marks)

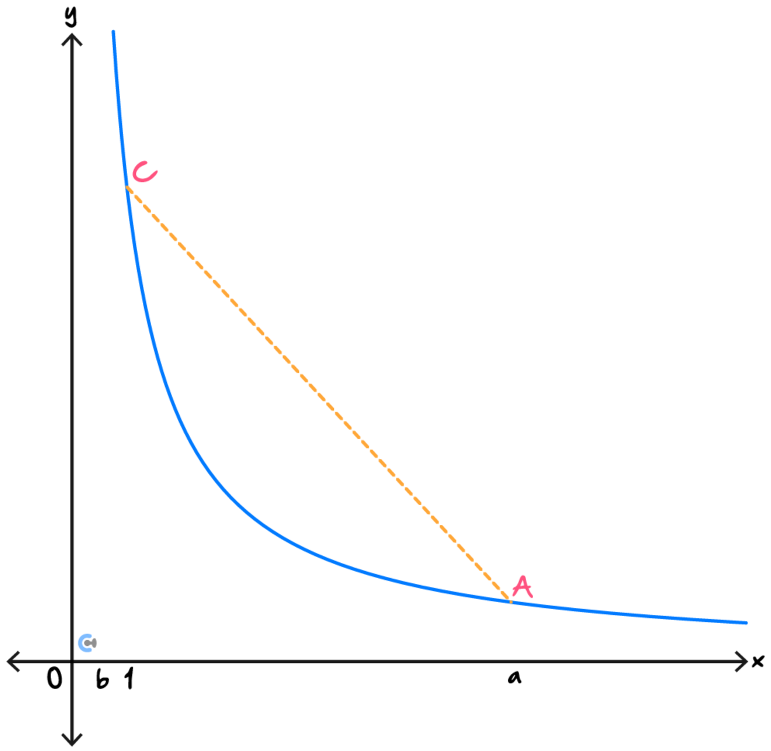
Area $= \int_{-1}^{1} -\frac{4}{3}(x - 1)(x + 1)\, dx$ (1M)

Area $= \frac{16}{9} \approx 1.78$. (1A)

Space for Personal Notes

Question 27 (12 marks)

The diagram below shows part of the graph of the function $f : \mathbb{R}^+ \to \mathbb{R}$, where $f(x) = \frac{8}{x}$. The line segment CA is drawn from the point $C(1, f(1))$ to the point $A(a, f(a))$, where $a > 1$.



a.

i. Calculate the gradient of CA in terms of a. (1 mark)

C is at $(1, 8)$.

Gradient $= \frac{\frac{8}{a} - 8}{a - 1} = -\frac{8}{a}$. (1A)

ii. At what value of x between 1 and a does the tangent to the graph of f have the same gradient as CA? (2 marks)

$f'(x) = -\frac{8}{x^2}$. (1M)

We solve $-\frac{8}{x^2} = -8 \Rightarrow x = \pm\sqrt{a}$.

But $x > 1$, so $x = \sqrt{a}$. (1A)

b.

i. Calculate $\int_1^e f(x)\,dx$. (1 mark)

8 (1A)

ii. Let b be a positive real number less than one. Find the exact value of b such that: (2 marks)

$$\int_b^1 f(x)dx = 8.$$

We have that $\int_b^1 f(x)\,dx = -8\log_e(b)$. (1M)
Solve $-8\log_e(b) = 8$. $b = e^{-1}$. (1A)

c.

i. Express the area of the region bounded by the line segment CA, the x-axis, the line $x = 1$ and the line $x = a$ in terms of a. (2 marks)

The area required is a trapezium. (1M, or other appropriate method)
$A = \frac{1}{2} \times \left(f(a) + f(1)\right) \times (a - 1) = \frac{4(a^2 - 1)}{a}$. (1A or equivalent)

ii. For what exact value of a does this area equal 8? (1 mark)

Solve $\frac{4(a^2 - 1)}{a} = 8 \Rightarrow a = 1 + \sqrt{2}$. (1A)

iii. Using the value for a determined in **part c. ii.**, explain in words, without evaluating the integral, why

$$\int_1^a f(x)dx < 8.$$

Use this result to explain why $a < e$. (1 mark)

The area under the curve is less than the area of the trapezium so we must have:

$$\int_1^a f(x)\, dx < 8$$

and from **part b.i.** we have $\int_1^a f(x)\, dx = 8$, so it must be that $a < e$. (1A)

d. Find the exact values of m and n such that: (2 marks)

$$\int_1^{mn} f(x)dx = 4 \text{ and } \int_1^{\frac{m}{n}} f(x)dx = 3.$$

We have that:

$$\int_1^{mn} f(x)dx = 8(\log_e(m) + \log_e(n)) = 4$$
$$\int_1^{mn} f(x)dx = 8(\log_e(\text{m}) - \log_e(\text{n})) = 3$$

Solving we get $m = e^{\frac{7}{16}}, n = e^{\frac{1}{16}}$ OR $m = -e^{\frac{7}{16}}, -n = e^{\frac{1}{16}}$. (1A for each pair of solutions)

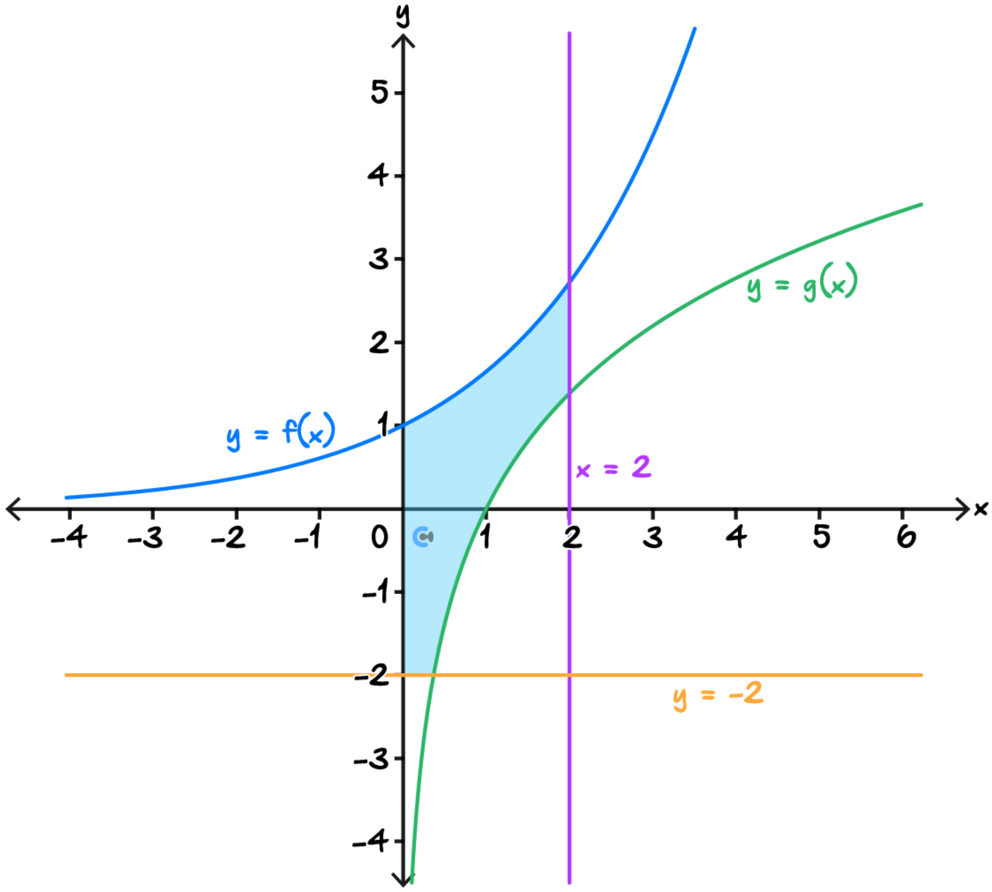
Space for Personal Notes

Question 28 (11 marks)

The shaded region in the diagram below represents part of the fencing boundary around a wildlife conservation zone in the Northern Territory. All distances are in kilometres.

Two of the boundaries of the conservation zone follow the graphs of the functions:

$$f: \mathbb{R} \to \mathbb{R}, f(x) = e^{\frac{x}{2}} \text{ and } g: \mathbb{R}^+ \to \mathbb{R}, g(x) = 2\log_e(x).$$



a.

i. Evaluate $\int_{-2}^{0} f(x)\, dx$. (1 mark)

$2 - \frac{2}{e}$

ii. **Hence**, or otherwise, find the area of the region bounded by the graph of g, the x and y-axes, and the line $y = -2$. (1 mark)

Bounded by y-axis and inverse function so same area as in previous part.
Area $= 2 - \frac{2}{e}$

iii. Find the **total** area of the shaded region. (2 marks)

Area $= 2 - \frac{2}{e} + \int_0^1 f(x)dx + \int_1^2 (f(x) - g(x))\, dx$. (1M)
Area $= 2 - \frac{2}{e} + 2e - 2\log_e(4)$. (1A, approx 3.93)

iv. The boundary at $x = 2$ is extended to $x = a$, where $a > 2$, such that the area of the conservation zone is 5 square kilometres. Find the value of a correct to three decimal places. (2 marks)

Area $= 2 - \frac{2}{e} + \int_0^1 f(x)\, dx + \int_1^a \left(f(x) - g(x)\right) dx = 5$. (1M)
Thus, $a = 2.692$. (1A)

The conservation committee decides that a better conservation zone will be the region bounded by the graph of g and that of a new function $k : (-\infty, b) \to \mathbb{R}$, where $k(x) = -2\log_e(b - x)$, and b is a positive real number.

b.

i. Find, in terms of b, the x-coordinates of the points of intersection of the graphs of g and k. (2 marks)

We solve $g(x) = k(x)$. (1M)

This yields $x = \frac{b \pm \sqrt{b^2 - 4}}{2}$. (1A)

ii. **Hence**, find the set of values of b for which the graphs of g and k have two distinct points of intersection. (1 mark)

We require that $b^2 - 4 > 0 \Rightarrow b > 2$. (1A)

iii. Determine the value of b, correct to two decimal places, such that this new conservation zone has an area of 5 square kilometres. (2 marks)

Let $x^1 = \frac{b - \sqrt{b^2 - 4}}{2}$ and $x^2 = \frac{b + \sqrt{b^2 - 4}}{2}$ be the two points of intersection of g and k as found in **part b.i.**, where $x_2 > x_1$.

We require that $\int_{x_1}^{x_2} g(x) - k(x)\, dx = 5$. (1M)

Thus $b = 3.56$.

Space for Personal Notes

VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make the Most of These (Free) Services!

- Experienced Contour tutors (45 + raw scores, 99 + ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726