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VCE Mathematical Methods ¾
Integration I [4.2]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 - Pg 22
Supplementary Questions	Pg 23 - Pg 36

Section A: Compulsory Questions

Sub-Section: Basics

Question 1

Consider the function $f(x) = x^2 + 1$.

a. Approximate the area under the curve $y = f(x)$ from $x = 0$ to $x = 2$ using two rectangles of equal width with left endpoints.

b. Is this approximation an overestimate or an underestimate of the true area? Explain briefly.

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Question 2

Find the area bounded by the curve $y = 1 - x^2$ and the x-axis.

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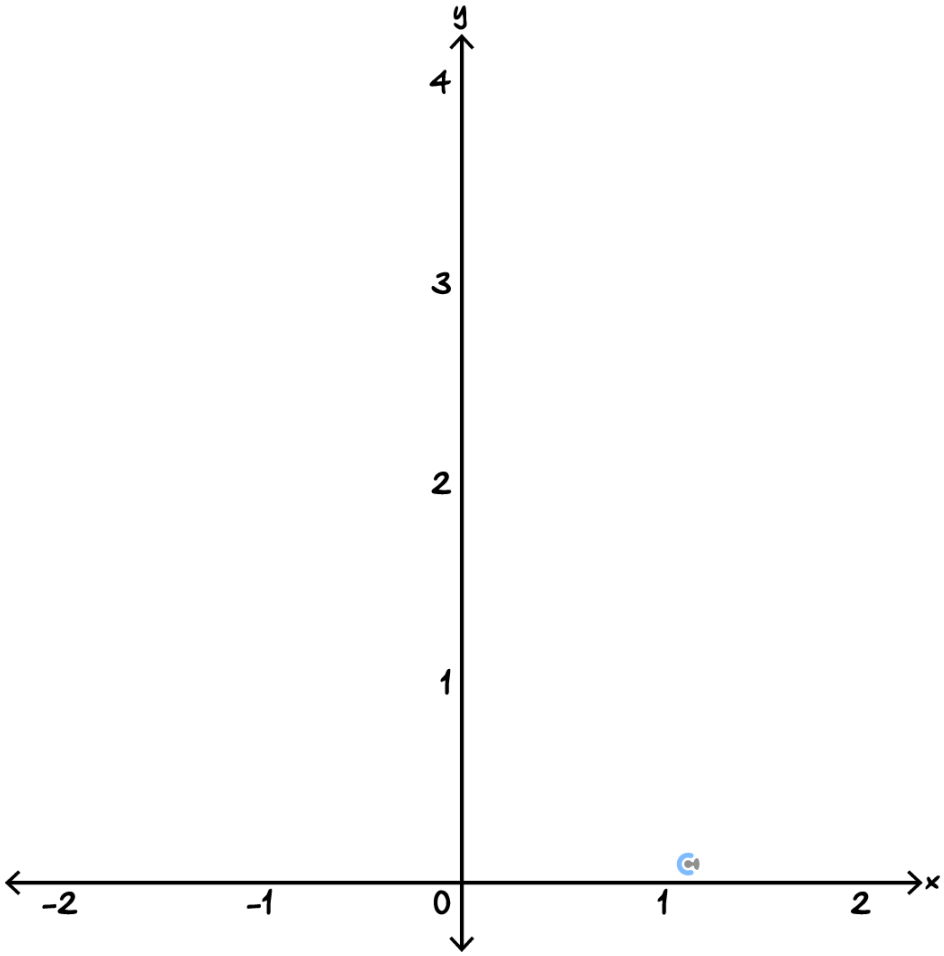
Question 3

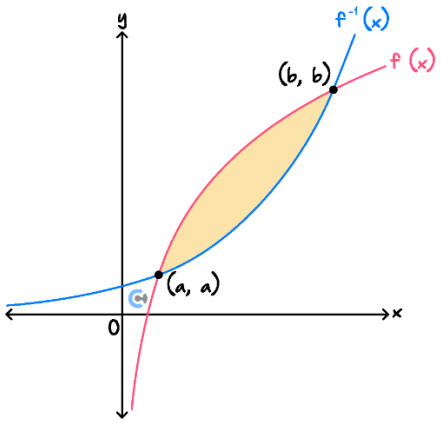
Find the area enclosed between the curve $y = x + 2$ and $y = x^2$.



Sub-Section: Problem Solving

Question 4

Construct the integral for the shaded region given in the diagram.



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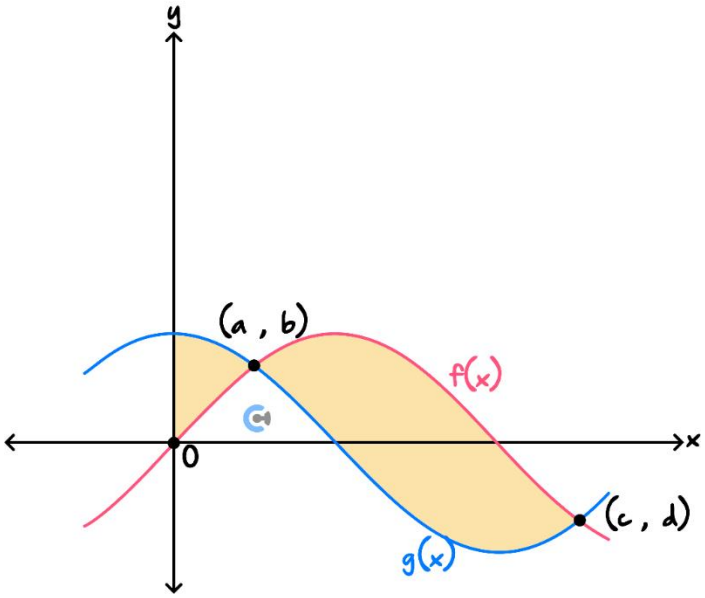
Question 5

Construct the integral for the shaded region given in the diagram.



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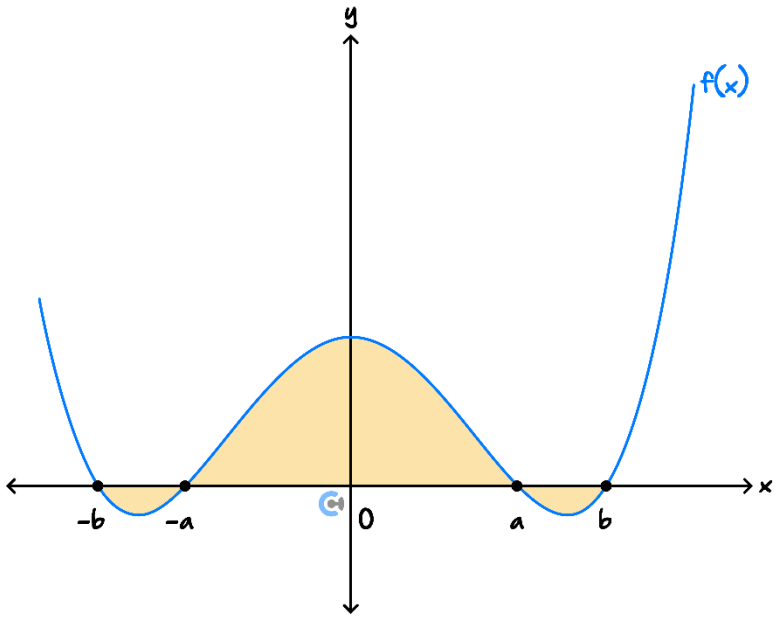
Question 6

Construct the integral for the shaded region given in the diagram.



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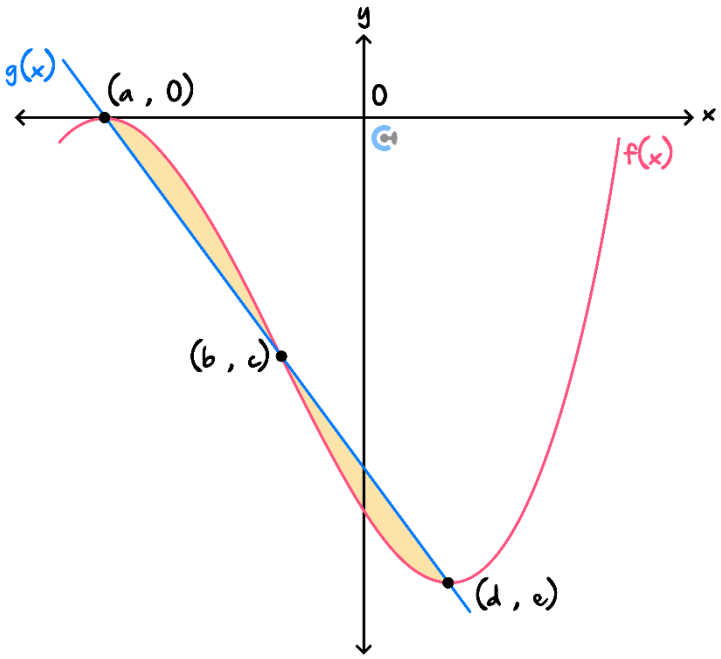
Question 7

Construct the integral for the shaded region given in the diagram.



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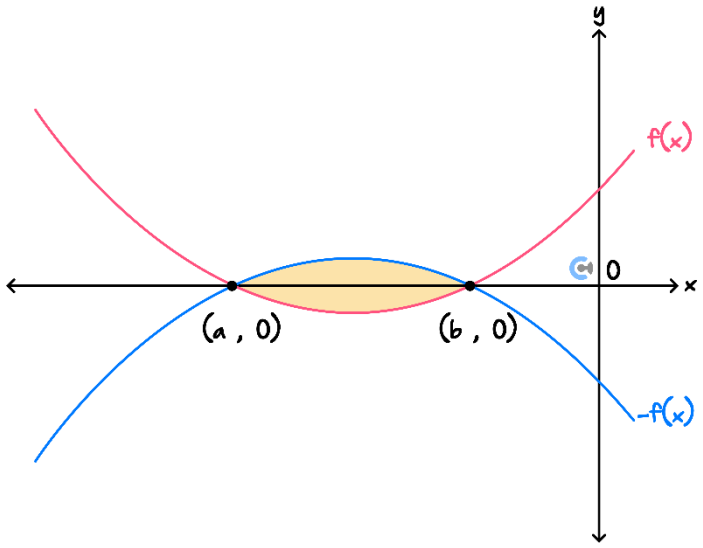
Question 8

Construct the integral for the shaded region given in the diagram.



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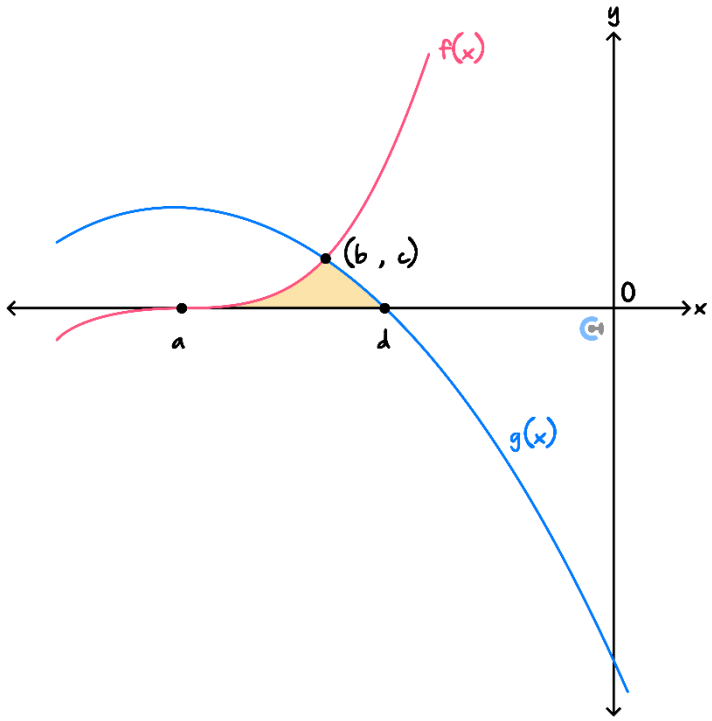
Question 9

Construct the integral for the shaded region given in the diagram.



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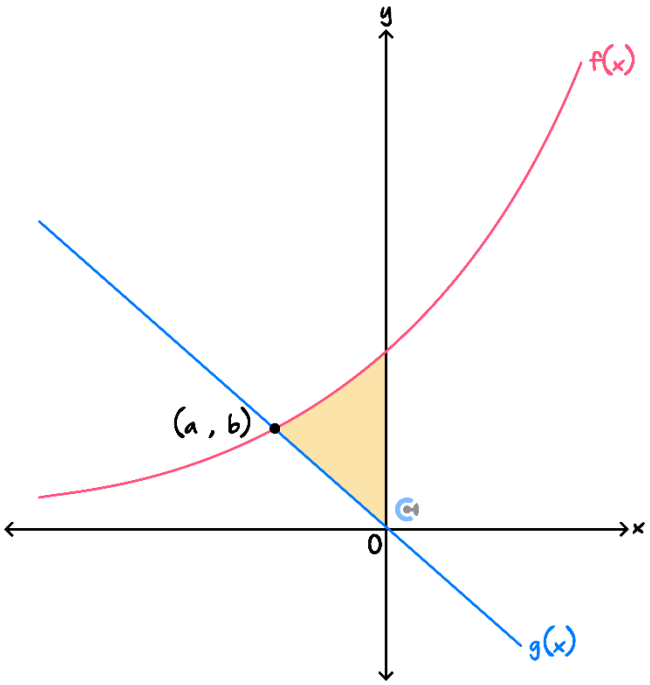
Question 10

Construct the integral for the shaded region given in the diagram.



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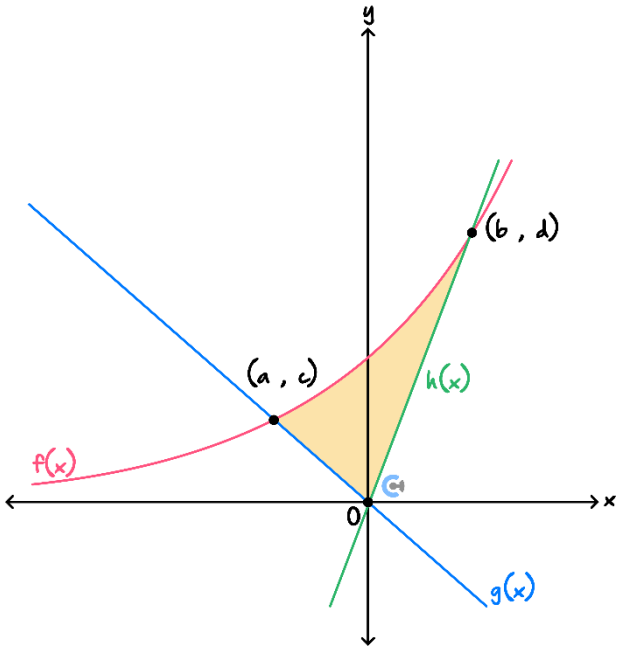
Question 11

Construct the integral for the shaded region given in the diagram.



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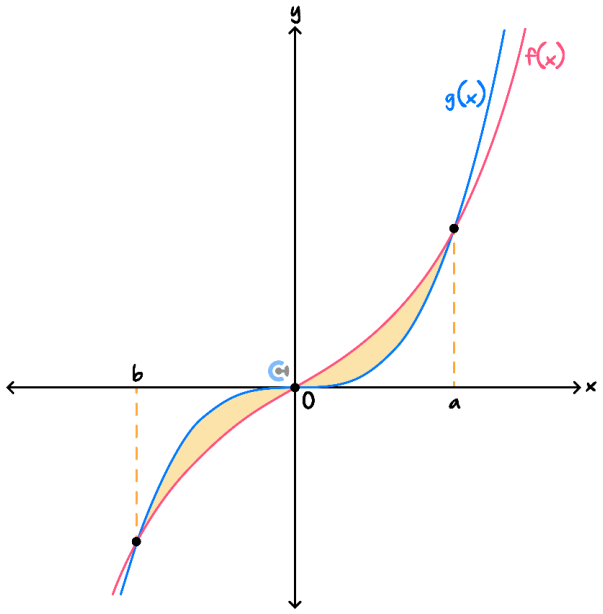
Question 12

Construct the integral for the shaded region given in the diagram.



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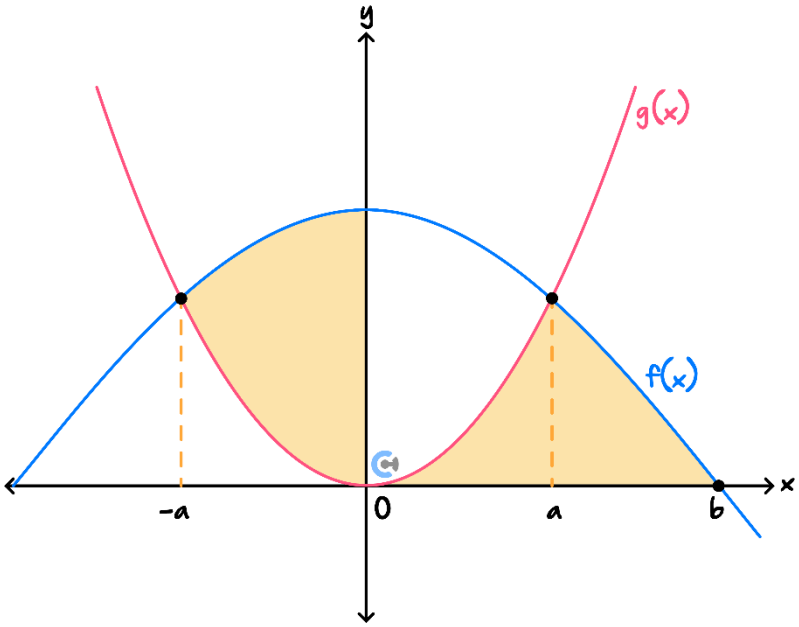
Question 13

Construct the integral for the shaded region given in the diagram.



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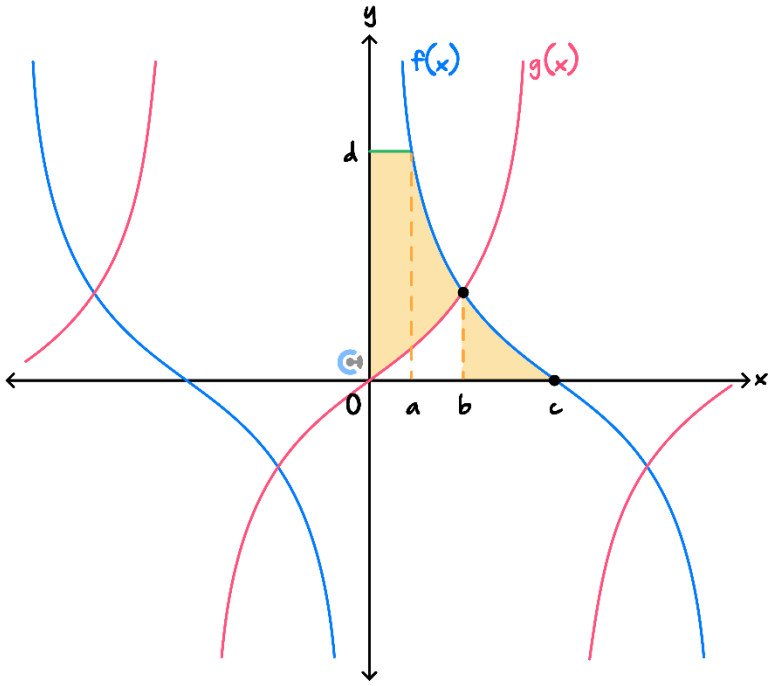
Question 14

Construct the integral for the shaded region given in the diagram.



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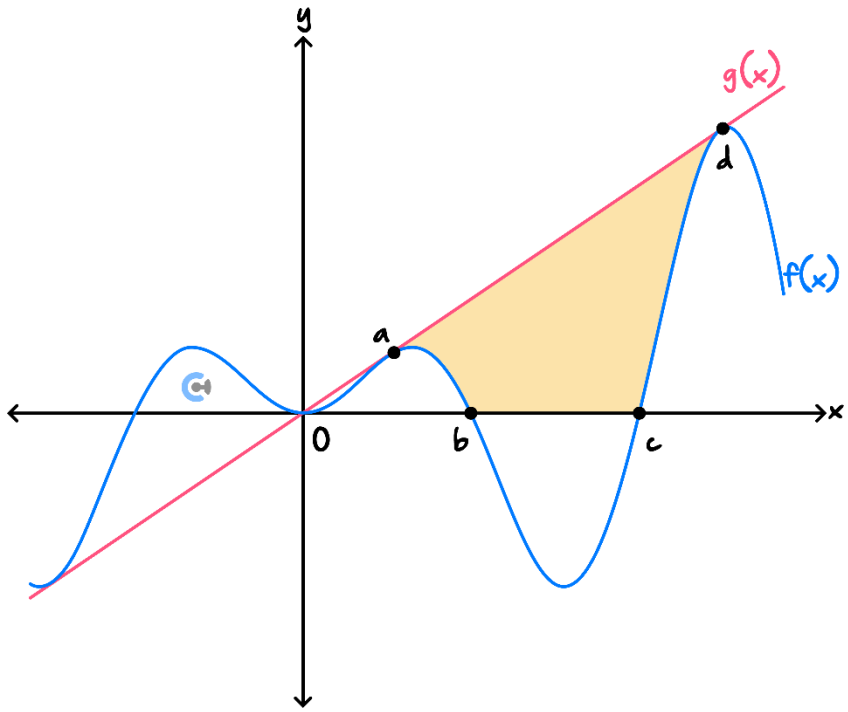
Question 15

Construct the integral for the shaded region given in the diagram.



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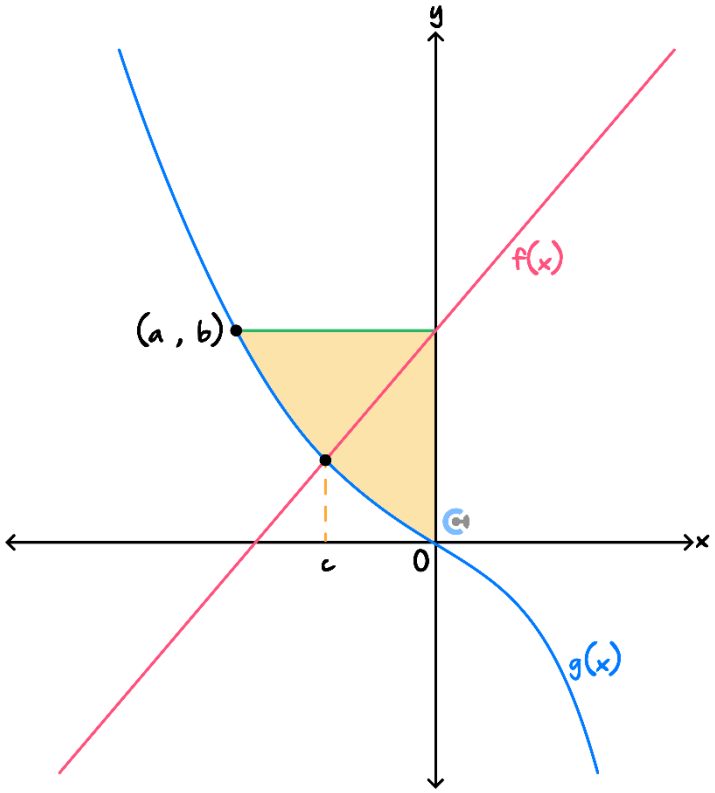
Question 16

Construct the integral for the shaded region given in the diagram.



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Question 17

Given that $\int_a^c f(x)\,dx = 10$ and the total area between $x = a$ and $x = b$ is 16 with $f(x) \geq 0$ on $[a, b]$ and $a < b < c$, find $\int_b^c f(x)\,dx$.

Question 18

Let $f(x)$ be a function such that $\int_0^4 f(x)\,dx = 5$ and the total area bounded by the graph of $y = f(x)$ and the x-axis on $[0, 4]$ is 13. The function $f(x)$ crosses the x-axis exactly once in $(0, 4)$ at $x = k$, with $f(x) \leq 0$ on $[0, k]$ and $f(x) \geq 0$ on $[k, 4]$.

a. Write two equations that relate $A = \int_0^k f(x)\,dx$ and $B = \int_k^4 f(x)\,dx$.

b. Hence, find the value of $\int_0^k f(x)\,dx$.

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Question 19

The graph of $y = f(x)$ consists of a straight-line segment from $(0, 1)$ to $(1, 2)$ and a quarter circle of radius 1 centred at $(1, 1)$ connecting $(1, 2)$ to $(2, 1)$. Evaluate $\int_0^2 f(x)\,dx$.

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Question 20

Consider the function $f(x) = x + 1$ on $[0, 2]$ using 2 equal subintervals.

a. Find the left endpoint approximation L.

b. Find the trapezoidal rule approximation T.

c. **Hence,** determine the right endpoint approximation R.

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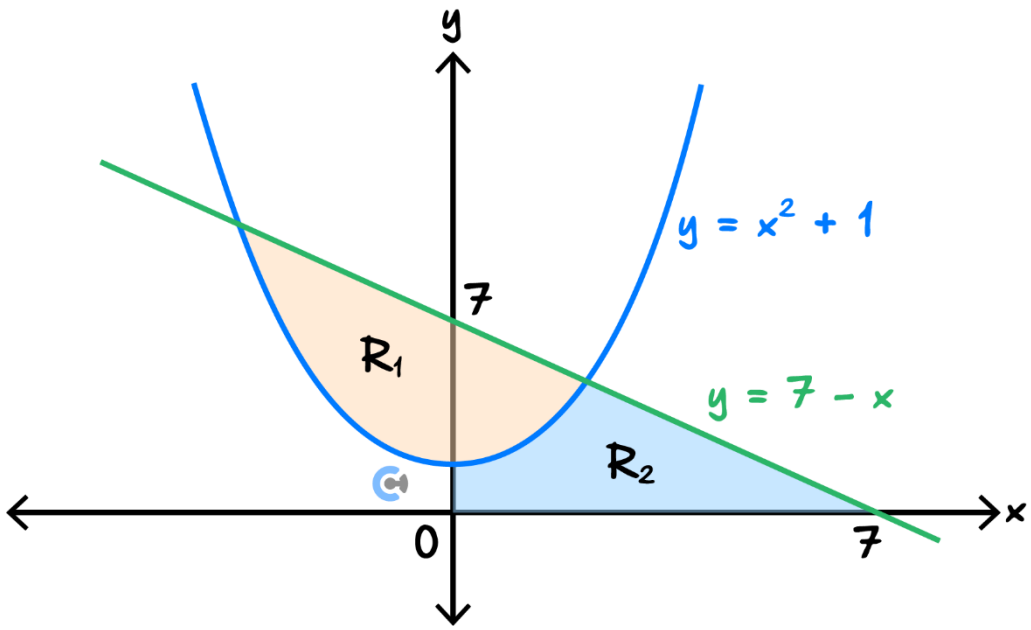
Question 21

The diagram shows the curve $y = x^2 + 1$ and the line $y = 7 - x$. These graphs enclose two finite regions. Region R_1 is bounded by the curve and the line. Region R_2 lies below both graphs and is also bounded by the positive x and y-axes.



a. Find the area of R_1.

b. Find the area of R_2.

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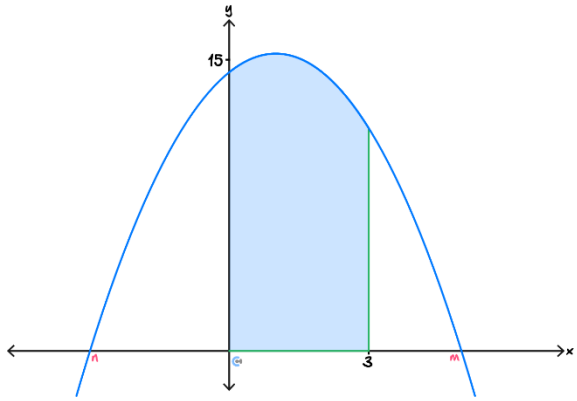
Section B: Supplementary Questions (47 Marks)

Sub-Section: Exam 1

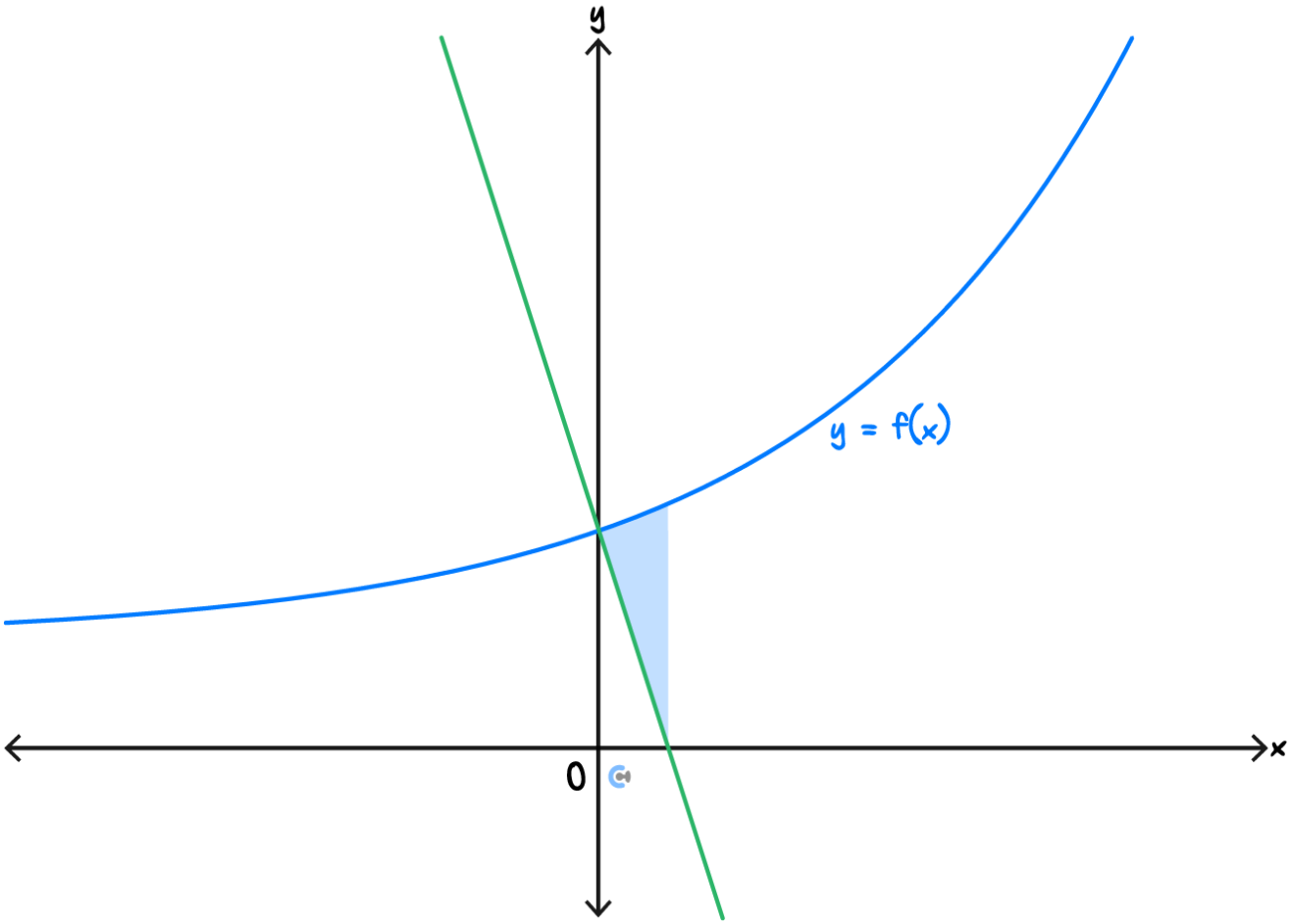
Question 22 (5 marks)

Part of the graph of $f : \mathbb{R} \to \mathbb{R}, f(x) = -x^2 + ax + 15$ is shown below. If the shaded area is 45 square units, find the values of a, m and n where m and n are the x-axis intercepts of the graph of $y = f(x)$.



Question 23 (5 marks)

The graph of $f: \mathbb{R} \to \mathbb{R}, f(x) = e^{\frac{x}{3}} + 1$ is shown. The normal to the graph of f where it crosses the y-axis is also shown.



a. Find the equation of the **normal** to the graph of f where it crosses the y-axis. (2 marks)

b. Find the exact area of the shaded region. (3 marks)

Question 24 (3 marks)

The area of the region bounded by the curve with equation $y = k\sqrt{x}$, where k is a positive constant, the x-axis and the line with equation $x = 9$ is 54. Find k.

Question 25 (3 marks)

The area of the region bounded by the y-axis, the x-axis, the curve $y = e^{3x}$ and the line $x = c$, where c is a positive real constant is $\frac{8}{3}$. Find c.

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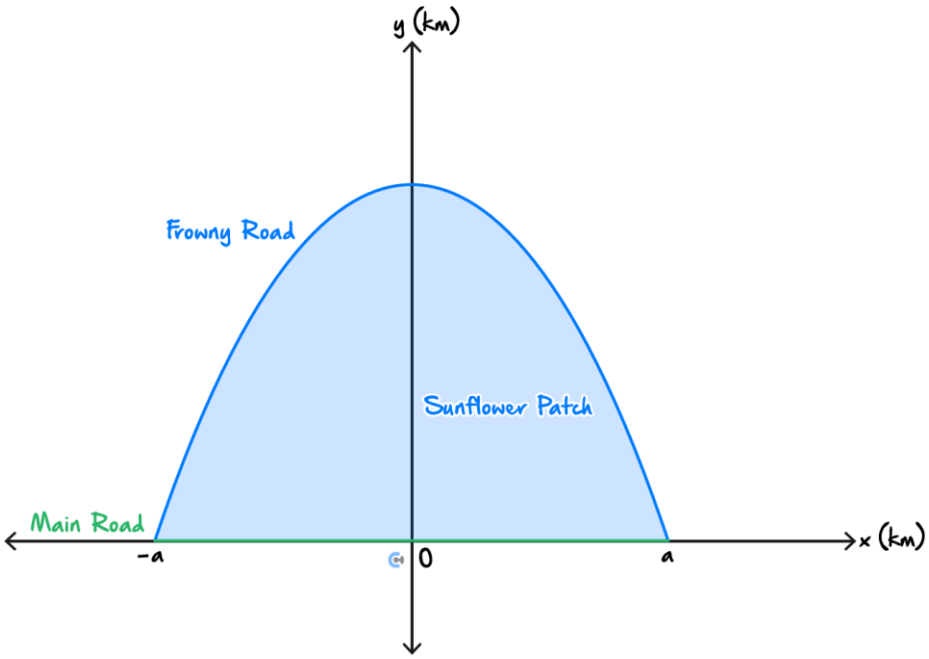
Sub-Section: Exam 2

Question 26 (8 marks)

Juliette's sunflower patch lies between two roads. Main Road lies along the x-axis and Frowny Road lies along the graph of the function:

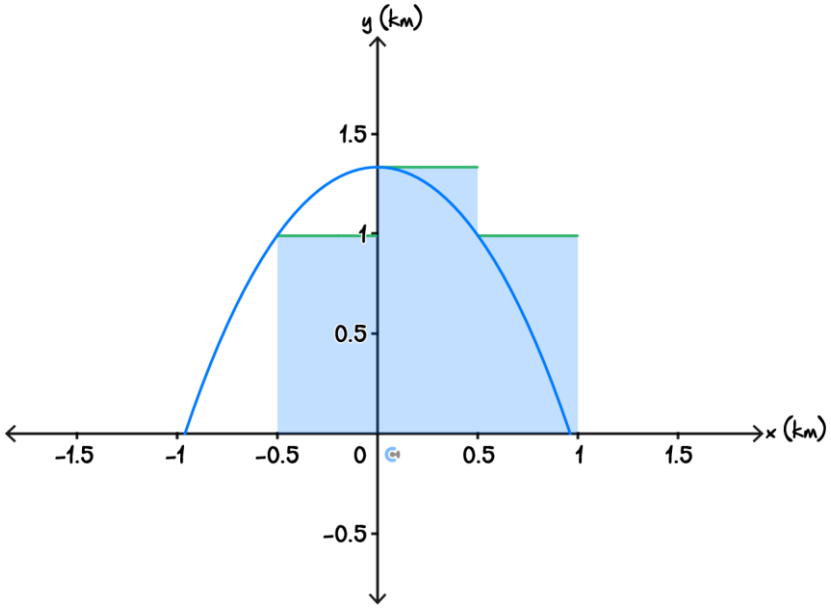
$$y = 4 - \frac{4}{3}(e^x + e^{-x})$$



a. Show that the y-axis intercept of the curve representing Frowny Road is $\left(0, \frac{4}{3}\right)$. (1 mark)

b. Find the exact value of a such that the sunflower patch lies between $x = -a$ and $x = a$. (1 mark)

c. Juliette wants to estimate the area of the sunflower patch. She uses a rectangle method with a width of 0.5 km and three rectangles between $x = -1$ and $x = 1$.



i. Complete the table of values for $y = 4 - \frac{4}{3}(e^x - e^{-x})$, correct to two decimal places. (1 mark)

x	-0.5	0	0.5
y			

ii. Use the table to estimate the area of the patch in square kilometres, correct to one decimal place. (1 mark)

iii. If she expects to harvest $w\ kg$ of sunflowers per square kilometre and she can sell them for $\$m$ per kilogram, write a formula for her expected revenue R. (Use your answer from the previous part.) (1 mark)

d. Juliette decides to try another approximation method by fitting a parabola through three known points: $(-1,0)$, $\left(0,\frac{4}{3}\right)$ and $(1,0)$. She then estimates the area under this parabola.

i. Find the equation of the parabola that fits these three points. (1 mark)

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ii. Find the area enclosed by the parabola and the x-axis, giving your answer correct to two decimal places. (2 marks)

_____

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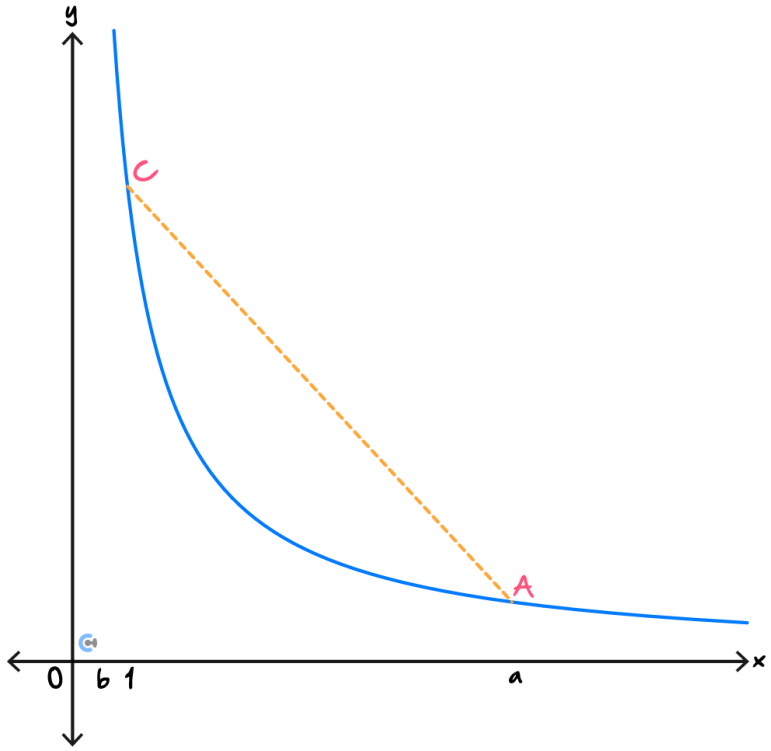
Question 27 (12 marks)

The diagram below shows part of the graph of the function $f : \mathbb{R}^+ \to \mathbb{R}$, where $f(x) = \frac{8}{x}$. The line segment CA is drawn from the point $C\big(1, f(1)\big)$ to the point $A\big(a, f(a)\big)$, where $a > 1$.



a.

i. Calculate the gradient of CA in terms of a. (1 mark)

ii. At what value of x between 1 and a does the tangent to the graph of f have the same gradient as CA? (2 marks)

b.

i. Calculate $\int_1^e f(x)\,dx$. (1 mark)

ii. Let b be a positive real number less than one. Find the exact value of b such that: (2 marks)

$$\int_b^1 f(x)dx = 8.$$

c.

i. Express the area of the region bounded by the line segment CA, the x-axis, the line $x = 1$ and the line $x = a$ in terms of a. (2 marks)

ii. For what exact value of a does this area equal 8? (1 mark)

iii. Using the value for a determined in **part c. ii.**, explain in words, without evaluating the integral, why

$$\int_1^a f(x)dx < 8.$$

Use this result to explain why $a < e$. (1 mark)

d. Find the exact values of m and n such that: (2 marks)

$$\int_1^{mn} f(x)dx = 4 \text{ and } \int_1^{\frac{m}{n}} f(x)dx = 3.$$

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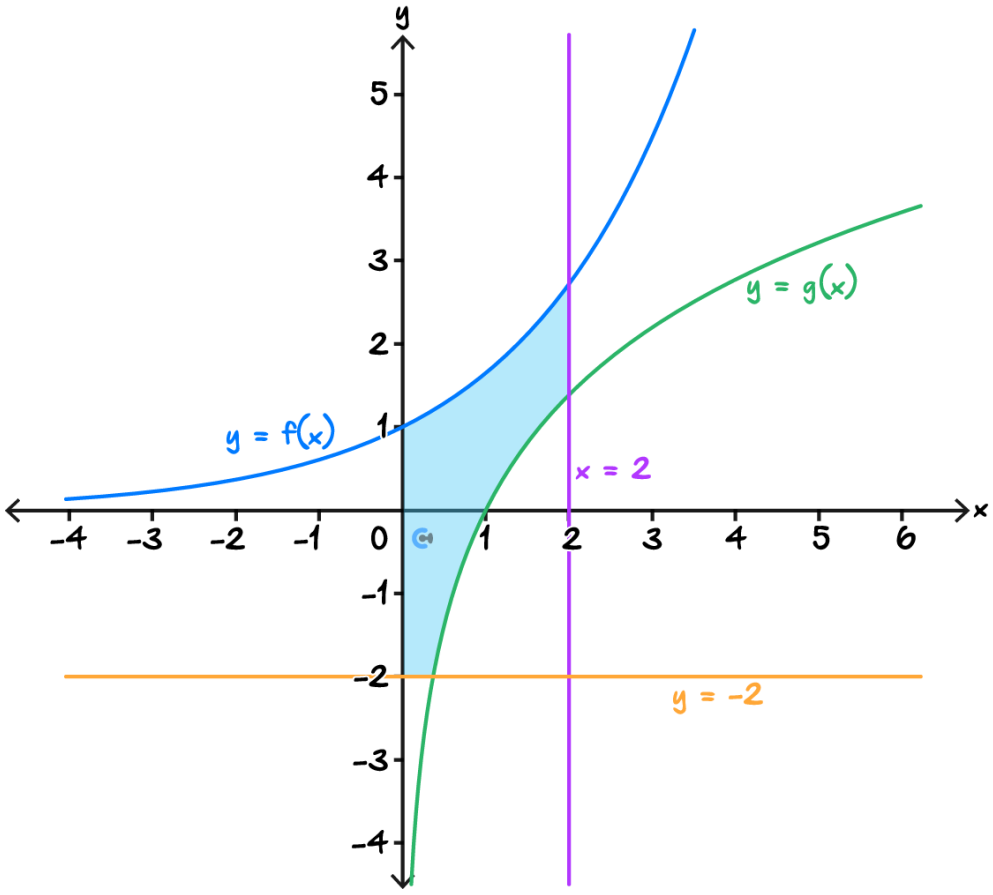
Question 28 (11 marks)

The shaded region in the diagram below represents part of the fencing boundary around a wildlife conservation zone in the Northern Territory. All distances are in kilometres.

Two of the boundaries of the conservation zone follow the graphs of the functions:

$$f: \mathbb{R} \to \mathbb{R}, f(x) = e^{\frac{x}{2}} \text{ and } g: \mathbb{R}^+ \to \mathbb{R}, g(x) = 2\log_e(x).$$



a.

i. Evaluate $\int_{-2}^{0} f(x)\,dx$. (1 mark)

ii. **Hence**, or otherwise, find the area of the region bounded by the graph of g, the x and y-axes, and the line $y = -2$. (1 mark)

iii. Find the **total** area of the shaded region. (2 marks)

iv. The boundary at $x = 2$ is extended to $x = a$, where $a > 2$, such that the area of the conservation zone is 5 square kilometres. Find the value of a correct to three decimal places. (2 marks)

The conservation committee decides that a better conservation zone will be the region bounded by the graph of g and that of a new function $k : (-\infty, b) \to \mathbb{R}$, where $k(x) = -2\log_e(b - x)$, and b is a positive real number.

b.

i. Find, in terms of b, the x-coordinates of the points of intersection of the graphs of g and k. (2 marks)

ii. **Hence**, find the set of values of b for which the graphs of g and k have two distinct points of intersection. (1 mark)

iii. Determine the value of b, correct to two decimal places, such that this new conservation zone has an area of 5 square kilometres. (2 marks)

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