CONTOUR EDUCATION

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VCE Mathematical Methods ¾
Antidifferentiation [4.1]
Workbook

Outline:

Learning Objectives:

- [ ] **MM34 [4.1.1]** - Find Antiderivative Functions
- [ ] **MM34 [4.1.2]** - Solve Definite Integrals
- [ ] **MM34 [4.1.3]** - Apply Integral Properties to Tackle Integration Questions

Section A: Recap of Differentiation

Let's do a quick recap of differentiation!

REMINDER

- Derivative of Standard Functions.
- Table of Standard Derivatives:

$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\log_e(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$

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REMINDER

- **Differentiation Rules:**

Rule	Function Notation	Leibniz Notation
The Chain Rule	$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
The Product Rule	$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + g'(x)f(x)$	$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$
The Quotient Rule	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Question 1

Find $f'(x)$ for each of the following functions:

a. $f(x) = e^x \sin(2x)$

b. $g(x) = x^2 \log_e(x)$

Section B: Indefinite Integrals

Sub-Section: Indefinite Integrals

How do we go back from $\frac{dy}{dx}$ to y?

Exploration: Understanding Anti-Differentiation

- Let's first multiply $\frac{dy}{dx}$ by dx.

$$\frac{dy}{dx} \cdot dx = \_\_\_\_\_\_\_\_\_\_$$

We can think of dy as a tiny piece of y!

- Given that dy is a small piece of y, how can we find the overall y-value?
  - We can find the __________.

$$y = S(\_\_\_\_\_\_) \text{ where } S = \text{Sum}$$

- Since we are adding **a lot** of small dy's, I think we need a better notation for sum!
  - Simply stretch the S vertically!

$$y = \int \_\_\_\_\_\_\_\_\_\_$$

- We call the vertically stretched sum: _______________.
  - $dy = \frac{dy}{dx} dx$.

$$y = \int \_\_\_\_\_\_\_\_\_\_$$

Indefinite Integrals

- The indefinite integral of $f(x)$ gives us the **Antiderivative Function.**

$$\int \frac{dy}{dx}\, dx = y + c$$

- We call dy/dx the ______________________ and y the ___________________________ function.
- c is an arbitrary real constant.

Discussion: Why do we need the $+\ c$?

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Table of Standard Antiderivatives

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	
$\sin(x)$	
$\cos(x)$	
$\sec^2(x)$	
e^x	
$\frac{1}{x}$	

NOTE: They are simply the opposite of differentiation.

Discussion: Why do we get two functions when antideriving $\frac{1}{x}$? Simply derive $\log_e(x)$ and $\log_e(-x)$ to find out!

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Let's quickly practice!

Question 2 Walkthrough.

Evaluate the following indefinite integrals:

a. $\int 6x^2 + 2x\,dx$

b. $\int 2\cos(x) - \sin(x)\,dx$

c. $\int \frac{1}{x-3} + 4\,dx$

d. $\int 2e^x - \sec^2(x)\,dx$

e. $\int \frac{1}{x^3} - 2e^x \, dx$

<u>Active Recall:</u> Table of Standard Antiderivatives

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	
$\sin(x)$	
$\cos(x)$	
$\sec^2(x)$	
e^x	
$\frac{1}{x}$	

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Question 3

Evaluate the following indefinite integrals:

a. $\int 3x^2\, dx$

b. $\int \frac{1}{2}x + 1\, dx$

c. $\int 2\sin(x) - 1\, dx$

d. $\int \frac{1}{x-2} - 3\, dx$

e. $\int \frac{1}{x-2} - \frac{e^x}{2}\,dx$

f. $\int \frac{1}{x^2} + 3e^x\,dx$

TIP: Don't forget the $+\,c$.

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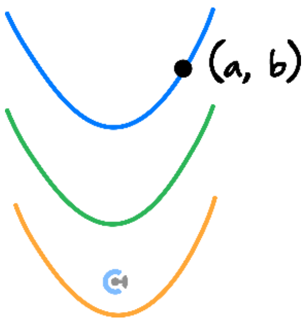
Sub-Section: Solving for $+c$

Discussion: Why do we need to find $+c$?

Solving for $+c$



- Substitute a point that the antiderivative function goes through.

Question 4 Walkthrough.

Find the rule for the function f given that $f'(x) = 2\cos(x)$ and $f(\pi) = -2$.

Your turn!

Question 5

Find the rule for the function f given that $f'(x) = 2x - e^x$ and $f(0) = 5$.

Question 6 Extension.

Find the rule for the function g given that $g'(x) = \frac{2}{\sqrt{x}} + \frac{1}{(x-1)^3}$ and $g(4) = 8$.

Section C: Definite Integrals

Sub-Section: Definite Integrals

What do definite integrals give us?

Exploration: Understanding Definite Integrals

- Consider the following definite integral:

$$\int_a^b \_\_\_\_\_\_\_\_\_\_\_\_$$

- Definite integrals have __________________ written on the integral.
- Simplify and cancel the dx.

$$\int_a^b \_\_\_\_\_\_\_\_$$

- dy stands for instantaneous change of y: ________________ in y.

$$\int_a^b \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- What would happen if we added up all the small changes in y? _________________________________.
- Definite integrals give us the change of the antiderivative function from _______________________.

$$\int_a^b \frac{dy}{dx}\,dx = Change\ in\ y\ as\ x: a \to b$$

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Definite Integrals

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

- a and b are called ______________.
- Definite Integrals give us the ________________ in the antiderivative as x changes from ____________.

NOTE: We always substitute in the top terminal first!

Question 7 Walkthrough.

Evaluate the following definite integral:

a. $\int_0^2 x^3 + x^2\,dx$

b. Describe in words what you have found in **part a.**

Active Recall: Definite Integrals

$$\int_a^b f(x)\,dx = [F(x)]_a^b = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Question 8

Evaluate the following definite integrals:

a. $\int_{-1}^{2} \frac{1}{2}x^2 - 3x \ dx$

b. $\int_0^{\frac{\pi}{6}} \sin(x) - 2\,dx$

c. $\int_3^5 \frac{1}{x-2} \, dx$

d. $\int_1^2 \frac{1}{x+2} - 2x^2 \, dx$

e. $\int_{-4}^{-1} \frac{1}{x^2} + e^{-3x} \, dx$

Question 9 Extension.

Evaluate:

$$\int_1^2 \frac{x^3 - 3x + 2}{x^2 - 2x + 1} + \frac{2}{x} + e^{-2x} dx$$

Calculator Commands: Indefinite Integrals and Definite Integrals

Mathematica

- Indefinite Integrals: "ESC + Intt."
- Or simply "Integrate."

$\int \square \, d\square$

```
Integrate[function, x]
```

- Definite Integrals: "ESC + Dintt."
- Or simply "Integrate."

```
Integrate[function, {x, lower, upper}]
```

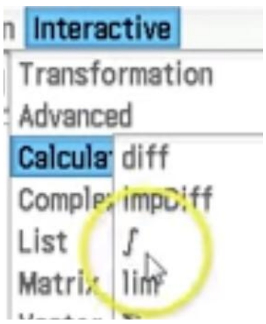
TI-Nspire

- "Shift Plus"
- Terminals are optional.

$\int_{\square}^{\square} \square \, d\square$

Casio Classpad

- Action/Interactive → Calculation → "Integral Sign."



$\int_{\square}^{\square} \square \, d\square$

Question 10 Tech-Active.

Using technology, evaluate the following:

a. $\int \sin(x) + x^3 dx$

b. $\int_0^1 e^x + \frac{1}{x+1} dx$

Question 11 Tech-Active.

a. Evaluate $\int_0^2 x^2 - 3\,dx$.

b. Find $f(x)$, the antiderivative of $x^2 - 3$ in terms of c.

c. Find $f(2) - f(0)$.

d. Hence, what $\int_0^2 x^2 - 3\,dx$ gives us in words?

Sub-Section: Integral Properties

Remember that definite integrals represent change!

Discussion: Consider $\int_b^a f(x)\,dx$. What would happen if we flipped the terminal values? $\int_a^b f(x)\,dx$.

Discussion: Consider $\int_a^a f(x)\,dx$. What would the definite integral (change) equal to if the initial and final states are both the same? $(x = a)$

Discussion: If we add change from $x = a$ to $x = b$ and change from $x = b$ to $x = c$, what would it be the same as?

In summary!

<u>Integral Properties</u>

- The following rules also hold for definite integrals:

$$\int_b^a f(x)\,dx = \underline{\hspace{5cm}}$$

$$\int_a^a f(x)\,dx = \underline{\hspace{3cm}}$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \underline{\hspace{5cm}}$$

$$\int_a^b c \cdot f(x)\,dx = \underline{\hspace{5cm}}$$

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Question 12 Walkthrough.

Evaluate the following integrals of unknown functions given that,

$$\int_{-1}^{2} f(x)\ dx = 5 \text{ and } \int_{0}^{2} f(x)\ dx = -3.$$

a. $\int_{2}^{0} 2f(x)\,dx$

b. $\int_{-1}^{0} f(x)\ dx$

<u>Active Recall:</u> Integral Properties

- The following rules also hold for definite integrals:

$$\int_{b}^{a} f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\int_{a}^{a} f(x)\,dx = \_\_\_\_\_\_\_\_$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\int_a^b c \cdot f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_$$

Your turn!

Question 13

Evaluate the following integrals of unknown functions given that,

$$\int_0^3 f(x)\ dx = 2 \text{ and } \int_1^3 f(x)\ dx = 5$$

a. $\int_0^1 f(x)\,dx$

b. $\int_0^3 2f(x) - 1\ dx$

c. $\int_3^1 2f(x)\ dx$

Section D: Reverse Chain Rule

How do we anti-derive a function when the inside function is ax+b instead of x?

Exploration: Understanding Reverse Chain Rule

- Consider $f(ax+b)$, where the inside function is **linear.**
- What does the derivative look like?

$$f'(ax+b) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- We have to multiply by ________________ via ____________.
- Now, let's try the other way around: Antidifferentiation.

$$\int af'(ax+b)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- What did you have to divide by?
- This process is called ________________.

The Reverse Chain Rule

$$\int f(ax+b)\,dx = \frac{1}{a}f(ax+b) + c$$

- Process of undoing the chain rule (where $u =$ Linear).
  - We divide by the coefficient of x.

NOTE: This only works when the inside is linear. For non-linear chain rules, we will need another technique (more on this later).

Table of Standard Integrals with Reverse Chain Rule

$f(x)$	$\int f(x)dx$
$(ax+b)^n$	
$\sin(ax+b)$	
$\cos(ax+b)$	
$\sec^2(ax+b)$	
e^{ax+b}	
$\frac{1}{ax+b}$	

- Simply divide by the coefficient of x.

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Question 14 Walkthrough.

Evaluate the following indefinite integral:

$$\int (2x + 1)^3 \, dx$$

Active Recall: Table of Standard Integrals with Reverse Chain Rule

$f(x)$	$\int f(x)dx$
$(ax + b)^n$	
$\sin(ax + b)$	
$\cos(ax + b)$	
$\sec^2(ax + b)$	
e^{ax+b}	
$\frac{1}{ax + b}$	

Your turn!

Question 15

Evaluate the following indefinite integrals:

a. $\int (3x+3)^{\frac{1}{2}}\,dx$

b. $\int e^{4x+1}\,dx$

c. $\int \frac{2}{3x}\,dx$

d. $\int \frac{-3}{2x+5}\, dx$

e. $\int \cos\left(\frac{3x+1}{4}\right)\, dx$

f. $\int \frac{1}{3}e^{-3x} + e^{-\frac{x}{2}+1} dx$

Question 16 Extension.

Evaluate:

$$\int_1^2 (3x-4)^4 - \frac{2}{x-3}\,dx$$

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Contour Check

Learning Objective: [4.1.1] – Find antiderivative functions

Key Takeaways

- **Indefinite Integrals:**
  - The indefinite integral of $f(x)$ gives us the **Antiderivative Function.**

$$\int \frac{dy}{dx}\, dx = y + c$$

  - We call dy/dx the ____________________ and y the ________________________ function.
  - c is an arbitrary real constant.

- **Table of Standard Integrals with Reverse Chain Rule:**

$f(x)$	$\int f(x)dx$
$(ax+b)^n$	
$\sin(ax+b)$	
$\cos(ax+b)$	
$\sec^2(ax+b)$	
e^{ax+b}	
$\frac{1}{ax+b}$	

Learning Objective: [4.1.2] - Solve definite integrals

Key Takeaways

- **Definite Integrals:**

$$\int_a^b f(x)\,dx = [F(x)]_a^b = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - a and b are called ________________.
  - Definite Integrals give us the ________________ in the antiderivative as x changes from ______________.

Learning Objective: [4.1.3] – Apply integral properties to tackle integration questions

Key Takeaways

- **Integral Properties**

  - The following rules also hold for definite integrals:

$$\int_b^a f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\int_a^a f(x)\,dx = \_\_\_\_\_\_\_$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\int_a^b c \cdot f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_$$

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