Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Antidifferentiation [4.1]
Test Solutions

34 Marks. 1 Minute Reading. 23 Minutes Writing.

Results:

Test Questions	____________ / 23
Extension Questions	____________ / 11

Section A: Test Questions (23 Marks)

Question 1 (4 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. If $F(x)$ is an antiderivative of $f(x)$, then $F(x)+c$ is also an antiderivative of $f(x)$.	✓	
b. The integral of x^n is $\frac{x^{n+1}}{n+1}+c$.	✓	
c. The reverse chain rule method of integration works for all factorised expressions. The inside must be linear.		✓
d. The definite integral of a function gives the change in the value of the antiderivative function over a given interval.	✓	
e. If $F(x)$ is an antiderivative of $f(x)$, then $\int_a^b -f(x)\,dx = F(b)-F(a)$.		✓
f. The integral of $\frac{1}{x}$ is always $\log_e(x)+c$. It can be $\log_e(-x)+c$		✓
g. If $\int_0^a f(x)\,dx = b$, then $\int_0^{2a} f\left(\frac{x}{2}\right)dx$ dilates the change in the antiderivative by a factor of $\frac{1}{2}$. The change is doubled.		✓
h. The definite integral of a sine function over an interval of one period is zero.	✓	

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Question 2 (6 marks)

Evaluate each of the following integrals:

a. $\int \frac{1}{2}x^3 + 2x^2 - 3x \; dx$. (2 marks)

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Integrate[x^3/2 + 2 x^2 - 3 x, x]
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$$-\frac{3x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{8}$$

b. $\int \frac{1}{2x+5} - 3 \, dx$. (2 marks)

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Integrate[1/(2 x + 5) - 3, x]
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$$-3x + \frac{1}{2}\text{Log}[5 + 2x]$$

c. $\int \frac{1}{2}\sin(5 - 2x) + \frac{1}{e^{2x}} \; dx$. (2 marks)

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Integrate[1/2 Sin[5 - 2 x] + 1/E^(2 x), x] // FullSimplify
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$$\frac{1}{4}\left(-2e^{-2x} + \text{Cos}[5 - 2x]\right)$$

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Question 3 (3 marks)

Find the rule of the antiderivative function of $y = 5e^{x+1} - 2x^2$ given that the antiderivative passes through the points $\left(-1, \frac{22}{3}\right)$.

$$\text{Integrate}\left[5\,E^{x+1} - 2\,x^2,\ x\right]$$

$$5\,e^{1+x} - \frac{2\,x^3}{3}$$

$$\text{Solve}\left[\frac{22}{3} == 5\,e^{1+x} - \frac{2\,x^3}{3} + c\ /.\ x \to -1,\ c\right]$$

$$\left\{\left\{c \to \frac{5}{3}\right\}\right\}$$

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Question 4 (5 marks)

Evaluate each of the following definite integrals:

a. $\int_{-6}^{-5} \cos\left(\pi x + \frac{\pi}{4}\right) - \frac{1}{x+3}\, dx$. (2 marks)

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Integrate[Cos[π * x + π/4] - 1/(x + 3), {x, -6, -5}]
```

$$-\frac{\sqrt{2}}{\pi} + \text{Log}\left[\frac{3}{2}\right]$$

b. $\int_{1}^{3} \frac{2}{e^{\frac{x}{2}-4}} + \frac{x^3}{4} - 2\, dx$. (3 marks)

```
Integrate[2/E^(x/2-4) + x^3/4 - 2, {x, 1, 3}] // Expand
```

$$1 - 4\, e^{5/2} + 4\, e^{7/2}$$

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Question 5 (5 marks)

Given that $\int_0^1 f(x)\,dx = 4$ and $\int_0^{-3} f(x)\,dx = -9$, evaluate:

a. $\int_1^{-3} 2f(x) - 1\,dx$. (2 marks)

$$\int_1^{-3} 2f(x) - 1\,dx = -2\int_{-3}^{1} f(x)\,dx + \int_{-3}^{1} 1\,dx$$
$$= -2(4+9) + 1(1-(-3)) = -22$$

b. $\frac{1}{2}\int_{-3}^{3} f\left(\frac{x-3}{2}\right) + 3\,dx$. (3 marks)

$$\frac{1}{2}\int_{-3}^{3} f\left(\frac{x-3}{2}\right) + 3\,dx = \frac{1}{2}\int_{-3}^{3} f\left(\frac{x-3}{2}\right)\,dx + \frac{1}{2}\int_{-3}^{3} 3\,dx$$
$$= \frac{1}{2}\int_{-6}^{0} f\left(\frac{x}{2}\right)\,dx + \frac{1}{2}\times 3(3-(-3))$$
$$= \frac{1}{2}\times 2\int_{-3}^{0} f(x)\,dx + 9$$
$$= 9 + 9 = 18$$

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Section B: Extension Questions (11 Marks)

Question 6 (2 marks)

If

$$\int \frac{m - ax^n + 2x}{bx}\,dx = \frac{3}{2}\log_e(x) - 2x^3 + x + c$$

Determine the values of a, b, n and m.

$b = 2, m = 3, n = 3$ and $a = 12$

In[43]:= $\text{Integrate}\left[\frac{3 - 12\,x\text{^}3 + 2\,x}{2\,x}, x\right]$

Out[43]= $x - 2\,x^3 + \frac{3\,\text{Log}[x]}{2}$

In[44]:= $\text{D}\left[x - 2\,x^3 + \frac{3\,\text{Log}[x]}{2}, x\right]$

Out[44]= $1 + \frac{3}{2\,x} - 6\,x^2$

In[45]:= $\text{Together}\left[1 + \frac{3}{2\,x} - 6\,x^2\right]$

Out[45]= $\frac{3 + 2\,x - 12\,x^3}{2\,x}$

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Question 7 (4 marks)

a. Find the derivative with respect to x, of $\log_e(f(x))$. (1 mark)

$$\boxed{\frac{f'(x)}{f(x)}}$$

b. Hence, evaluate the integral. (3 marks).

$$\int_1^3 \frac{6-4x}{2x^2-6x-3}\,dx$$

In[37]:= $\text{Integrate}\left[\frac{6-4x}{2x\text{^}2-6x-3}, x\right]$

Out[37]= $-\text{Log}\left[-3-6x+2x^2\right]$

In[38]:= $\text{Integrate}\left[\frac{6-4x}{2x\text{^}2-6x-3}, \{x, 1, 3\}\right]$

Out[38]= $\text{Log}\left[\frac{7}{3}\right]$

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Question 8 (5 marks)

a. Differentiate $y = \log_e(\cos(3x))$. (1 mark).

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In[47]:= D[Log[Cos[3 x]], x]
Out[47]= -3 Tan[3 x]
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b. A function f is such that $f'(x) = \tan(3x)$ and $f\left(\frac{2\pi}{3}\right) = 3$. Find a possible rule for $f(x)$. (4 marks)

$$g(x) = -\frac{1}{3}\log_e(\cos(3x)) + 3$$

```
In[48]:= Integrate[Tan[3 x], x]
Out[48]= -1/3 Log[Cos[3 x]]

In[49]:= g[x_] := -1/3 Log[Cos[3 x]] + c

In[50]:= Solve[g[2 Pi / 3] == 3, c]
Out[50]= {{c → 3}}
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