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VCE Mathematical Methods $\frac{3}{4}$
Antidifferentiation [4.1]
Test Solutions

34 Marks. 1 Minute Reading. 23 Minutes Writing.

Results:

Test Questions	_____ / 23
Extension Questions	_____ / 11



Section A: Test Questions (23 Marks)

Question 1 (4 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. If $F(x)$ is an antiderivative of $f(x)$, then $F(x) + c$ is also an antiderivative of $f(x)$.	☒	
b. The integral of x^n is $\frac{x^{n+1}}{n+1} + c$.	☒	
c. The reverse chain rule method of integration works for all factorised expressions.		☒
The inside must be linear.		
d. The definite integral of a function gives the change in the value of the antiderivative function over a given interval.	☒	
e. If $F(x)$ is an antiderivative of $f(x)$, then $\int_a^b f(x) dx = F(b) - F(a)$.		☒
f. The integral of $\frac{1}{x}$ is always $\log_e(x) + c$.		☒
It can be $\log_e(-x) + c$		
g. If $\int_0^a f(x) dx = b$, then $\int_0^{2a} f\left(\frac{x}{2}\right) dx$ dilates the change in the antiderivative by a factor of $\frac{1}{2}$.		☒
The change is doubled.		
h. The definite integral of a sine function over an interval of one period is zero.	☒	

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Question 2 (6 marks)

Evaluate each of the following integrals:

a. $\int \frac{1}{2}x^3 + 2x^2 - 3x \, dx$. (2 marks)

$$\text{Integrate} \left[\frac{x^3}{2} + 2x^2 - 3x, x \right]$$

$$-\frac{3x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{8}$$

b. $\int \frac{1}{2x+5} - 3 \, dx$. (2 marks)

$$\text{Integrate} \left[\frac{1}{2x+5} - 3, x \right]$$

$$-3x + \frac{1}{2} \text{Log}[5 + 2x]$$

c. $\int \frac{1}{2} \sin(5 - 2x) + \frac{1}{e^{2x}} \, dx$. (2 marks)

$$\text{Integrate} \left[\frac{1}{2} \text{Sin}[5 - 2x] + \frac{1}{e^{2x}}, x \right] // \text{FullSimplify}$$

$$\frac{1}{4} \left(-2e^{-2x} + \text{Cos}[5 - 2x] \right)$$

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Question 3 (3 marks)

Find the rule of the antiderivative function of $y = 5e^{x+1} - 2x^2$ given that the antiderivative passes through the points $\left(-1, \frac{22}{3}\right)$.

Integrate $[5 E^{x+1} - 2 x^2, x]$

$$5 e^{1+x} - \frac{2 x^3}{3}$$

Solve $\left[\frac{22}{3} == 5 e^{1+x} - \frac{2 x^3}{3} + c /. x \rightarrow -1, c\right]$

$$\left\{\left\{c \rightarrow \frac{5}{3}\right\}\right\}$$

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Question 4 (5 marks)

Evaluate each of the following definite integrals:

a. $\int_{-6}^{-5} \cos\left(\pi x + \frac{\pi}{4}\right) - \frac{1}{x+3} dx$. (2 marks)

$$\text{Integrate} \left[\cos \left[\pi * x + \frac{\pi}{4} \right] - \frac{1}{x+3}, \{x, -6, -5\} \right]$$

$$-\frac{\sqrt{2}}{\pi} + \text{Log} \left[\frac{3}{2} \right]$$

b. $\int_1^3 \frac{2}{e^{\frac{x}{2}-4}} + \frac{x^3}{4} - 2 dx$. (3 marks)

$$\text{Integrate} \left[\frac{2}{E^{\frac{x}{2}-4}} + \frac{x^3}{4} - 2, \{x, 1, 3\} \right] // \text{Expand}$$

$$1 - 4 e^{5/2} + 4 e^{7/2}$$

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Question 5 (5 marks)

Given that $\int_0^1 f(x) dx = 4$ and $\int_0^{-3} f(x) dx = -9$, evaluate:

a. $\int_1^{-3} 2f(x) - 1 dx$. (2 marks)

$$\begin{aligned}\int_1^{-3} 2f(x) - 1 dx &= -2 \int_{-3}^1 f(x) dx + \int_{-3}^1 1 dx \\ &= -2(4 + 9) + 1(1 - (-3)) = -22\end{aligned}$$

b. $\frac{1}{2} \int_{-3}^3 f\left(\frac{x-3}{2}\right) + 3 dx$. (3 marks)

$$\begin{aligned}\frac{1}{2} \int_{-3}^3 f\left(\frac{x-3}{2}\right) + 3 dx &= \frac{1}{2} \int_{-3}^3 f\left(\frac{x-3}{2}\right) dx + \frac{1}{2} \int_{-3}^3 3 dx \\ &= \frac{1}{2} \int_{-6}^0 f\left(\frac{x}{2}\right) dx + \frac{1}{2} \times 3(3 - (-3)) \\ &= \frac{1}{2} \times 2 \int_{-3}^0 f(x) dx + 9 \\ &= 9 + 9 = 18\end{aligned}$$

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Section B: Extension Questions (11 Marks)

Question 6 (2 marks)

If

$$\int \frac{m - ax^n + 2x}{bx} dx = \frac{3}{2} \log_e(x) - 2x^3 + x + c$$

Determine the values of a , b , n and m .

$$b = 2, m = 3, n = 3 \text{ and } a = 12$$

```
In[43]:= Integrate[ $\frac{3 - 12x^3 + 2x}{2x}$ , x]
```

$$\text{Out[43]} = x - 2x^3 + \frac{3 \log[x]}{2}$$

```
In[44]:= D[x - 2 x^3 +  $\frac{3 \log[x]}{2}$ , x]
```

```
Out[44]= 1 +  $\frac{3}{2x}$  - 6 x^2
```

```
In[45]:= Together[1 +  $\frac{3}{2x}$  - 6 x^2]
```

```
Out[45]=  $\frac{3 + 2x - 12x^3}{2x}$ 
```

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Question 7 (4 marks)

- a. Find the derivative with respect to x , of $\log_e(f(x))$. (1 mark)

$$\frac{f'(x)}{f(x)}$$

- b. Hence, evaluate the integral. (3 marks).

$$\int_1^3 \frac{6 - 4x}{2x^2 - 6x - 3} dx$$

```
In[37]:= Integrate[ $\frac{6 - 4x}{2x^2 - 6x - 3}$ , x]
Out[37]= -Log[-3 - 6x + 2x^2]

In[38]:= Integrate[ $\frac{6 - 4x}{2x^2 - 6x - 3}$ , {x, 1, 3}]
Out[38]= Log[ $\frac{7}{3}$ ]
```

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Question 8 (5 marks)

- a. Differentiate $y = \log_e(\cos(3x))$. (1 mark).

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In[47]:= D[Log[Cos[3 x]], x]
```

```
Out[47]= -3 Tan[3 x]
```

- b. A function f is such that $f'(x) = \tan(3x)$ and $f\left(\frac{2\pi}{3}\right) = 3$. Find a possible rule for $f(x)$. (4 marks)

$$g(x) = -\frac{1}{3} \log_e(\cos(3x)) + 3$$

```
In[48]:= Integrate[Tan[3 x], x]
```

```
Out[48]= -1/3 Log[Cos[3 x]]
```

```
In[49]:= g[x_] := -1/3 Log[Cos[3 x]] + c
```

```
In[50]:= Solve[g[2 Pi / 3] == 3, c]
```

```
Out[50]= {{c -> 3}}
```

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