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VCE Mathematical Methods ¾
Antidifferentiation [4.1]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 16
Supplementary Questions	Pg 17 – Pg 30

Section A: Compulsory Questions

Sub-Section [4.1.1]: Find an Antiderivative Function

Question 1

Evaluate each of the following integrals:

a. $\int (3x^2 - 4x + 1)\, dx$

$$\int (3x^2 - 4x + 1)\, dx = \int 3x^2\, dx - \int 4x\, dx + \int 1\, dx$$
$$= x^3 - 2x^2 + x + c$$

b. $\int (\sin x + \cos x)\, dx$

$$\int (\sin x + \cos x)\, dx = \int \sin x\, dx + \int \cos x\, dx$$
$$= -\cos x + \sin x + c$$

c. $\int \left(e^x + \frac{1}{x}\right) dx$, where $x > 0$.

$$\int (e^x + \frac{1}{x})\, dx = \int e^x\, dx + \int \frac{1}{x}\, dx$$
$$= e^x + \log_e(x) + c$$

d. $\int (\sec^2 x - 3)\, dx$

$$\int (\sec^2 x - 3)\, dx = \int \sec^2 x\, dx - \int 3\, dx$$
$$= \tan x - 3x + c$$

e. $\int \left(\frac{2}{x} + 5x^3\right) dx$, where $x < 0$.

$$\int \left(\frac{2}{x} + 5x^3\right) dx = \int \frac{2}{x}\, dx + \int 5x^3\, dx$$
$$= 2\log_e(-x) + \frac{5}{4}x^4 + c$$

f. $\int (4e^x - \sin x)\, dx$

$$\int (4e^x - \sin x)\, dx = 4e^x + \cos x + c$$

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Question 2

Evaluate each of the following integrals using the reverse chain rule.

a. $\int (3x+4)^5 \, dx$

$$\boxed{\begin{aligned}\int (3x+4)^5 \, dx &= \frac{1}{3\cdot 6}(3x+4)^6 + c \\ &= \frac{1}{18}(3x+4)^6 + c\end{aligned}}$$

b. $\int \cos(2x-\pi) \, dx$

$$\boxed{\int \cos(2x-\pi)\, dx = \frac{1}{2}\sin(2x-\pi) + c = -\frac{1}{2}\sin(2x) + c}$$

c. $\int e^{5x-7} \, dx$

$$\boxed{\int e^{5x-7}\, dx = \frac{1}{5}e^{5x-7} + c}$$

d. $\int \sec^2(4x) \, dx$

$$\boxed{\int \sec^2(4x)\, dx = \frac{1}{4}\tan(4x) + c}$$

e. $\int \frac{1}{3x-1}\, dx$, where $x > \frac{1}{3}$.

$$\int \frac{1}{3x-1}\, dx = \frac{1}{3}\log_e(3x-1) + c$$

f. $\int \frac{1}{-2x+5}\, dx$, where $x < \frac{5}{2}$.

$$\int \frac{1}{-2x+5}\, dx = -\frac{1}{2}\log_e(5-2x) + c$$

Question 3

For each of the following, the derivative $f'(x)$ is given, and the function f passes through a specific point. Find the rule for $f(x)$.

a. $f'(x) = 4x^3 + 1$, and $f(1) = 6$.

$$f(x) = \int (4x^3 + 1)\, dx = x^4 + x + c$$

$$\text{Substitute } f(1) = 6: \quad 6 = 1^4 + 1 + c = 2 + c \implies c = 4$$

$$\therefore f(x) = x^4 + x + 4$$

b. $f'(x) = \sin(2x)$, and $f\left(\frac{\pi}{2}\right) = 0$.

$$f(x) = \int \sin(2x)\,dx = -\frac{1}{2}\cos(2x) + c$$

$$0 = -\frac{1}{2}\cos(\pi) + c = \frac{1}{2} + c \implies c = -\frac{1}{2}$$

$$\therefore f(x) = -\frac{1}{2}\cos(2x) - \frac{1}{2}$$

c. $f'(x) = \frac{1}{x+4}$ and $f(0) = 2$, with $x > -4$.

$$f(x) = \int \frac{1}{x+4}\,dx = \log_e(x+4) + c$$

$$2 = \log_e(4) + c \implies c = 2 - \log_e(4)$$

$$\therefore f(x) = \log_e(x+4) + 2 - \log_e(4)$$

Question 4 Tech-Active.

Find $f(x)$ if $f'(x) = 3x^2 + 9x + \frac{1}{x}$ and $f(1) = 6$ and $x > 0$.

$$x^3 + \frac{9x^2}{2} + \log_e(x) + \frac{1}{2}$$

```
In[144]:= f[x_] := c + 9 x^2/2 + x^3 + Log[x]
In[147]:= Solve[f[1] == 6]
Out[147]= {{c -> 1/2}}
In[148]:= f[x] /. c -> 1/2
Out[148]= 1/2 + 9 x^2/2 + x^3 + Log[x]
```

```
∫ 3*x^2+9*x+1/x dx          x^3+9·x^2/2+ln(|x|)
define f(x) = x^3+9·x^2/2+ln(|x|)+c          done
solve(f(1)=6,c)          {c=1/2}
f(x)|c=1/2          x^3+9·x^2/2+ln(|x|)+1/2
```

```
∫ 3·x^2+9·x+1/x dx          ln(x)+x^3+9·x^2/2
Define f(x)=ln(x)+x^3+9·x^2/2+c          Done
solve(f(1)=6,c)          c=1/2
f(x)|c=1/2          ln(x)+x^3+9·x^2/2+1/2
```

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Sub-Section [4.1.2]: Evaluate Definite Integrals

Question 5

Evaluate the following definite integrals:

a. $\int_1^3 (2x+5)\,dx$

$$\int_1^3 (2x+5)\,dx = \left[x^2+5x\right]_1^3$$
$$= (9+15)-(1+5) = 24-6 = 18$$

b. $\int_0^\pi \sin(x)\,dx$

$$\int_0^\pi \sin(x)\,dx = [-\cos(x)]_0^\pi$$
$$= (-\cos(\pi)) - (-\cos(0)) = 1-(-1) = 2$$

c. $\int_1^2 (3x^2+4x)\,dx$

$$\int_1^2 (3x^2+4x)\,dx = \left[x^3+2x^2\right]_1^2$$
$$= (8+8)-(1+2) = 16-3 = 13$$

d. $\int_0^1 \frac{1}{x+2}\,dx$

$$\int_0^1 \frac{1}{x+2}\,dx = [\log_e(x+2)]_0^1$$
$$= \log_e(3) - \log_e(2) = \log_e\left(\frac{3}{2}\right)$$

e. $\int_0^{\frac{\pi}{12}} \sec^2(3x)\,dx$

$$\int_0^{\frac{\pi}{12}} \sec^2(3x)\,dx = \frac{1}{3}\int_0^{\frac{\pi}{12}} \sec^2(3x)\cdot 3\,dx$$
$$= \frac{1}{3}[\tan(3x)]_0^{\frac{\pi}{12}} = \frac{1}{3}\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right) = \frac{1}{3}$$

f. $\int_1^4 (x^2 + 2x + 1)\,dx$

$$\int_1^4 (x^2+2x+1)\,dx = \left[\frac{1}{3}x^3 + x^2 + x\right]_1^4$$
$$= \left(\frac{64}{3} + 16 + 4\right) - \left(\frac{1}{3} + 1 + 1\right)$$
$$= \left(\frac{64}{3} + 20\right) - \left(\frac{1}{3} + 2\right) = \frac{64-1}{3} + 18 = \frac{63}{3} + 18 = 21 + 18 = 39$$

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Question 6

Evaluate each of the following definite integrals:

a. $\int_0^1 (2x+1)^3\,dx$

$$\int (2x+1)^3\,dx = \frac{1}{2\cdot 4}(2x+1)^4 = \frac{1}{8}(2x+1)^4$$
$$\left[\frac{1}{8}(2x+1)^4\right]_0^1 = \frac{1}{8}(3^4-1^4) = \frac{80}{8} = 10$$

b. $\int_\pi^{2\pi} \cos(3x-\pi)\,dx$

$$\int \cos(3x-\pi)\,dx = \frac{1}{3}\sin(3x-\pi)$$
$$\left[\frac{1}{3}\sin(3x-\pi)\right]_\pi^{2\pi} = \frac{1}{3}(\sin(\pi)-\sin(0)) = 0$$

c. $\int_0^{\frac{\pi}{12}} \sec^2(4x)\,dx$

$$\int \sec^2(4x)\,dx = \frac{1}{4}\tan(4x)$$
$$\left[\frac{1}{4}\tan(4x)\right]_0^{\frac{\pi}{12}} = \frac{1}{4}\left(\tan\left(\frac{\pi}{3}\right)-\tan(0)\right) = \frac{\sqrt{3}}{4}$$

d. $\int_0^1 e^{2x+1}\,dx$

$$\int e^{2x+1}\,dx = \frac{1}{2}e^{2x+1}$$
$$\left[\frac{1}{2}e^{2x+1}\right]_0^1 = \frac{1}{2}(e^3-e)$$

e. $\int_0^2 \frac{1}{3x+1}\, dx$

$$\int \frac{1}{3x+1}\, dx = \frac{1}{3}\ln|3x+1|$$
$$\left[\frac{1}{3}\ln|3x+1|\right]_0^2 = \frac{1}{3}(\ln(7) - \ln(1)) = \frac{1}{3}\ln(7)$$

f. $\int_0^1 \frac{1}{2x-5}\, dx$

$$\int \frac{1}{2x-5}\, dx = \frac{1}{2}\ln|2x-5|$$
$$\left[\frac{1}{2}\ln|2x-5|\right]_0^1 = \frac{1}{2}(\ln|-3| - \ln|-5|) = \frac{1}{2}(\ln(3) - \ln(5)) = \frac{1}{2}\ln\left(\frac{3}{5}\right)$$

Question 7

Evaluate the following definite integrals:

a. $\int_0^{\log_e(2)} 4e^{2x} - \frac{3}{1+2x}\, dx$

$$\begin{aligned}\int_0^{\log_e(2)} \left(4e^{2x} - \frac{3}{1+2x}\right) dx &= \int_0^{\log_e(2)} 4e^{2x}\, dx - \int_0^{\log_e(2)} \frac{3}{1+2x}\, dx \\ &= \left[2e^{2x}\right]_0^{\log_e(2)} - \left[\frac{3}{2}\log_e(1+2x)\right]_0^{\log_e(2)} \\ &= \left(2e^{2\log_e(2)} - 2e^0\right) - \frac{3}{2}(\log_e(1+2\log_e(2)) - \log_e(1)) \\ &= (2\cdot 4 - 2) - \frac{3}{2}\log_e(1+2\log_e(2)) \\ &= 6 - \frac{3}{2}\log_e(1+2\log_e(2))\end{aligned}$$

b. $\int_1^2 \frac{6x+5}{(3x^2+5x+1)^2}\,dx$

HINT: What is the derivative of $\frac{1}{f(x)}$?

Let the numerator be the derivative of the denominator:

$$\frac{d}{dx}(3x^2+5x+1) = 6x+5.$$

Then this fits the form $\frac{f'(x)}{(f(x))^2}$, whose antiderivative is $\frac{-1}{f(x)}$. So:

$$\begin{aligned}\int_1^2 \frac{6x+5}{(3x^2+5x+1)^2}\,dx &= \left[\frac{-1}{3x^2+5x+1}\right]_1^2 \\ &= \frac{-1}{3(2)^2+5(2)+1} + \frac{1}{3(1)^2+5(1)+1} \\ &= \frac{-1}{12+10+1} + \frac{1}{3+5+1} = \frac{-1}{23} + \frac{1}{9} \\ &= \frac{-9+23}{207} = \frac{14}{207}\end{aligned}$$

c. $\int_0^{\frac{\pi}{4}} \left(\frac{3}{4}\cos\left(2x+\frac{\pi}{3}\right) + \frac{1}{2}\sec^2(x+\pi)\right) dx$

$$\begin{aligned}&\int_0^{\frac{\pi}{4}} \left(\frac{3}{4}\cos\left(2x+\frac{\pi}{3}\right) + \frac{1}{2}\sec^2(x+\pi)\right) dx \\ &= \left[\frac{3}{8}\sin\left(2x+\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{4}} + \left[\frac{1}{2}\tan(x+\pi)\right]_0^{\frac{\pi}{4}} \\ &= \frac{3}{8}\left(\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)\right) + \frac{1}{2}\left(\tan\left(\frac{\pi}{4}+\pi\right) - \tan(\pi)\right) \\ &= \frac{3}{8}\left(\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{3}\right)\right) + \frac{1}{2}\left(\tan\left(\frac{5\pi}{4}\right) - 0\right) \\ &= \frac{3}{8}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) + \frac{1}{2} \\ &= \frac{3}{8}\cdot\left(\frac{1-\sqrt{3}}{2}\right) + \frac{1}{2} \\ &= \frac{3(1-\sqrt{3})}{16} + \frac{1}{2} = \frac{11-3\sqrt{3}}{16}\end{aligned}$$

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Question 8 Tech-Active.

Evaluate the following definite integral using technology:

$$\int_0^{\frac{\pi}{4}} \sin^2(x)\cos(x)\,dx$$

$\frac{1}{6\sqrt{2}}$

In[149]:= `Integrate[Sin[x]^2 Cos[x], {x, 0, Pi/4}]`

Out[149]= $\frac{1}{6\sqrt{2}}$

$$\int_0^{\frac{\pi}{4}} \left((\sin(x))^2 \cdot \cos(x)\right)dx \qquad \frac{\sqrt{2}}{12}$$

$\int_0^{\pi/4} (\sin(x))\text{^}2*\cos(x)\,dx$

$\frac{\sqrt{2}}{12}$

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Sub-Section [4.1.3]: Apply Integral Properties to Tackle Integration Questions

Question 9

Evaluate the following integrals given that:

$$\int_0^4 f(x)\,dx = 6 \text{ and } \int_2^4 f(x)\,dx = 5$$

a. $\int_0^2 f(x)\,dx$

$$\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$$
$$6 = \int_0^2 f(x)\,dx + 5$$
$$\implies \int_0^2 f(x)\,dx = 1$$

b. $\int_0^4 3f(x) - 2dx$

$$\int_0^4 3f(x) - 2\,dx = 3\int_0^4 f(x)\,dx - \int_0^4 2\,dx$$
$$= 3(6) - 2(4) = 18 - 8 = 10$$

c. $\int_2^4 2f(x) + x\,dx$

$$\int_2^4 2f(x) + x\,dx = 2\int_2^4 f(x)\,dx + \int_2^4 x\,dx$$
$$= 2(5) + \left[\frac{1}{2}x^2\right]_2^4 = 10 + (8-2) = 10 + 6 = 16$$

Question 10

Evaluate the following integrals given that:

$$\int_1^5 g(x)\,dx = 7 \text{ and } \int_3^5 g(x)\,dx = 4$$

a. $\int_1^3 g(x)\,dx$

$$\int_1^5 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^5 g(x)\,dx$$
$$7 = \int_1^3 g(x)\,dx + 4$$
$$\implies \int_1^3 g(x)\,dx = 3$$

b. $\int_1^5 5g(x) - 4\,dx$

$$\int_1^5 5g(x) - 4\,dx = 5\int_1^5 g(x)\,dx - \int_1^5 4\,dx$$
$$= 5(7) - 4(4) = 35 - 16 = 19$$

c. $\int_1^3 2g(x) + 3x\,dx$

$$\int_1^3 2g(x) + 3x\,dx = 2\int_1^3 g(x)\,dx + \int_1^3 3x\,dx$$
$$= 2(3) + \left[\frac{3}{2}x^2\right]_1^3 = 6 + \left(\frac{27}{2} - \frac{3}{2}\right) = 6 + 12 = 18$$

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Question 11

Given that:

$$\int_1^4 g(x)\,dx = 6 \text{ and } \int_4^7 g(x)\,dx = -3$$

Evaluate the following integrals:

a. $\int_2^8 2g\left(\frac{x}{2}\right)\,dx$

The function has been dilated by factor 2 from the x-axis and also a factor of 2 from the y axis. So $6 \times 2\times = 24$. Alternatively,
let $u = \frac{x}{2} \Rightarrow x = 2u$. When $x = 2$, $u = 1$; when $x = 8$, $u = 4$

$$\int_2^8 2g\left(\frac{x}{2}\right)\,dx = 2 \cdot \int_2^8 g\left(\frac{x}{2}\right)\,dx = 2 \cdot 2\int_1^4 g(u)\,du = 4 \cdot 6 = 24$$

b. $\int_1^7 2g(x) + 1\,dx$

$$\begin{aligned}\int_1^7 2g(x) + 1\,dx &= \int_1^7 2g(x)\,dx + \int_1^7 1\,dx \\ &= 2 \cdot \int_1^7 g(x)\,dx + (7-1) \\ &= 2 \cdot (6 + (-3)) + 6 = 2 \cdot 3 + 6 = 6 + 6 = 12\end{aligned}$$

c. $\int_6^9 2g(x-5)\,dx$

This is just a translation 5 units right of the function then scaled by a factor of 2. Thus $6 \times 2 = 12$. Alternatively,
let $u = x - 5 \Rightarrow du = dx$. When $x = 6$, $u = 1$; when $x = 9$, $u = 4$

$$\int_6^9 2g(x-5)\,dx = 2\int_1^4 g(u)du = 12$$

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Sub-Section: The 'Final Boss'

Question 12

Let $f(x) = 2e^x + \cos(2x)$, and let $F(x)$ be an antiderivative of $f(x)$.

a. Given that $F(0) = 5$, find an explicit expression for $F(x)$.

$$F(x) = \int (2e^x + \cos(2x))\, dx$$
$$= 2e^x + \frac{1}{2}\sin(2x) + c$$

Using $F(0) = 5$:

$$F(0) = 2 + 0 + c = 5 \implies c = 3$$
$$\therefore F(x) = 2e^x + \frac{1}{2}\sin(2x) + 3$$

b. Hence, evaluate $\int_0^{\log_e(2)} f(x)\, dx$.

$$\int_0^{\log_e(2)} f(x)\, dx = F(\log_e(2)) - F(0)$$
$$= \left(2e^{\log_e(2)} + \frac{1}{2}\sin(2\log_e(2)) + 3\right) - 5$$
$$= \left(4 + \frac{1}{2}\sin(2\log_e(2)) + 3\right) - 5 = 2 + \frac{1}{2}\sin(2\log_e(2))$$

Let g be a function such that:

$$\int_0^3 g(x)dx = 12$$

c. Evaluate $\int_0^6 \frac{1}{2} g\left(\frac{x}{2}\right)\, dx$.

The function $g(x) = 4$ satisfies the requirement.
Thus $\int_0^6 \frac{1}{2} g\left(\frac{x}{2}\right)\, dx = \int_0^6 \frac{1}{2} \times 4 dx = 12.$

Section B: Supplementary Questions

Sub-Section[4.1.1]: Find an Antiderivative Function

Question 13

Evaluate the following:

a. $\int \frac{2}{\sqrt{x}}\, dx$

$$\int \frac{2}{\sqrt{x}}\, dx = \int 2x^{-\frac{1}{2}}\, dx = 2 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 4\sqrt{x} + C \ \mathbf{1A}$$

b. $\int x^4 + 3x - 9\, dx$

$$\int x^4 + 3x - 9\, dx = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C \ \mathbf{1A}$$

c. $\int \left(\frac{1}{x} + e^{7x} + x^{\pi} + 7\right)\, dx$ where $x < 0$.

$$\int \left(\frac{1}{x} + e^{7x} + x^{\pi} + 7\right)\, dx = \ln\left(-\boldsymbol{x}\right) + \frac{1}{7}e^{7x} + \frac{x^{\pi+1}}{\pi+1} + 7x + C \ \mathbf{1A}$$

See restriction for $\boldsymbol{x}$

d. $\int (t-6)^2\, dt$

$$\int (t-6)^2\, dt = \int (t^2 - 12t + 36)\, dt = \frac{t^2}{3} - 6t^2 + 36t + C = \frac{t^3}{3} - 6t^2 + 36t + C \ \mathbf{1A}$$

Or $\frac{1}{3}(t-6)^3 + C \ \mathbf{1A}$

e. $\int (2\cos(x) + 4\sin(x))\, dx$

$$\int (2\cos(x) + 4\sin(x))\, dx = 2\sin(x) - 4\cos(x) + C \ \mathbf{1A}$$

f. $\int (3x - \sec^2(x))\, dx$

$$\int (3x - \sec^2(x))\, dx = \frac{3x^2}{2} - \tan(x) + C \text{ } \mathbf{1A}$$

Question 14

Find an antiderivative of the following functions:

a. $(3x+1)^5$

$$\int (3x+1)^5\, dx = \frac{1}{3} \cdot \frac{(3x+1)^6}{6} = \frac{(3x+1)^6}{18} + \boldsymbol{C} \text{ } (\mathbf{1A})$$

b. $5e^{7x}$

$$\int 5e^{7x}\, dx = 5 \cdot \frac{e^{7x}}{7} = \frac{5}{7}e^{7x} + \boldsymbol{C} \text{ } (\mathbf{1A})$$

c. $\sec^2(4x-2)$

$$\int \sec^2(4x-2)\, dx = \frac{1}{4}\tan(4x-2) + C \text{ } (\mathbf{1A})$$

d. $\sin(-2x+3)$

$$\int \sin(3-2x)\, dx = \frac{1}{2}\cos(3-2x) + C(\mathbf{1A})$$

e. $(7x-4)^{\frac{3}{2}}$

$$\int (7x-4)^{\frac{3}{2}}\, dx = \frac{1}{7} \cdot \frac{(7x-4)^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{35}(7x-4)^{\frac{5}{2}} + \boldsymbol{C} \text{ } (\mathbf{1A})$$

f. $\frac{3}{2x+3}$ where $x > -\frac{3}{2}$.

$$\int \frac{3}{2x+3}\,dx = \frac{3}{2}\log_e(2x+3) + C\ (\mathbf{1A})$$

Question 15

a. Given $f'(x) = 4x^3 - 2x + 7$, and $f(1) = 10$, find $f(x)$.

$$\int f'(x)\,dx = \int (4x^3 - 2x + 7)\,dx = x^4 - x^2 + 7x + C\ \mathbf{1M}$$
$$f(1) = 1^4 - 1^2 + 7(1) + C = 1 - 1 + 7 + C = 7 + C = 10 \Rightarrow C = 3\ \mathbf{1M}$$
$$\boxed{f(x) = x^4 - x^2 + 7x + 3}\,\mathbf{1A}$$

b. Given $f'(x) = \frac{5}{2x+1}$, and $f(0) = 2$, find $f(x)$ for $x > -\frac{1}{2}$.

$$\int f'(x)dx = \int \frac{5}{2x+1}\,dx = \frac{5}{2}\log_e(2x+1)\ \ +C\ \mathbf{1M}$$
$$f(0) = \frac{5}{2}\log_e(1)\ \ +C = 0 + C = 2 \Rightarrow C = 2\ \mathbf{1M}$$
$$\boxed{f(x) = \frac{5}{2}\log_e(2x+1) + 2}\,\mathbf{1A}$$

c. Given $f'(x) = 6e^{3x}$, and $f(0) = 4$, find $f(x)$.

$$\int f'(x)\,dx = \int 6e^{3x}\,dx = 6 \cdot \frac{1}{3}e^{3x} = 2e^{3x} + C \ \mathbf{1M}$$

$$f(0) = 2e^{0} + C = 2 + C = 4 \Rightarrow C = 2 \ \mathbf{1M}$$

$$\boxed{f(x) = 2e^{3x} + 2}\ \mathbf{1A}$$

Question 16 Tech-Active.

Find y in terms of x if $\frac{dy}{dx} = 6(2x+1)^2 - \frac{10}{(3x-4)^2}$ and $y = 5$ when $x = 1$.

$\int 6(2x+1)\text{^}2 - \frac{10}{(3x-4)\text{^}2}dx$

$8\cdot x^3+12\cdot x^2+6\cdot x+\frac{10}{3\cdot(3\cdot x-4)}+1$

Define $f(x)=8\cdot x^3+12\cdot x^2+6\cdot x+\frac{10}{3\cdot(3\cdot x-4)}+1+c$ 1M

done

solve(f(1)=5, c

$\left.\begin{matrix}\text{done}\\ \left\{c=-\frac{56}{3}\right\}\end{matrix}\right\}$ 1M

$y=f(x)|c=-\frac{56}{3}$

$y=8\cdot x^3+12\cdot x^2+6\cdot x+\frac{10}{3\cdot(3\cdot x-4)}-\frac{53}{3}$ 1A

$$\int \left(6\cdot(2\cdot x+1)^2-\frac{10}{(3\cdot x-4)^2}\right)dx$$

$$\frac{10}{3\cdot(3\cdot x-4)}+(2\cdot x+1)^3$$

Define $f(x)=\frac{10}{3\cdot(3\cdot x-4)}+(2\cdot x+1)^3+c$ *Done*

solve$(f(1)=5,c)$ $c=\frac{-56}{3}$

$y=f(x)|c=\frac{-56}{3}$

$$y=\frac{10}{3\cdot(3\cdot x-4)}+8\cdot x^3+12\cdot x^2+6\cdot x-\frac{53}{3}$$

In[151]:= Integrate$\left[6\ (2\ x+1)\text{^}2-\frac{10}{(3\ x-4)\text{^}2}, x\right]$

Out[151]= $-\frac{10}{3\ (4-3\ x)}+(1+2\ x)^3$

In[152]:= f[x_] := $-\frac{10}{3\ (4-3\ x)}+(1+2\ x)^3+c$

In[153]:= Solve[f[1] == 5, c]

Out[153]= $\left\{\left\{c\to-\frac{56}{3}\right\}\right\}$

In[154]:= f[x] /. $c\to\frac{-56}{3}$ // Expand

Out[154]= $-\frac{53}{3}-\frac{10}{3\ (4-3\ x)}+6\ x+12\ x^2+8\ x^3$

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Sub-Section [4.1.2]: Solve Definite Integrals

Question 17

Evaluate the following:

a. $\int_0^1 (3x^2 + 2x)\, dx$

$$\int_0^1 (3x^2 + 2x)\, dx = [x^3 + x^2]_0^1 \ \mathbf{1M} = (1^3 + 1^2) - (0^3 + 0^2) = 1 + 1 = 2 \ \mathbf{1A}$$

b. $\int_1^2 (\sqrt{x} + \frac{4}{3}x^3)\, dx$

$$\int_1^2 (\sqrt{x} + \frac{4}{3}x^3)\, dx = [\frac{2}{3}x^{\frac{3}{2}} + \frac{x^4}{3}]_1^2 \ \mathbf{1M} = \left(\frac{2}{3} \cdot 2^{\frac{3}{2}} + \frac{2^4}{3}\right) - \left(\frac{2}{3} \cdot 1^{\frac{3}{2}} + \frac{1^4}{3}\right) \mathbf{1M}$$
$$= \left(\frac{4\sqrt{2}}{3} + \frac{16}{3}\right) - \left(\frac{2}{3} + \frac{1}{3}\right) = \frac{4\sqrt{2} + 16}{3} - 1 = \frac{4\sqrt{2} + 16 - 3}{3} = \frac{4\sqrt{2} + 13}{3} \mathbf{1A}$$

c. $\int_0^2 (x^2 + 1)\, dx$

$$\int_0^2 (x^2 + 1)\, dx = [\frac{x^3}{3} + x]_0^2 \ \mathbf{1M} = \left(\frac{2^3}{3} + 2\right) - \left(\frac{0^3}{3} + 0\right) = \frac{8}{3} + 2 = \frac{14}{3} \ \mathbf{1A}$$

d. $\int_1^e \frac{1}{x}\, dx$

$$\int_1^e \frac{1}{x}\, dx = [\log_e(x)\ ]_1^e \mathbf{1}\boldsymbol{M} = \log_e(e) - \log_e(1) = 1 - 0 = 1\ \mathbf{1}\boldsymbol{A}$$

e. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(\theta)\ d\theta$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(\theta)\, d\theta = [-\cos(\theta)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\ \mathbf{1}\boldsymbol{M} = \left(-\cos\left(\frac{\pi}{2}\right)\right) - \left(-\cos\left(-\frac{\pi}{2}\right)\right) = (-0) - (-0) = 0\ \mathbf{1}\boldsymbol{A}$$

f. $\int_0^2\ (x+1)^7\, dx$

$$\int_0^2 (x+1)^7\, dx = [\frac{(x+1)^8}{8}]_0^2\ \mathbf{1}\boldsymbol{M} = \frac{(2+1)^8}{8} - \frac{(0+1)^8}{8} = \frac{3^8}{8} - \frac{1}{8}\mathbf{1}\boldsymbol{M} = \frac{6561}{8} - \frac{1}{8} = \frac{6560}{8} = 820\ \mathbf{1}\boldsymbol{A}$$

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Question 18

Evaluate the following:

a. $\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)+\sin(x)}{2}\, dx$

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)+\sin(x)}{2}\, dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2(x)\, dx + \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin(x)\, dx$$

$$= \frac{1}{2}[\tan(x)]_0^{\frac{\pi}{4}} + \frac{1}{2}[-\cos(x)]_0^{\frac{\pi}{4}} \mathbf{1M}$$

$$= \frac{1}{2}\left(\tan\left(\frac{\pi}{4}\right) - \right) + \frac{1}{2}\left[-\cos\left(\frac{\pi}{4}\right) + \right]$$

$$= \frac{1}{2}(1-0) + \frac{1}{2}\left(-\frac{\sqrt{2}}{2} + 1\right)\mathbf{1M}$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$= 1 - \frac{\sqrt{2}}{4}.\mathbf{1A}$$

b. $\int_\pi^{\frac{3\pi}{2}} (6\sin(2w) -$

$$\int_\pi^{\frac{3\pi}{2}} (6\sin(2w) - 10\cos(w))\, dw = 6\int_\pi^{\frac{3\pi}{2}} \sin(2w)\, dw - 10\int_\pi^{\frac{3\pi}{2}} \cos(w)\, dw.$$

$$\int \sin(2w)\, dw = -\frac{1}{2}\cos(2w)$$

$$6\int_\pi^{\frac{3\pi}{2}} \sin(2w)\, dw = 6\left[-\frac{1}{2}\cos(2w)\right]_\pi^{\frac{3\pi}{2}} = -3[\cos(2w)]_\pi^{\frac{3\pi}{2}}$$

$$= -3(\cos(3\pi) - \cos(2\pi)) = -3((-1)-1) = 6.$$

$$\int \cos(w)\, dw = \sin(w)$$

$$-10\int_\pi^{\frac{3\pi}{2}} \cos(w)\, dw = -10[\sin(w)]_\pi^{\frac{3\pi}{2}} = -10(-)$$

$$= -10((-1)-0) = 10.$$

1M for antiderivative, 1M substitution

$$6 + 10 = 16.\mathbf{1}\boldsymbol{A}$$

c. $\int_0^2 (10x^2+10)\, dx$

$$\int_0^2 (10x^2+10)\, dx = 10\int_0^2 x^2\, dx + 10\int_0^2 1\, dx$$

$$= 10\left[\frac{x^3}{3}\right]_0^2 + 10[x]_0^2\mathbf{1M}$$

$$= 10\left(\frac{2^3}{3} - 0\right) + 10(2-0)\mathbf{1M}$$

$$= 10\left(\frac{8}{3}\right) + 20$$

$$= \frac{80}{3} + 20$$

$$= \frac{80+60}{3}$$

$$= \frac{140}{3}.\mathbf{1A}$$

d. $\int_0^{\frac{\pi}{2}} \sin\left(2\left(\theta + \frac{\pi}{4}\right)\right)\, d\theta$

$$\int_0^{\frac{\pi}{2}} \, d\theta = \int_0^{\frac{\pi}{2}} \, d\theta.$$
$$\sin(2\theta + \frac{\pi}{2}) = \cos(2\theta).\, \mathbf{1M}$$
$$\int_0^{\frac{\pi}{2}} \cos(2\theta)\, d\theta = [\frac{1}{2}\sin(2\theta)]_0^{\frac{\pi}{2}} \mathbf{1M}$$
$$= \frac{1}{2}(\sin(\pi) - \sin(0))$$
$$= \frac{1}{2}(0 - 0) = 0.\, \mathbf{1A}$$

e. $\int_0^{\frac{\pi}{4}} \cos(2\theta)\, d\theta$

$$\int_0^{\frac{\pi}{4}} \cos(2\theta)\, d\theta = [\frac{1}{2}\sin(2\theta)]_0^{\frac{\pi}{4}} \mathbf{1M}$$
$$= \frac{1}{2}(-)$$
$$= \frac{1}{2}(1 - 0) = \frac{1}{2}.\, \mathbf{1A}$$

f. $\int_0^{\pi} \sin(4\theta)\, d\theta$

$$\int_0^{\pi} \sin(4\theta)\, d\theta = [-\frac{1}{4}\cos(4\theta)]_0^{\pi} \mathbf{1M}$$
$$= -\frac{1}{4}(\cos(4\pi) - \cos(0))$$
$$= -\frac{1}{4}(1 - 1)$$
$$= -\frac{1}{4}(0) = 0.\, \mathbf{1A}$$

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Question 19

a. Evaluate $\int_1^3 \left(2x^3 - \frac{6}{x+1}\right) dx$.

$$\int_1^3 \left(2x^3 - \frac{6}{x+1}\right) dx$$

$$= [\frac{x^4}{2} - 6\ \log_e(x+1)\ ]_1^3\ \mathbf{1} = (\frac{3^4}{2} - 6\ \log_e 4) - (\frac{1^4}{2} - 6\ \log_e 2).\mathbf{1M}$$

$$= \left(\frac{81}{2} - 6\log_e 4\right) - \left(\frac{1}{2} - 6\log_e 2\right) = \frac{81}{2} - 6\log_e 4 - \frac{1}{2} + 6\log_e 2.$$

$$= 40 - 6\log_e 2.\mathbf{1A}$$

b. Evaluate $\int_0^1 4e^{2x+1}\, dx$.

$$\int_0^1 4e^{2x+1}\, dx.$$

$$= [2e^{2x+1}]_0^1 \mathbf{1M} = 2e^{2\cdot1+1} - 2e^{2\cdot0+1} = 2e^3 - 2e^1 = 2(e^3 - e).\mathbf{1A}$$

c. Evaluate $\int_0^{\frac{\pi}{2}} (\sin(3x) + 4\cos(x))\, dx$.

$$\int_0^{\frac{\pi}{2}} (\sin(3x) + 4\cos(x))\, dx$$

$$= \left[-\frac{1}{3}\cos(3x) + 4\sin(x)\right]_0^{\frac{\pi}{2}} \mathbf{1M} = \left(-\frac{1}{3}\cos\left(\frac{3\pi}{2}\right) + 4\sin\left(\frac{\pi}{2}\right)\right) - \left(-\frac{1}{3}\cos(0) + 4\sin(0)\right).\ \mathbf{1M}$$

$$= \left(-\frac{1}{3}\cdot0 + 4\cdot1\right) - \left(-\frac{1}{3}\cdot1 + 0\right) = (4) - \left(-\frac{1}{3}\right) = 4 + \frac{1}{3} = \frac{13}{3}.\ \mathbf{1A}$$

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Question 20 Tech-Active.

Evaluate $\int_0^1 3(4x + x^4)(10x^2 - 2)\, dx$.

$\int_0^1 3(4x+x^4)(10x^2-2)dx$

$\frac{738}{35}$ 1A

$$\int_0^1 \left(3\cdot\left(4\cdot x+x^4\right)\cdot\left(10\cdot x^2-2\right)\right)dx \qquad \frac{738}{35}$$

```
In[1]:= Integrate[3 * (4 x + x^4) * (10 x^2 - 2), {x, 0, 1}]
```

Out[1]= $\frac{738}{35}$

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Sub-Section [4.1.3]: Apply Integral Properties to Tackle Integration Questions

Question 21

Let $f(x) = g'(x)$ and $h(x) = k'(x)$, where $g(x) = (x^2 + 1)^4$ and $k(x) = \sin(x^2)$.

Find:

a. $\int f(x)\, dx$

$$\int f(x)\, dx = \int g'(x)\, dx = g(x) + C = (x^2 + 1)^4 + C \ \mathbf{1A}$$

b. $\int f(x) + h(x)\, dx$

$$\int (f(x) + h(x))\, dx = \int f(x)\, dx + \int h(x)\, dx$$
$$= g(x) + k(x) + C = (x^2 + 1)^4 + \sin(x^2) + C \ \mathbf{1A}$$

c. $\int 3h(x)\, dx$

$$\int 3h(x)\, dx = 3\int h(x)\, dx = 3k(x) + C = 3\sin(x^2) + C \ \mathbf{1A}$$

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Question 22

Suppose $\int_1^4 f(x)\,dx = 2$ and $\int_1^4 g(x)\,dx = 7$. Evaluate the following integrals:

a. $\int_1^4 \big(5\,f(x) + 3\,g(x)\big)\,dx$

$$\int_1^4 \big(5f(x) + 3g(x)\big)\,dx = 5\int_1^4 f(x)\,dx + 3\int_1^4 g(x)\,dx = 5\cdot 2 + 3\cdot 7\mathbf{1M} = 10 + 21 = 31.\,\mathbf{1A}$$

b. $\int_1^4 \big(6 - 2\,f(x)\big)\,dx$

$$\int_1^4 (6 - 2f(x))\,dx = \int_1^4 6\,dx - 2\int_1^4 f(x)\,dx.$$

$$= [6x]_1^4 - 2\times 2\ \mathbf{1M} = 6(4-1) - 4 = 18 - 4 = 14\ \mathbf{1A}$$

c. $\int_1^4 (2\,f(x) + 10\,g(x) + 3)\,dx$

$$\int_1^4 (2f(x) + 10g(x) + 3)\,dx = 2\int_1^4 f(x)\,dx + 10\int_1^4 g(x)\,dx + \int_1^4 3\,dx.$$

$$2\int_1^4 f(x)\,dx = 2\cdot 2 = 4,$$

$$10\int_1^4 g(x)\,dx = 10\cdot 7 = 70$$

$$\int_1^4 3\,dx = [3x]_1^4 = 3(4-1) = 9.$$

1M Evaluating each term

$$4 + 70 + 9 = 83.\,\mathbf{1A}$$

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Question 23

Suppose it is known that $\int_1^5 f(x)\,dx = -3$ and $\int_2^5 f(x)\,dx = 4$.

a. Evaluate $\int_1^2 f(x)\,dx$.

$$\int_1^5 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx.$$

$$-3 = \int_1^2 f(x)\,dx + 4\ \mathbf{1M} \Longrightarrow \int_1^2 f(x)\,dx = -3 - 4 = -7.\ \mathbf{1A}$$

b. Hence, evaluate $\int_1^2 \big(3 - 4f(x)\big)\,dx$

$$\int_1^2 (3 - 4f(x))\,dx = \int_1^2 3\,dx - 4\int_1^2 f(x)\,dx.$$

$$= [3x]_1^2 - 4 \times (-7)\ \mathbf{1M} = 3(2-1) + 28 = 3 + 28 = 31\ \mathbf{1A}$$

c. Evaluate $\int_2^1 f(x)\,dx$.

$$\int_2^1 f(x)\,dx = -\int_1^2 f(x)\,dx = -(-7) = 7.\ \mathbf{1A}$$

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Question 24 Tech-Active.

Find $\int_0^1 f(3x+1)\,dx$ given that $\int_1^4 f(x)\,dx = 5$.

Working out:

$$\int_0^1 f(3x+1)\,dx = \int_0^1 \frac{1}{3}\frac{d}{dx}[F(3x+1)]\,dx = \frac{1}{3}[F(3x+1)]_0^1.\ (reverse\ chain\ rule)$$

$$= \frac{1}{3}(F(4) - F(1)).\ \mathbf{1M}$$

$$\int_1^4 f(x)\,dx = F(4) - F(1) = 5.$$

$$\int_0^1 f(3x+1)\,dx = \frac{1}{3}\cdot 5 = \frac{5}{3}.\ \mathbf{1A}$$

MCQ technique:

Define f(x)=a

done

solve($\int_1^4$ f(x)dx=5, a

$\{a=\frac{5}{3}\}$

Define f(x)=$\frac{5}{3}$

done

$\int_0^1$ f(3x+1)dx

$\frac{5}{3}$

Define $f(x)=a$ Done

solve$\left(\int_1^4 f(x)\,dx=5, a\right)$ $a=\frac{5}{3}$

Define $f(x)=\frac{5}{3}$ Done

$\int_0^1 f(3\cdot x+1)\,dx$ $\frac{5}{3}$

```
In[155]:= f[x_] := a
In[156]:= Solve[Integrate[f[x], {x, 1, 4}] == 5]
Out[156]= {{a → 5/3}}
In[157]:= f[x_] := 5/3
In[158]:= Integrate[f[3 x + 1], {x, 0, 1}]
Out[158]= 5/3
```

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