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VCE Mathematical Methods ¾
Circular Functions Exam Skills [3.4]
Workbook

Outline:

Section A: Introduction

Let's quickly go over last week's content.

Contour Check

Learning Objective: [3.3.1] – Solve advanced trigonometric equations

Key Takeaways

- General Solutions with domain restriction
  - Steps:
    1. Make the trigonometric function the subject.
    2. Find the necessary ___________ for one period.
    3. Solve for x by equating the necessary angles to the _______________ of the trigonometric functions.
    4. Add $period \cdot n$ where the_________________ of n is appropriately restricted.
- Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A =$______

Learning Objective: [3.3.2] - Graph sine, cosine, and tangent functions

Key Takeaways

- **Amplitude, Period and Average Value**

$$\text{For } y = A\sin/\cos\,(nx+b)+k$$

$$Amplitude = \_\_\_\_\_\_$$

$$Period = \_\_\_\_\_\_$$

$$Average\ Value = \_\_\_$$

- **Graphing of $\sin$ and $\cos$ Functions**
  - Steps:
    1. Identify, ____.
    2. Create a "mini version" of the graph you are about to draw.
    3. Start plotting the function from when the angle = ____.
    4. Draw the start and end of the periods, and plot the halves (turning points).
    5. Find any_____________________________.
    6. Join all the points!

- **Steps For Sketching $\tan$ Functions**

1. Identify
   - The period =______.
2. Find the vertical asymptotes by solving for angle =______.
   Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
3. Plot the inflection point (h, k) (Midpoint of the two ________________________).
   - x-value of inflection point $=$ x-value which makes angle $= 0$.
   - y-value of inflection point $=$ vertical translation of the function.
4. Find any ___________________.
5. Sketch a ______________ shape.

<u>Learning Objective</u>: [3.3.3] – Fraction of periods

Key Takeaways

- **Fraction of Period**

$$\boldsymbol{Fraction\ of\ Period} = \underline{\hspace{6cm}}$$

$$\boldsymbol{\%\ of\ Period} = \underline{\hspace{6cm}} \times \mathbf{100\%}$$

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Section B: Recap

Sub-Section: General Solutions with Domain Restrictions

If you were here last week, skip to Section C – Warmup Test.

Misconception

"When there is a domain restriction, we always get particular solutions"

TRUTH: If the domain restriction has either ∞ or $-\infty$, we can still have general solutions

Question 1

Solve for the following trigonometric equation.

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

<u>**General Solution with Domain Restriction**</u>

$$\text{E.g., trig}\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

- We can have infinite solutions for restricted domains.
- The value of n is also restricted.

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Sub-Section: Hidden Quadratics

Let's have a look at hidden quadratics for circular functions!

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

$$\text{Let } A = f(x)$$

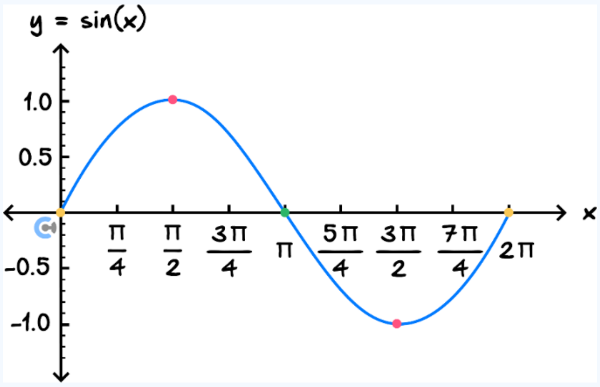
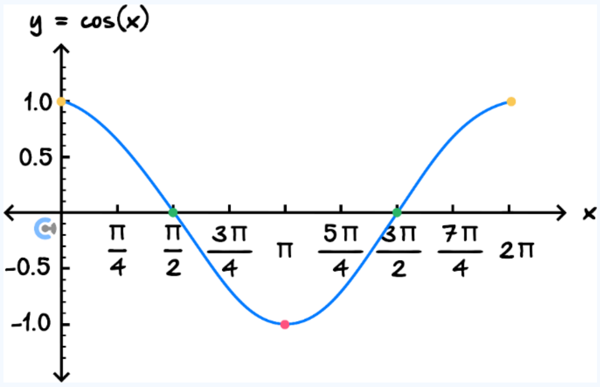
Question 2 Walkthrough.

Solve the following for the values of x.

$$\sin^2\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{\pi}{3}\right) = 2, 0 \le x \le 3\pi$$

Sub-Section: Graphing Sine and Cosine Functions

Sine and Cosine Graphs

Sine	Cosine
	

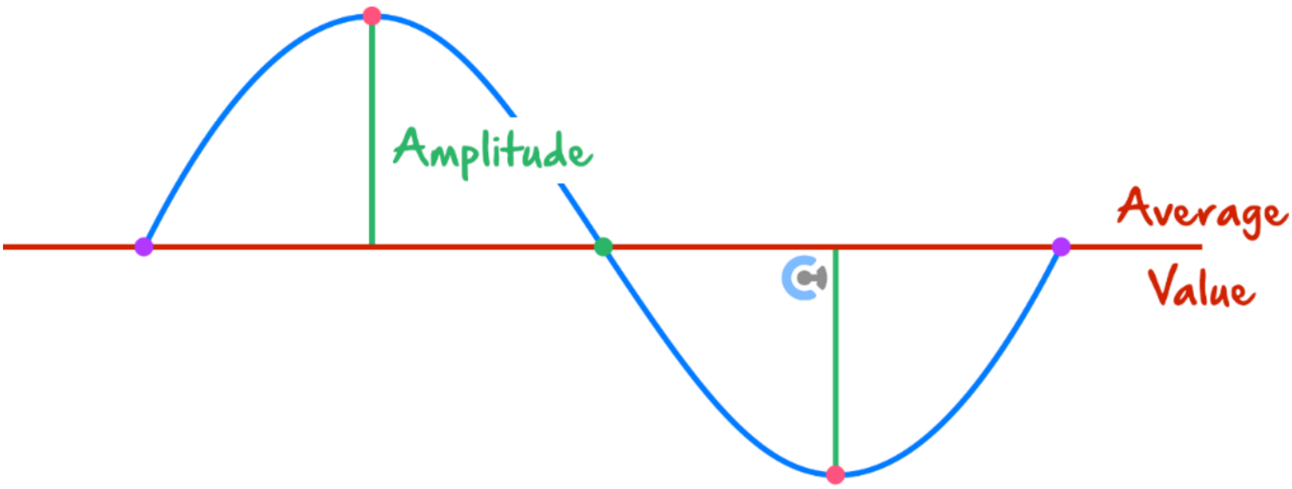
Discussion: Is $\cos(x)$ an even function or an odd function. What about $\sin(x)$?

Discussion: What does $\sin\left(\frac{\pi}{2}+x\right)$ equal to? So, how can we translate sine function to cosine function?

Definition

Amplitude, Period and Average Value

For $y = A\sin/\cos\,(nx+b)+k$



Consider the sign of our graph

Amplitude $= |A|$

Period $= \frac{2\pi}{n}$

Average Value $= k$

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Question 3

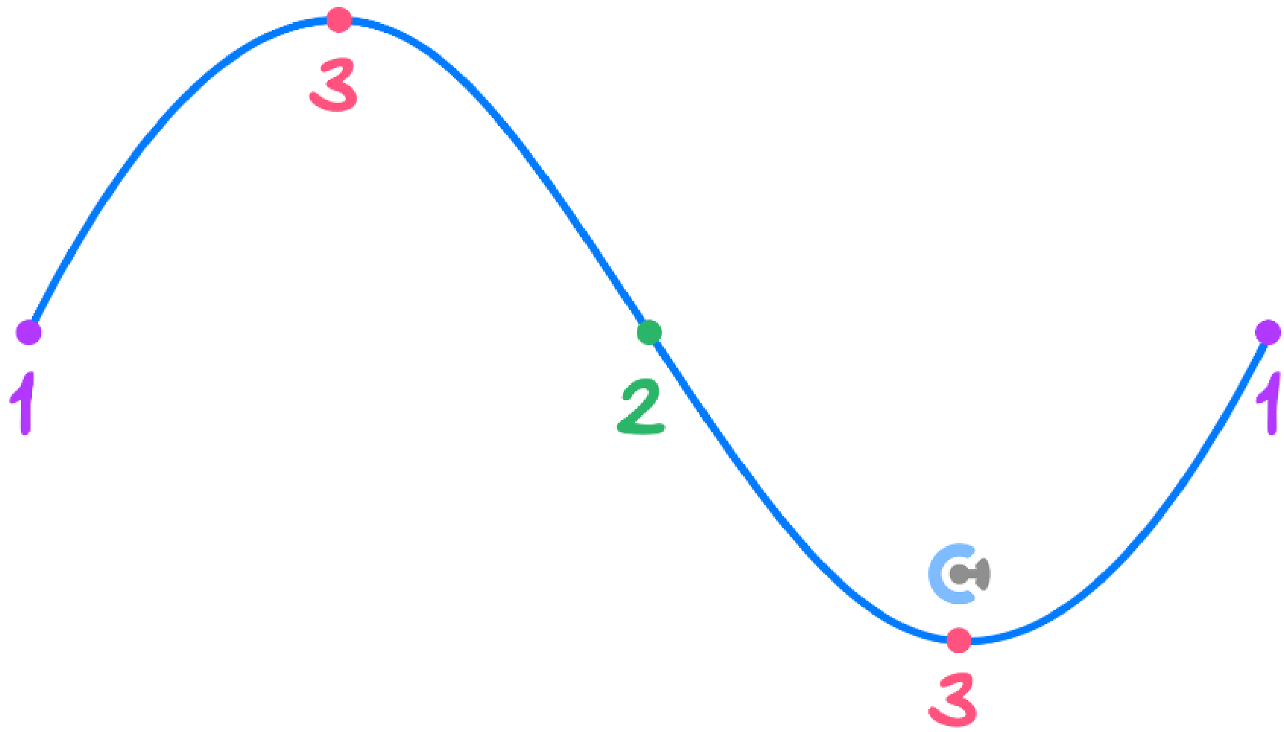
Identify the amplitude, period and average value of the following functions:

a. $f(x) = 2\sin\left(\frac{\pi}{3} - 3x\right) + 1$

b. $g(x) = -3\cos(2x + 3) - 4$

Definition

Graphing of $\sin$ And $\cos$ Functions



1. Identify amplitude, period, mean value, and positive/negative shape.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle$= 0$.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

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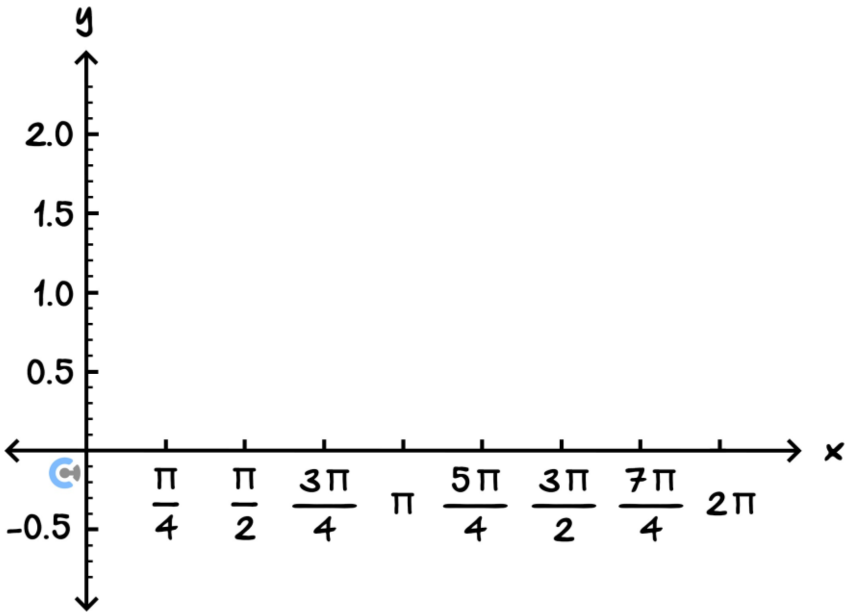
Question 4

Sketch the graph of $f(x) = -\sin(2x) + 1$ for $x \in [0, 2\pi]$ on the axes below, labelling all intercepts and endpoints with their coordinates.



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Sub-Section: Finding the Rule

Finding the Rule

$$\text{Amplitude } (A) = \frac{max - min}{2}$$

$$\text{Average } (k) = \frac{max + min}{2}$$

Question 5

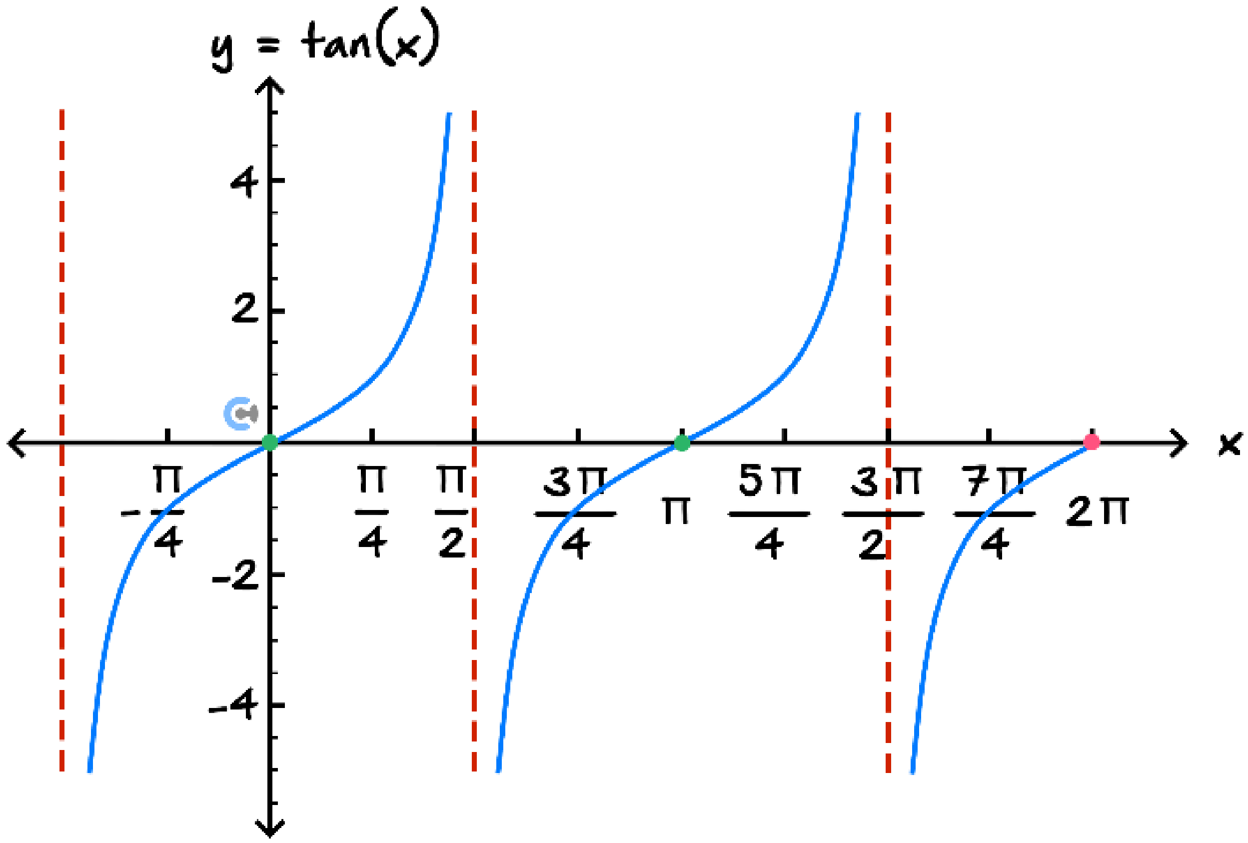
A function with rule $y = A\sin(nt) + b$ where $A > 0$ has a range $[-5,3]$ and period 4. Find A, n and b.

TIP: Graphing helps!

Sub-Section: Understanding Tangent Graphs

Definition

Graph of Tangent



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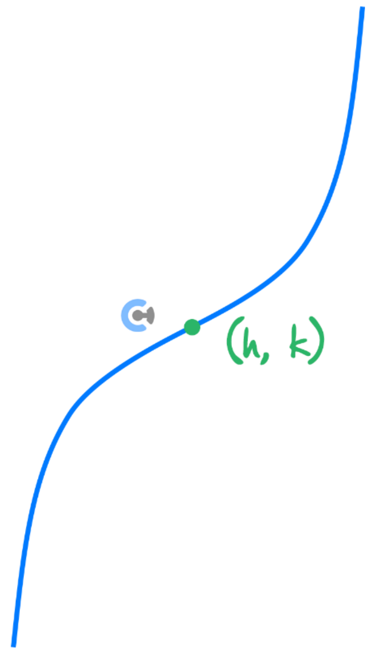
Sub-Section: Graphing Tangent Functions

Steps for Sketching tan Functions

- Identify:
  - The period $= \frac{\pi}{n}$.
- Find the vertical asymptotes by solving for angle $= \frac{\pi}{2}$.
  Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
  - For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x.
- Plot the inflection point (h, k) (Midpoint of the two vertical asymptotes).
  - x-value of inflection point $=$ x-value, which makes angle $= 0$.
  - y-value of inflection point $=$ vertical translation of the function.

eg: $\tan(x-h)+k$

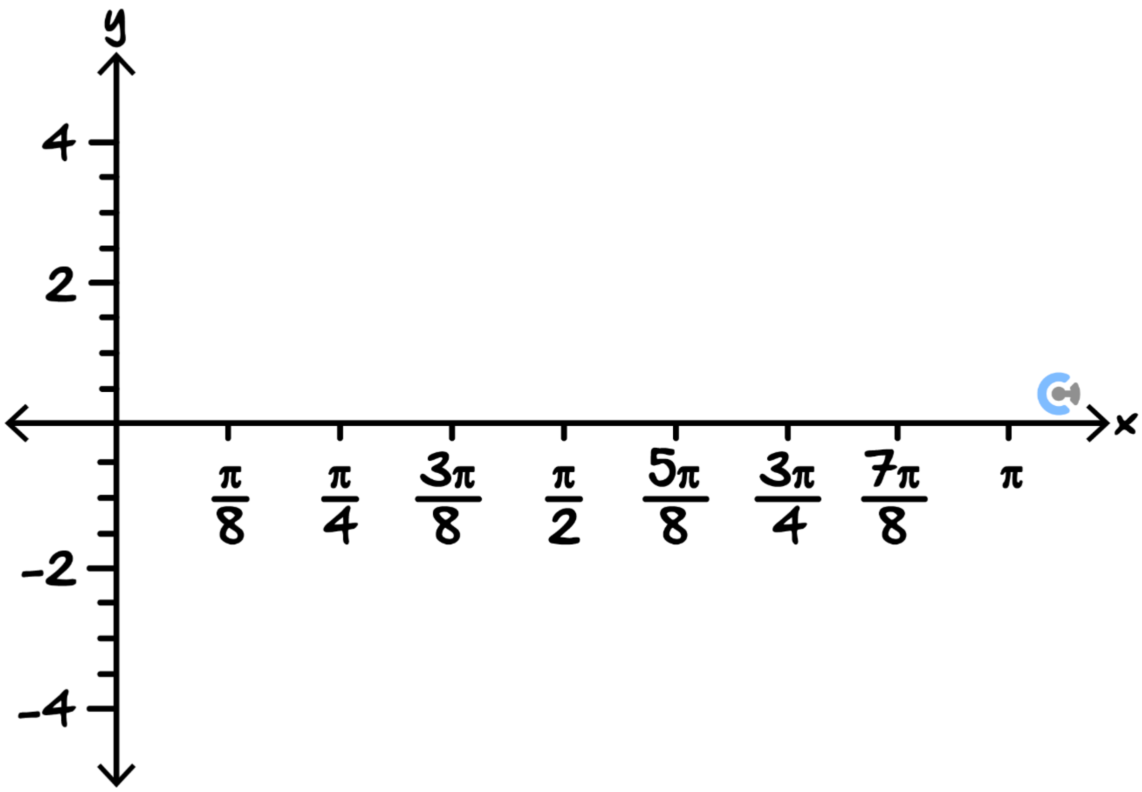


- Find any x-intercepts.
- Sketch a "cubic-like" shape.

Question 6

Sketch the following on the axes below, labelling all intercepts, points of inflection, and endpoints with their coordinates, and all asymptotes with their equations.

$$y = \tan\left(2x + \frac{\pi}{2}\right) + 1 \text{ for } x \in (0, \pi)$$



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Sub-Section: Fraction of Period

Fraction of Period

Definition

$$\text{Fraction of Period} = \frac{\text{Duration}}{\text{Period}}$$

$$\% \text{ of Period} = \frac{\text{Duration}}{\text{Period}} \times 100\%$$

Question 7 Walkthrough.

The population of dogs in a certain household is modelled by $P(t)$.

$$P(t) = 5 - 2\cos\left(\frac{\pi}{4}t\right)$$

Where $P(t)$ is the number of dogs t years since 2024. Find the fraction of time where the population is above 4.

Section C: Warm Up Test (16 Marks)

INSTRUCTION: 16 Marks. 16 Minutes Writing.

Question 8 (8 marks)

Consider the function $f(x) = -2\sin\left(2x - \frac{\pi}{6}\right) + 1$.

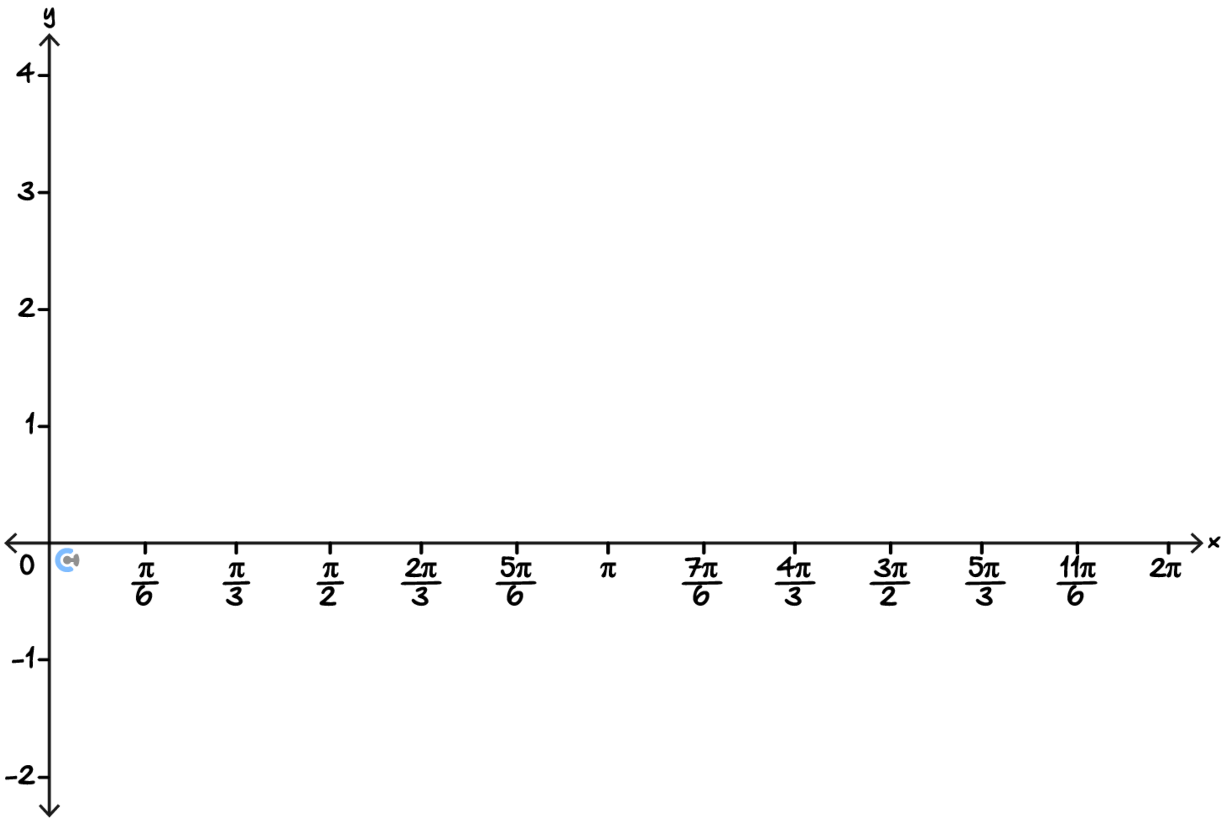
a.

i. Find the general solution to $f(x) = 0$. (3 marks)

ii. Hence, state the general solution to $f(x) = 0$ for $x > 0$. (1 mark)

b. Find all solutions to $f(x) = 0$ for $x \in [0,2\pi]$. (1 mark)

c. Sketch the graph of $y = f(x)$ for $x \in [0, 2\pi]$. Label all endpoints, axial intercepts, and turning points with coordinates. (3 marks)



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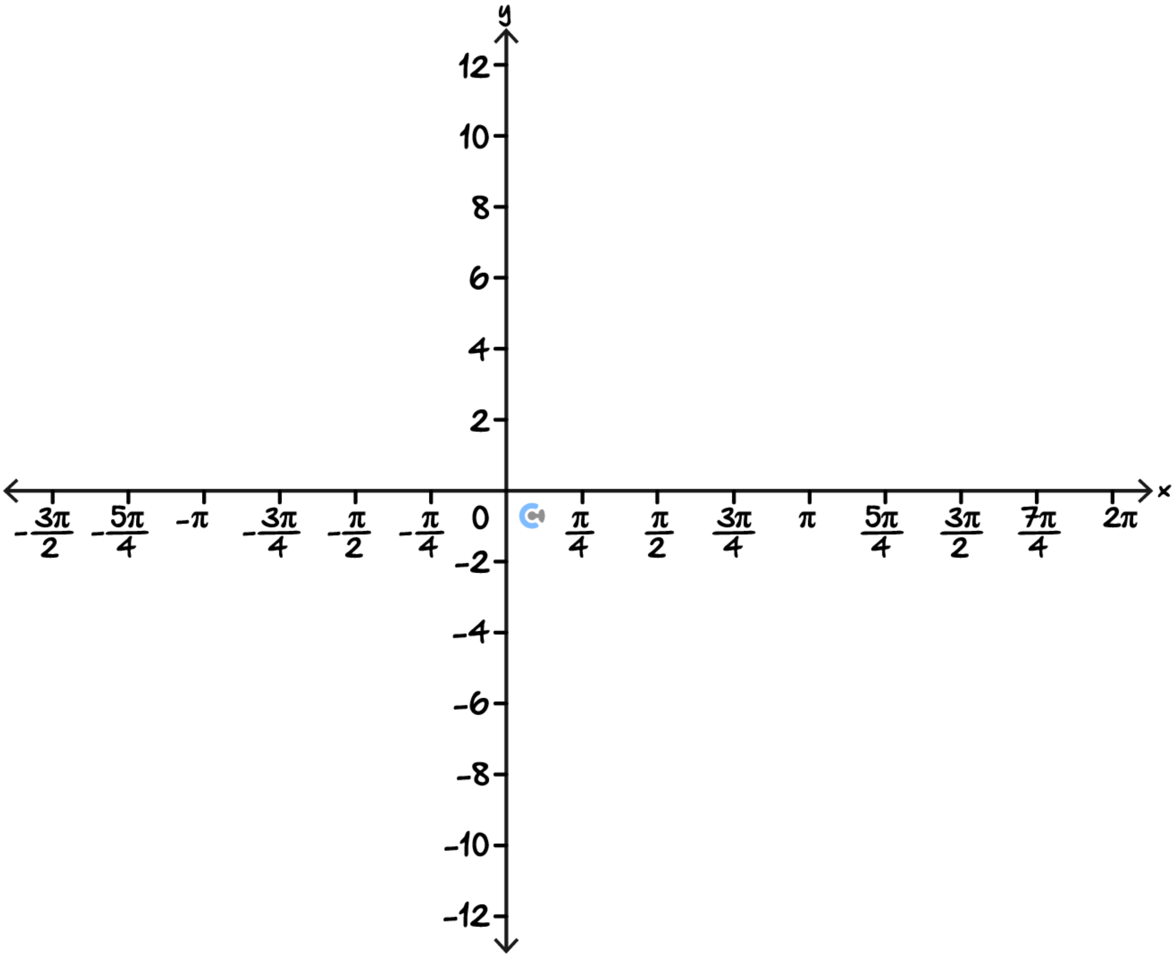
Question 9 (5 marks)

Consider the function $g(x) = 2\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) - 2$.

a. Find the general solution to $g(x) = 0$. (2 marks)

b. Hence, sketch the graph of $y = g(x)$ for $x \in \left(-\frac{3\pi}{2}, 2\pi\right]$. Label all axes intercepts, endpoints, and points of inflection with coordinates and asymptotes with their equations. (3 marks)



Question 10 (3 marks)

Solve the equation $\cos^2(2x) + 7\cos(2x) = 4$ for $x \in [0, 2\pi]$.

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Section D: Circular Functions Exam Skills

Sub-Section: Equivalent General Solutions

Discussion: Is $0 + 6k, k \in Z$ the same as $6 + 6k, k \in Z$?

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Question 11 (1 mark)

Which one of the following is **not** the same as the rest?

A. $\frac{5\pi}{6} + \frac{\pi}{3}n, n \in Z$

B. $\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

C. $-\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

D. $\frac{5\pi}{3} + \frac{\pi}{3}n, n \in Z$

E. $\frac{\pi}{6} - \frac{\pi}{3}n, n \in Z$

NOTE: Very important for multiple choice questions in VCAA exams!

Sub-Section: Sum and Difference of Trigonometric Functions

Discussion: Consider $\sin(x)$ and $\cos\left(\frac{2}{3}x\right)$. What would be the period of $\sin(x) + \cos\left(\frac{2}{3}x\right)$?

Period For Sum/Difference of Circular Functions

- When we add two circular functions,

$$\textit{Period of the sum} = \textit{LCM of two periods}$$

Question 12

Find the period of $\sin(2x) - \cos(4x)$.

NOTE: This only works for sum and difference. Multiplication does not work due to the compound angle formula (only in Specialist Maths).

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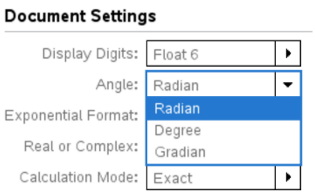
Section E: Technology Exam Skills

Calculator Commands: Degrees And Radians

- **TI**
  - Doc → 7 → 2



- **Casio**
  - Change at the bottom of the screen

- **Mathematica**
  - In radians by default.
  - Write "Degree"

In[27]:= **Sin[30 Degree]**

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving Trigonometric Functions.

- **TI**
  - solve($trig(..) = a, x$) | domain restriction
  - | is under control equal.

- **Casio**
  - solve($trig(..) = a, x$) | domain restriction
  - | is under maths 3.

- **Mathematica**
  - Solve[$trig[] == a$ && domain restriction, x]

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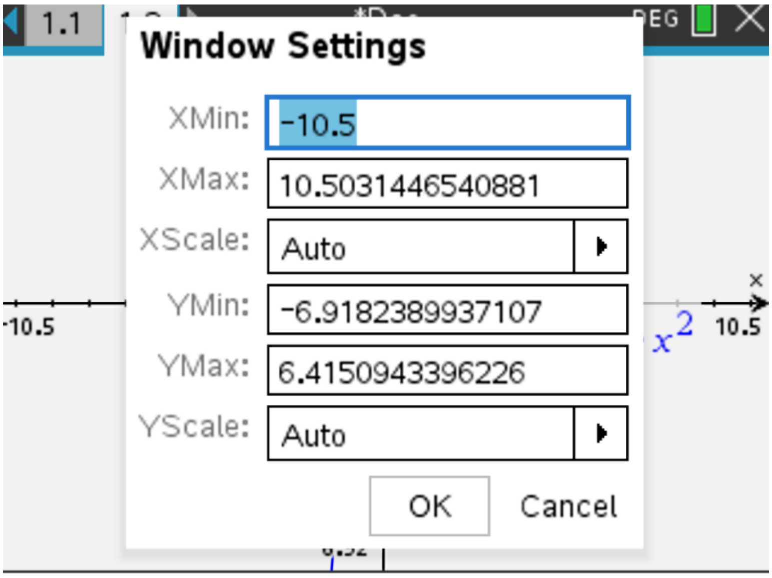
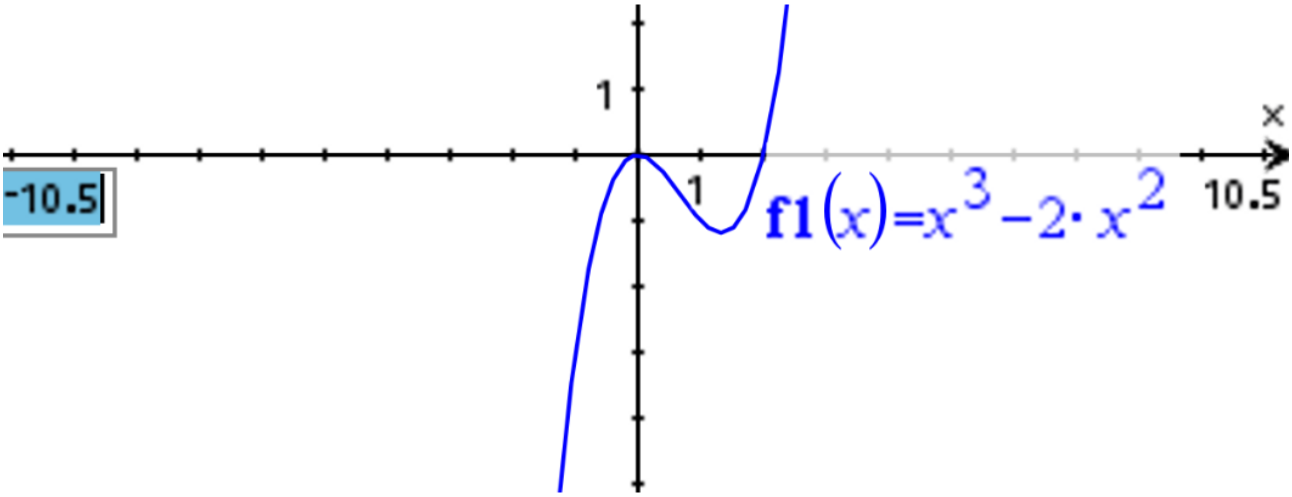
<u>Calculator Commands:</u> Graphing

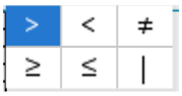
- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window/zoom) → 1 enter your x and y-ranges.

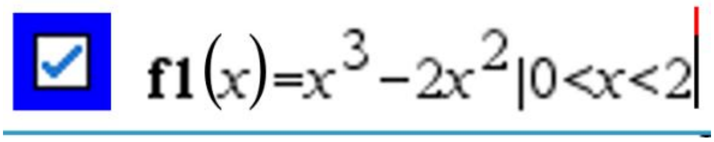


- Can also click the axis numbers on the graph and alter them directly.

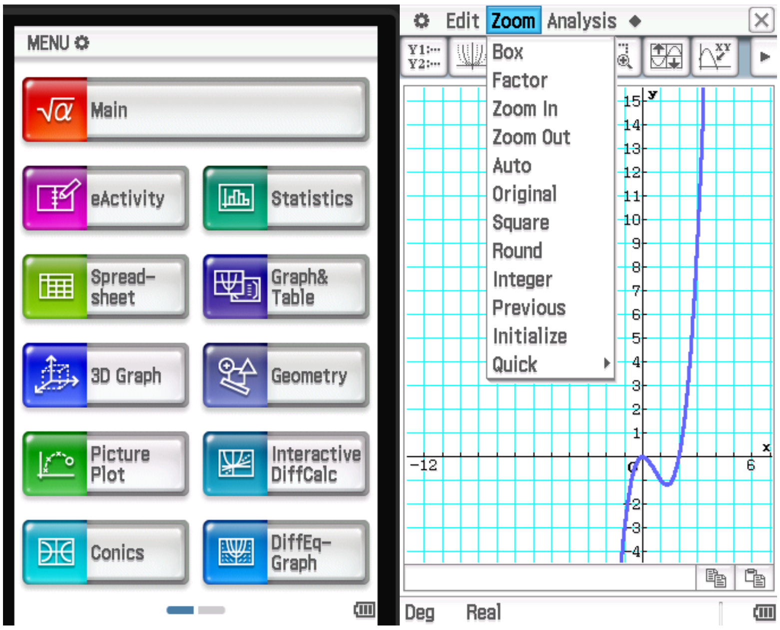


- Menu → 6 (Analyse) to find min/max x and y-intercepts.
- Restrict domain to $0 < x < 2$ use the bar can get it from $\mathrm{ctrl}+=$

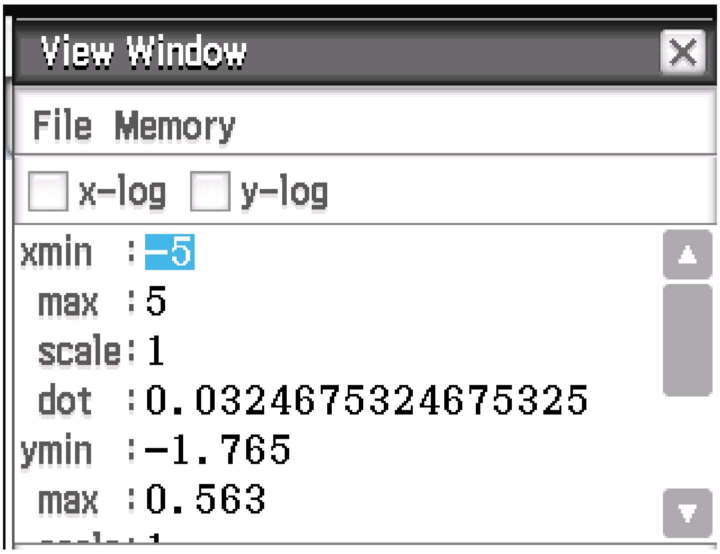




- **Casio:** Click Graph & Table, and enter the function.



- Analysis → G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict domain → find it in Math 3.

$$y1 = x^3 - 2 \cdot x^2 \,|\, 0 < x < 2$$

- **Mathematica:** Plot[function, {x, $x$min, $x$max}, PlotRange → {$y$min, $y$max}]

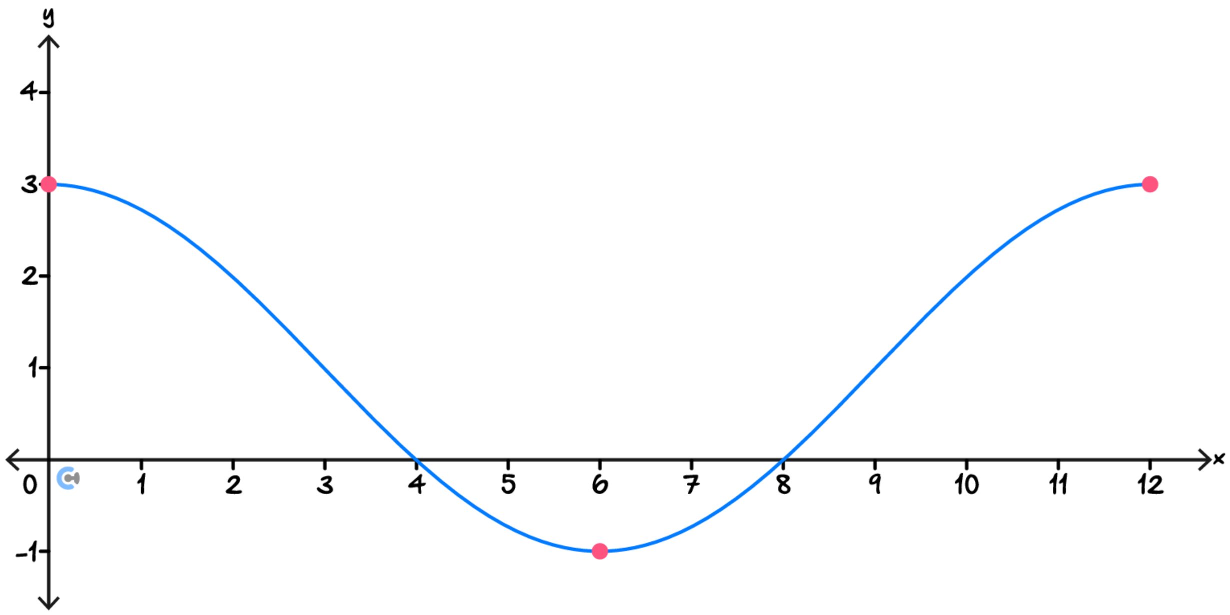
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Section F: Exam 2 (30 Marks)

INSTRUCTION: 30 Marks. 5 Minutes Reading. 40 Minutes Writing.

Question 13 (1 mark)

The diagram below shows one cycle of the graph of a circular function.



This graph could have the rule:

A. $y = 2\cos\left(\frac{1}{6}x\right) + 1$

B. $y = 2\cos\left(\frac{\pi}{6}x\right) + 1$

C. $y = -2\sin\left(\frac{\pi}{6}x\right) + 1$

D. $y = 2\cos\left(\frac{\pi}{12}x\right) + 1$

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Question 14 (1 mark)

For the equation $2\cos(3x) = 1$, the **sum** of the solutions in the interval $[0, \pi]$ is equal to:

A. $\frac{2\pi}{3}$

B. 2π

C. $\frac{13\pi}{9}$

D. 6π

Question 15 (1 mark)

For the function $f : \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right], f(x) = a\sin(2x - b) + c$, where a, b, and c are positive constants. The minimum and maximum values of f respectively are:

A. $-\frac{\pi}{3}$ and $\frac{2\pi}{3}$

B. $c - a$ and $a + c$

C. $a - c$ and $a + c$

D. $a - b$ and $b + c$

Question 16 (1 mark)

If m is the smallest solution and n the largest solution to $\sqrt{3}\cos(3x) - \sin(3x) = 0$ for $x \in [-\pi, \pi]$, then $m + n$ is:

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $-\frac{\pi}{9}$

D. $\frac{\pi}{6}$

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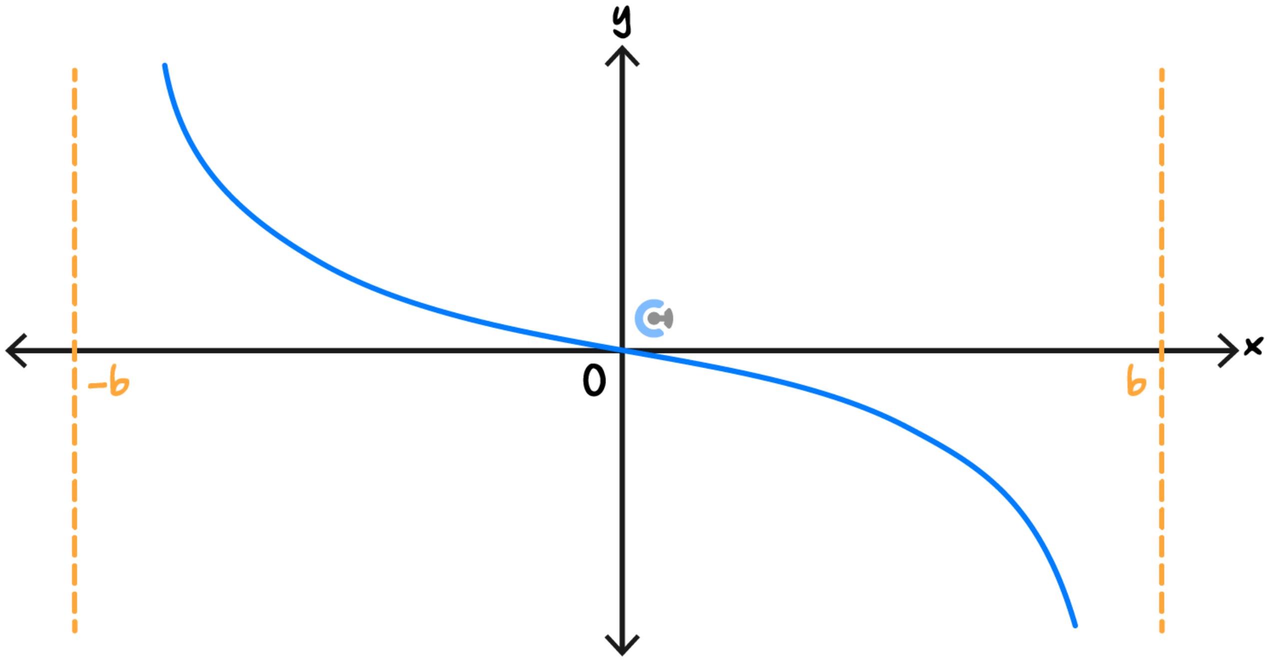
Question 17 (1 mark)

The diagram below shows one period of the graph with equation $y = \tan(ax)$.

Vertical asymptotes have the equations $x = b$ and $x = -b$.



Possible values of a and b are:

A. $a = -3, b = \frac{\pi}{6}$

B. $a = -3, b = \frac{2\pi}{3}$

C. $a = 3, b = \frac{\pi}{6}$

D. $a = -\frac{1}{3}, b = \frac{\pi}{6}$

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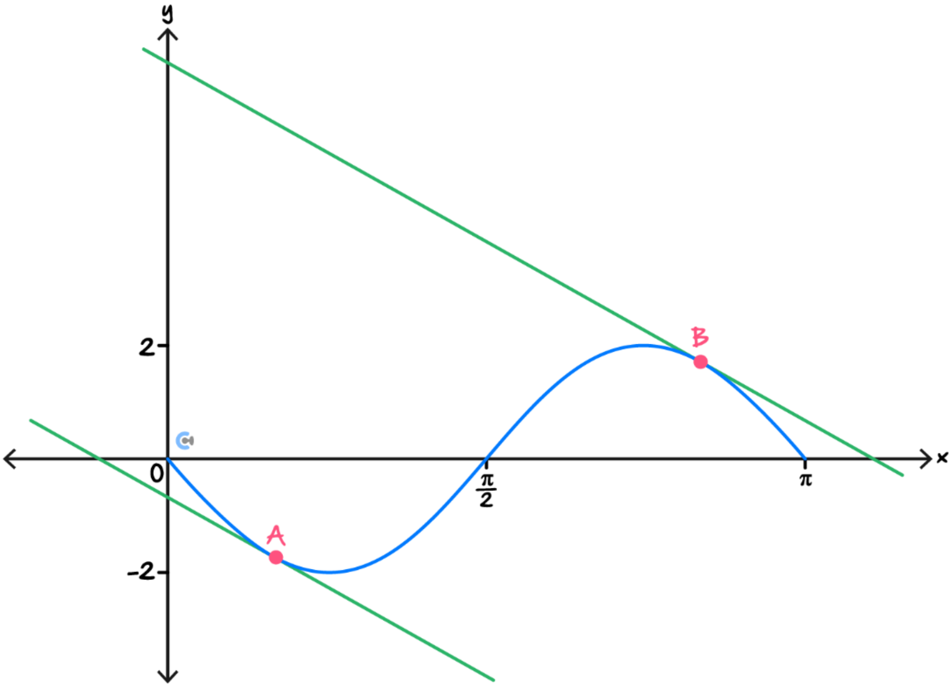
Question 18 (13 marks)

Consider the function $f : [0, \pi] \to \mathbb{R}$, where $f(x) = -2\cos\left(2x - \frac{\pi}{2}\right)$. The graph of f is shown with tangents drawn at points A and B.



a.

i. Write a rule for $f(x)$ in the form $a\sin(bx)$, where a and b are integers. (1 mark)

ii. Find $f'(x)$. (1 mark)

iii. State the maximum and minimum values of $f'(x)$ for $x \in \left[0, \frac{\pi}{4}\right]$. (2 marks)

b.

i. The gradient of the curve $y = f(x)$ when $x = \frac{5\pi}{6}$ is -2. Find the other value of x for which the gradient is also -2. (1 mark)

ii. Find the equation of the tangent to the curve at $x = \frac{5\pi}{6}$. (1 mark)

iii. Find the x-and y-intercepts of the tangent line found in **part b.ii.** (2 marks)

c. The two tangents to the curve at points A and B both have a gradient -2. A horizontal translation of m units moves the tangent at A to the tangent at B. Find the exact value of m. (2 marks)

d. Let $h: \mathbb{R} \to \mathbb{R}$ be defined by $h(x) = 2\sin(2x)$.

i. State the maximum vertical distance between the functions f and h. (1 mark)

ii. Find a general form for the coordinates of all turning points of h. (2 marks)

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Question 19 (12 marks)

Leila is monitoring the temperature in a temperature-controlled aquatic tank. During a 24-hour period, the water temperature $T(t)$ in degrees Celsius is modelled by:

$$T(t) = 20 + 2\cos\left(\frac{\pi t}{8}\right), 0 \leq t \leq 24,$$

Where t is the number of hours from the beginning of the 24-hour time interval.

a. State the maximum temperature in the tank and the value(s) of t when this occurs. (2 marks)

b. State the period of the function T. (1 mark)

c. Find the smallest value of t for which the temperature is exactly $21°C$. (2 marks)

d. For how many hours during the 24-hour period is the temperature greater than or equal to $21°C$? (2 marks)

e. Find the values of t, when the water is **cooling down** the fastest. (2 marks)

The water temperature is now modelled over a 48 hours period instead, with the rule for $T(t)$ being unchanged. Thus,

$$T(t) = 20 + 2\cos\left(\frac{\pi t}{8}\right), \qquad 0 \leq t \leq 48$$

f. The water temperature is **below** K°C for exactly a quarter of the time. Find the exact value of K. (3 marks)

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