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VCE Mathematical Methods ¾
Circular Functions Exam Skills [3.4]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 3-Pg 12
Supplementary Questions	Pg 13-Pg 22

Section A: Recap

Contour Check

- Learning Objective: [3.4.1] – Period for sum of trigonometric functions and equivalent general solutions

Key Takeaways

- Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

- Period For Sum/Difference of Circular Functions
  - When we add two circular functions,

$$Period\ of\ the\ sum = LCM\ of\ two\ periods$$

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Section B: Compulsory Questions

Sub-Section: Basics (Tech Free)

Question 1 [3.4.1]

Find the period of $\sin(2x) - \cos(3x)$.

Question 2 [3.3.3]

Consider the function $f(x) = 2\sin(2x) + 2$. Find the fraction of a period that $f(x) > 3$.

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Question 3 [3.2.3]

Solve the equation $\sin\left(\frac{x}{3}\right) = -\frac{1}{2}$ for $x \in [2\pi, 6\pi]$.

Question 4

a. Find a general solution to the equation $2\sin\left(x - \frac{\pi}{3}\right) = \sqrt{3}$. **[3.2.3]**

b. Subu is marking a student's answer to **part a.,** the student gave the answer:

$$x = 2n\pi - \frac{4\pi}{3}, 2n\pi - \pi, \text{ where } n \in \mathbb{Z}.$$

Is the student's answer correct? Explain. **[3.4.1]**

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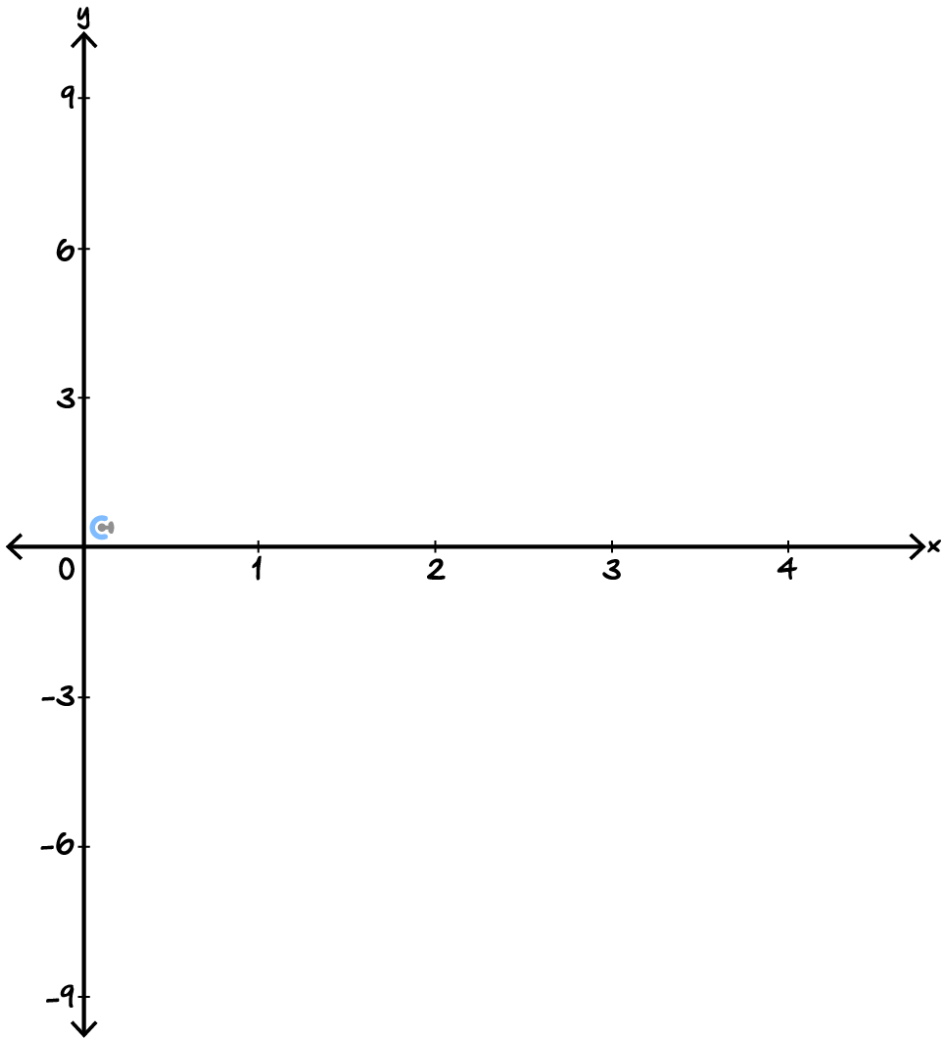
Question 5 [3.3.2]

Sketch the graph of $y = \tan\left(\frac{\pi}{2}(x-1)\right) - 1$ for $x \in (0,4)$. Label all axes intercepts with coordinates and asymptotes with their equations.



Sub-Section: Problem Solving (Tech-Active)

Question 6

The temperature T (in degrees Celsius) inside a greenhouse at t hours after midnight on a typical November day is modelled by the formula:

$$T = 25 - 4\cos\left(\frac{\pi(t-3)}{12}\right), \text{ for } 0 \leq t \leq 24.$$

Use this model to answer the following:

a. State the maximum and minimum temperatures reached inside the greenhouse during the day. **[3.3.2]**

b. At what time is the maximum temperature reached? **[3.2.3]**

c. Determine the time(s) when the temperature is exactly 23°C. **[3.2.3]**

d. Consider the hottest continuous 4-hour period in the greenhouse. What is the minimum temperature reached during this period? Give your answer to two decimal places. **[3.3.2]**

e. In the greenhouse, there is an automatic watering system that activates when the rate of change of temperature with respect to time is at least $+0.2$°C per hour. It switches off once the rate drops below this threshold.

i. Use calculus to find an expression for the rate of change of temperature with respect to time.

ii. Hence, determine the time interval(s) (correct to two decimal places) during which the watering system will be on. **[3.2.3]**

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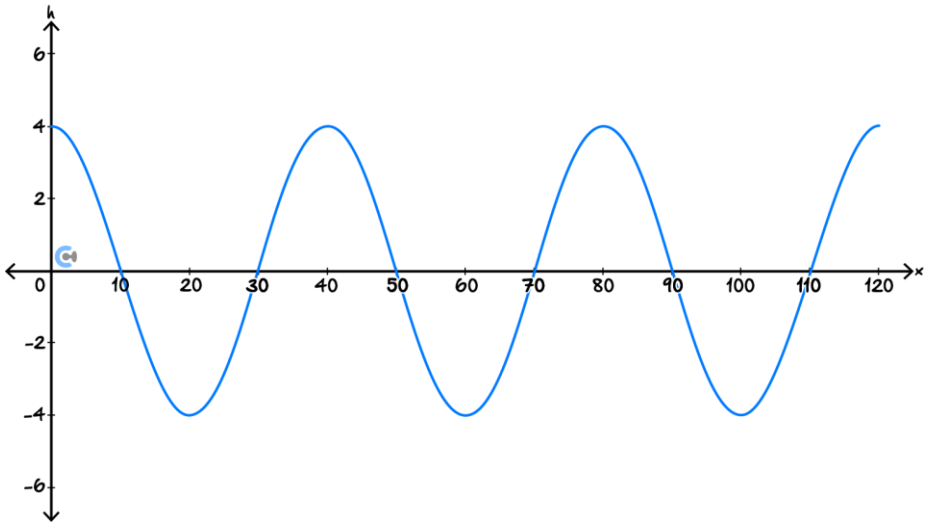
Question 7

An environmental scientist is monitoring the shape of sand dunes along a stretch of beach in Western Australia. She observes that the profile of the dunes can be modelled by a sine curve. Let $h\ cm$ represent the height of the sand above sea level and $x\ m$ represent the horizontal distance along the beach.



a. What is the period of this curve? **[3.3.2]**

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b. What is the amplitude of the curve? **[3.3.2]**

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c. The height of the dunes can be modelled by a function of the form $h = a\sin(n(x - b))$. Write down an equation for the height of the dunes. **[3.3.2]**

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d. If this pattern continues for 2 km, how many dune troughs would a person walk through? **[3.3.2]**

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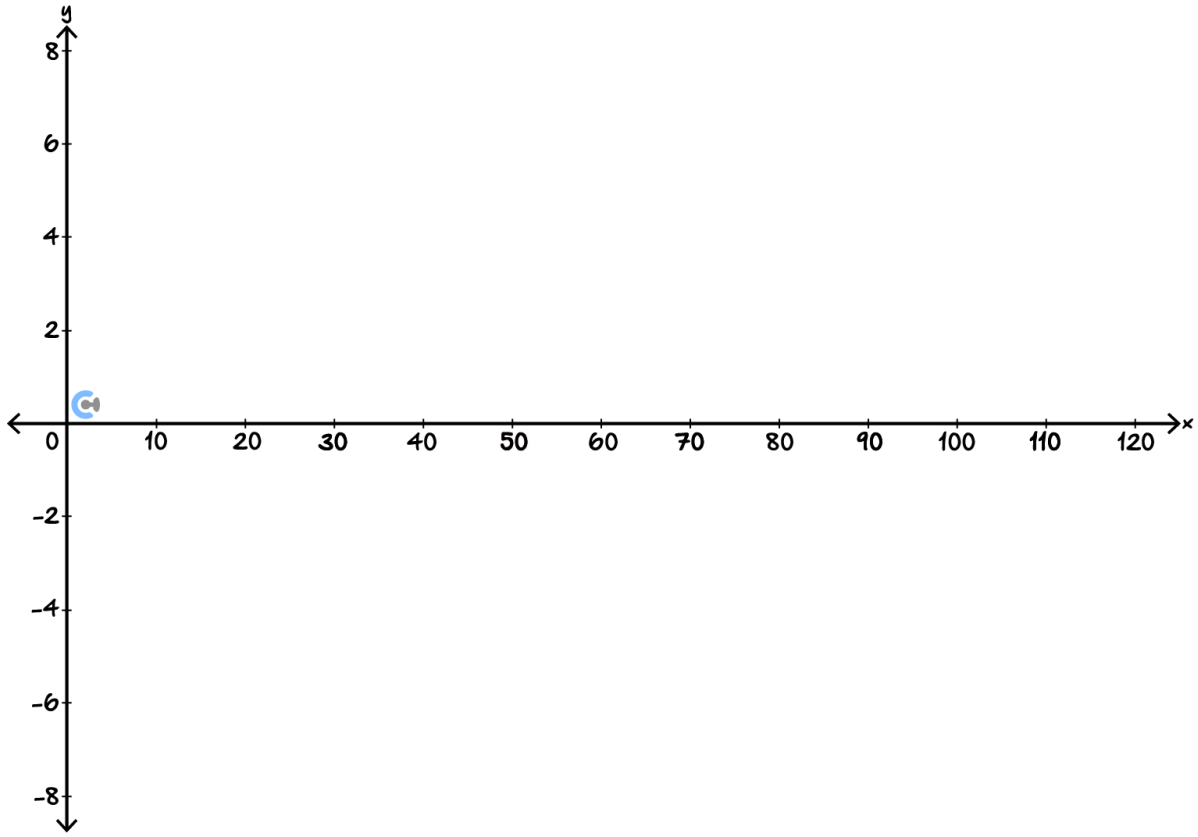
Further along the beach, the wind pattern changes, causing the shape of the dunes to be modelled by a more complex function:

$$h = 3\sin\left(\frac{\pi}{10}x\right) + 5\cos\left(\frac{\pi}{30}x\right)$$

e. Sketch this function over the domain $0 \leq x \leq 120$. You only need to label the endpoints with coordinates. **[3.3.2]**



f. What is the period of this new function? **[3.4.1]**

g. What are the absolute maximum and minimum values of this function? Give your answers correct to two decimal places. **[3.3.2]**

h. Give the coordinates, correct to two decimal places, of all local maximum points during the first cycle. **[3.2.3]**

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Question 8

The temperature, A℃, inside a cabin at t hours after 3 PM is given by the rule:

$$A = 20 - 4\cos\left(\frac{\pi}{12}t\right), \text{ for } 0 \leq t \leq 24.$$

The temperature, B℃, outside the cabin at the same time is given by:

$$B = 14 + 6\cos\left(\frac{\pi}{12}t\right), \text{ for } 0 \leq t \leq 24.$$

a. Find the temperature inside the cabin at 9 AM. **[3.2.1]**

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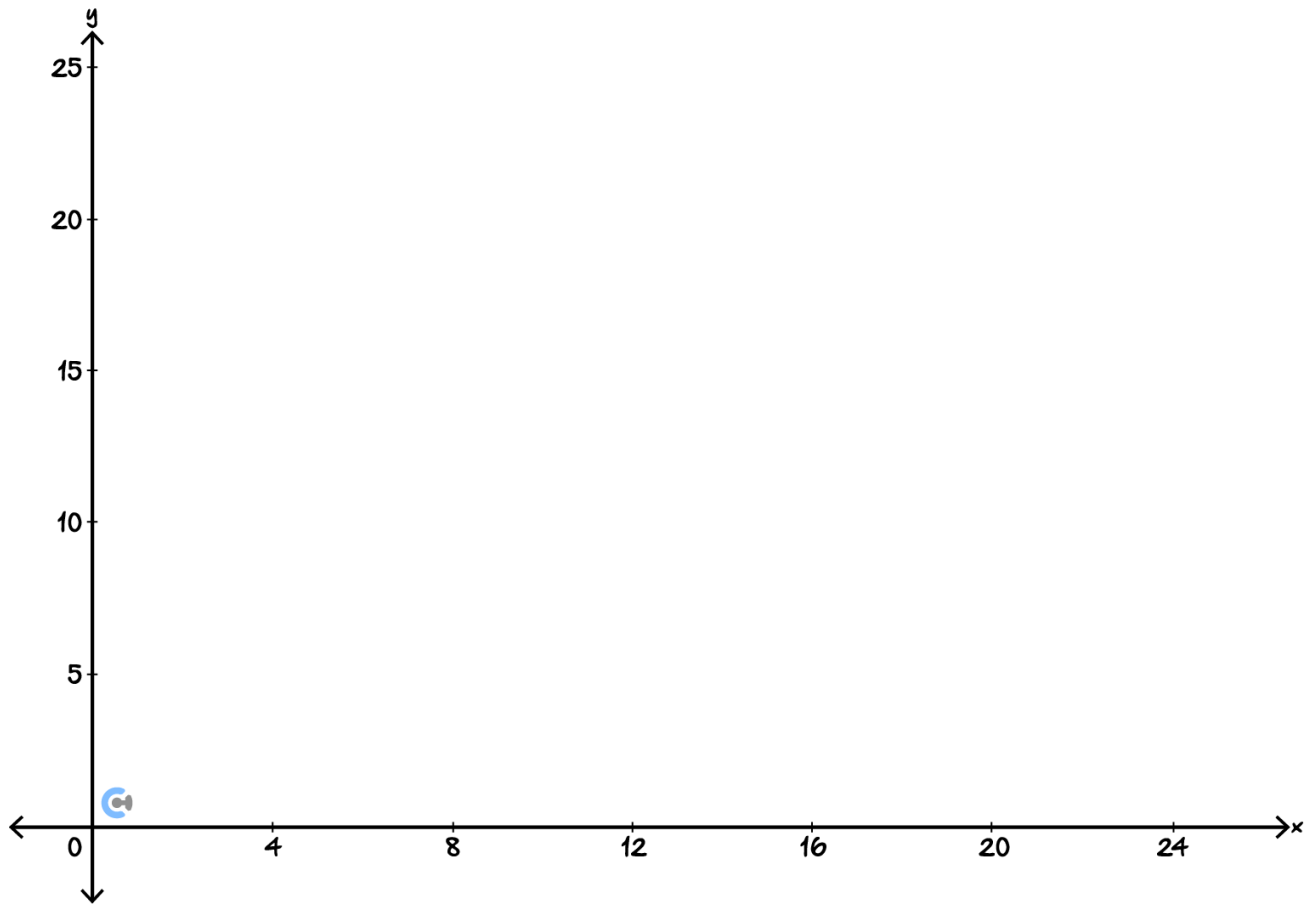
b. Sketch the graphs of $y = A(t)$ and $y = B(t)$ on the axes below. Label all endpoints and stationary points with coordinates. **[3.3.2]**



c. Hence, state the values of t for when the temperature inside the cabin is greater than the temperature outside. **[3.2.3]**

d. Let $D(t)$ be the function that represents the difference between the temperature inside the cabin and outside the cabin.

i. State the rule for $D(t)$.

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ii. A sudden heat wave occurs and now the temperature outside is given by $K + 6\cos\left(\frac{\pi}{12}t\right)$.
The temperature inside the cabin remains unchanged.

Find the value of K if the temperature outside is warmer than inside the cabin for exactly 16 hours. **[3.3.3]**

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Section C: Supplementary Questions

Sub-Section: Exam 1 Questions

Question 9

a. State the range and period of the function.

$$h : \mathbb{R} \rightarrow \mathbb{R}, h(x) = 5 + 6\cos\left(\frac{\pi x}{2}\right)$$

b. Solve the equation:

$$\sin\left(3x - \frac{\pi}{6}\right) = \frac{1}{2}, \text{ for } x \in [0, \pi].$$

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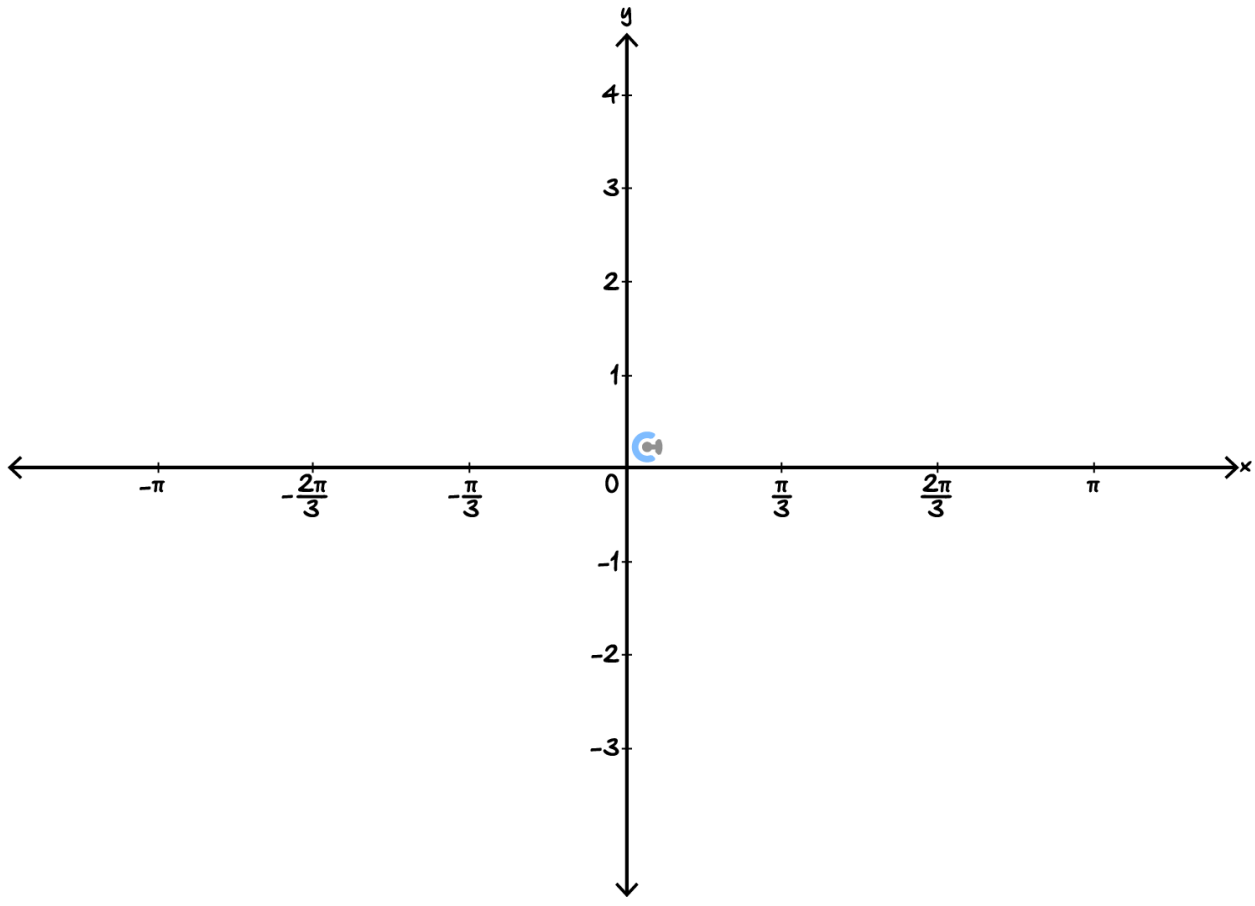
Question 10

For the function $f : [-\pi, \pi] \to \mathbb{R}, f(x) = -3\sin\left(2x + \frac{\pi}{3}\right)$.

a. Write down the amplitude and period of the function.

b. Sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.



Question 11

a. Show that $(2x - 1)(4x^2 + 2x - 1) = 8x^3 - 4x + 1$.

b. Show that $\frac{\tan^2(x)+1}{\tan^2(x)-1} = \frac{1}{1-2\cos^2(x)}$.

c. Hence, using the previous two results, solve the equation:

$$\frac{\tan^2(x)+1}{\tan^2(x)-1} = 4\cos(x) \text{ for } 0 \leq x \leq \frac{\pi}{2}.$$

You may use the fact that $\cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right) = \frac{2\pi}{5}$.

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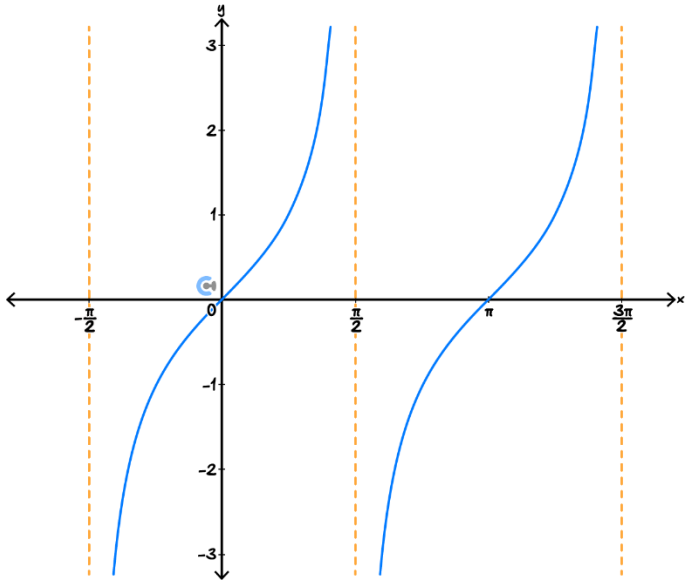
Sub-Section: Exam 2 Questions

Question 12

The graph of $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \to \mathbb{R}$, where $f(x) = \tan(x)$ is shown below.



a.

i. Find $f'\left(\frac{\pi}{4}\right)$.

ii. Find the equation of the **normal** to the graph of $y = f(x)$ at the point where $x = \frac{\pi}{4}$.

iii. Sketch the graph of this normal on the axes above. Give the exact axis intercepts.

b. Find the exact values of $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ such that $f'(x) = f'\left(\frac{\pi}{4}\right)$.

c. Let $g(x) = f(x - a)$. Find the exact value of $a \in (-1, 1)$ such that $g(1) = 1$.

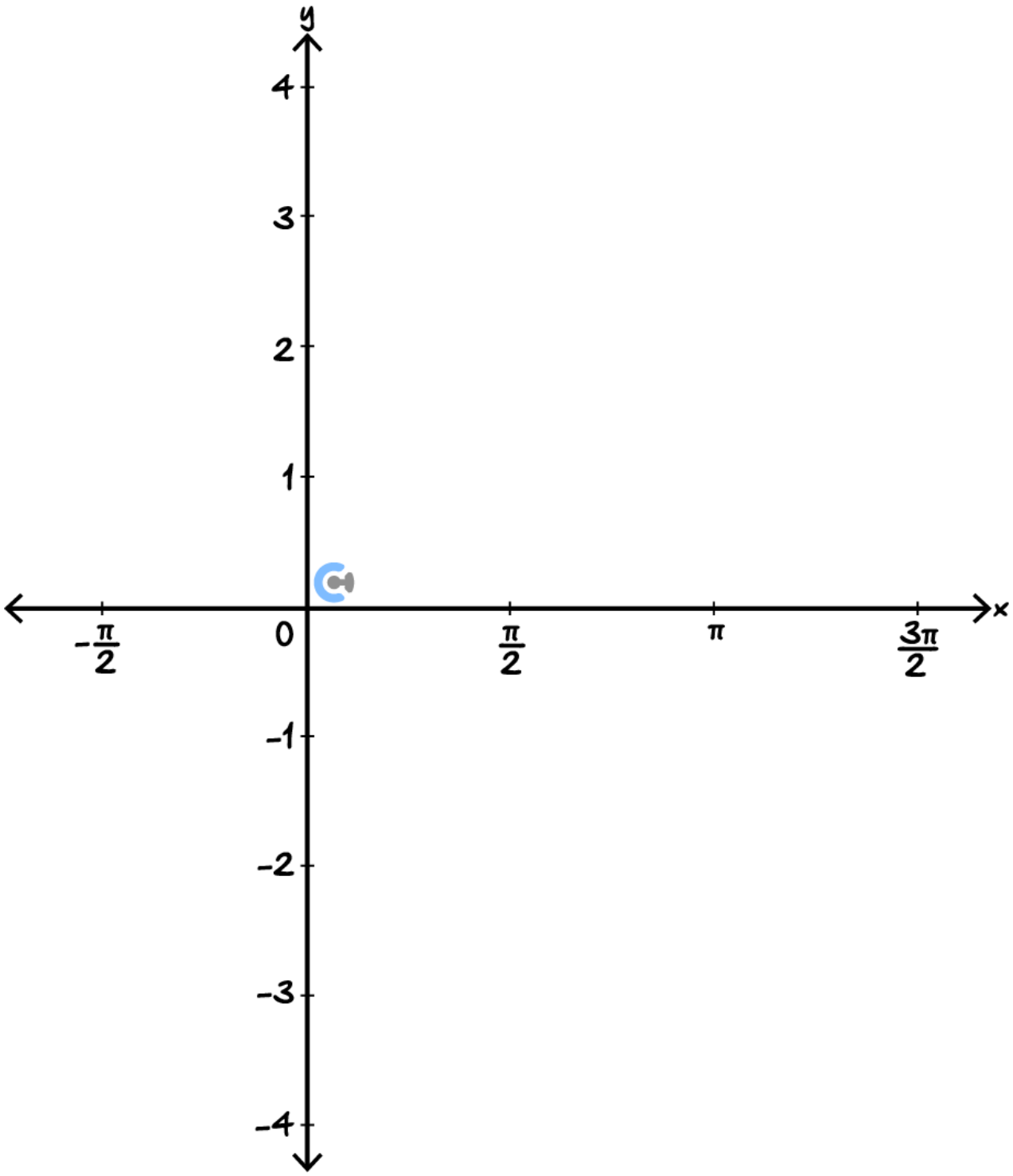
d. Let $h: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \to \mathbb{R}, h(x) = \sin(x) + \tan(x) + 2$.

i. Find $h'(x)$.

ii. Solve the equation $h'(x) = 0$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. (Give exact values.)

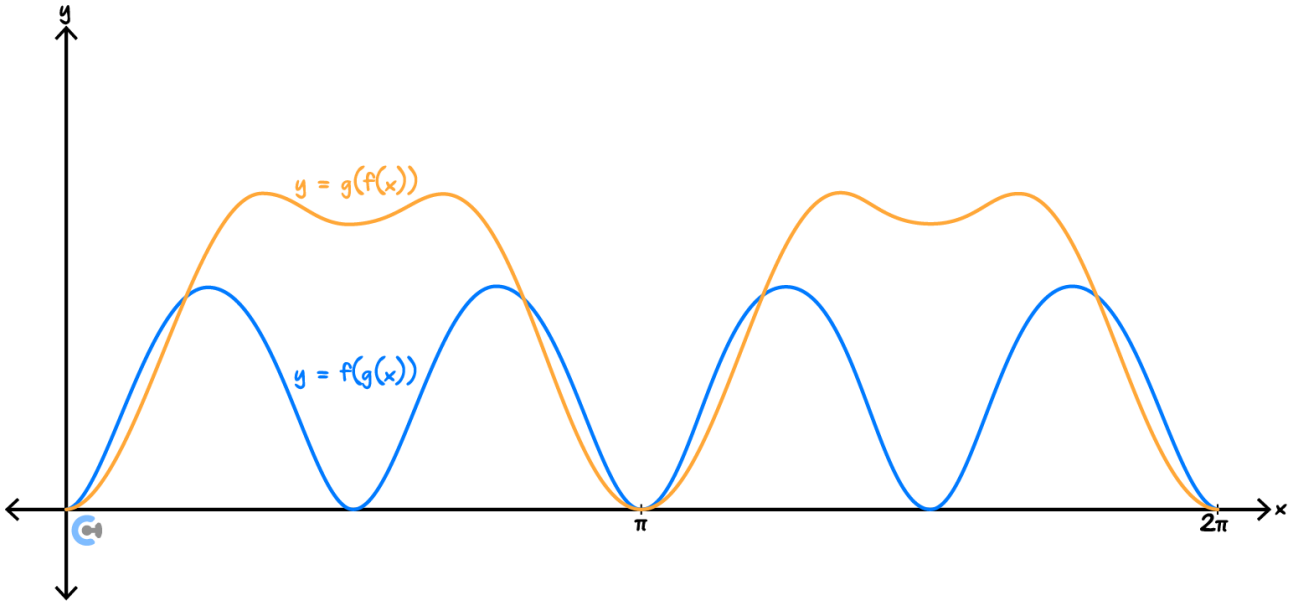
e. Sketch the graph of $y = h(x)$ on the axes below.

- Give the exact coordinates of any stationary points.
- Label each asymptote with its equation.
- Give the exact value of the y-intercept.



Question 13

The graph below shows the compositions $g \circ f$ and $f \circ g$, where $f(x) = \sin^2(x)$ and $g(x) = \sin(2x)$.



a.

i. The graph of $y = (g \circ f)(x)$ has a local maximum whose x-value lies in the interval $\left[0, \frac{\pi}{2}\right]$.

Find the coordinates of this local maximum, correct to one decimal place.

ii. State the range of $g \circ f$ for $x \in [1, 2]$. Give your answers correct to one decimal place.

b.

i. State the period of $f \circ g$.

ii. Find the derivative of $f \circ g$.

iii. Hence, find two equations that when solved give the $x$coordinates for turning points of $f \circ g$.

iv. Hence, find the x-values of the stationary points of $f \circ g$ where $x \in [0, \pi]$.

v. Find the range of $f \circ g$ where $x \in [0, 2\pi]$.

c. Let $f_1 : (-\pi, 0) \to \mathbb{R}, f_1(x) = \sin^2(x)$.

Find all values of x in the interval $(0, 2\pi)$ for which the composition $f_1 \circ g$ is defined.

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