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VCE Mathematical Methods ¾
Circular Functions II [3.3]
Workbook

Outline:

Learning Objectives:

- ☐ **MM34 [3.3.1]** - Solve Advanced Trigonometric Equations
- ☐ **MM34 [3.3.2]** - Graph Sine, Cosine and Tangent Functions
- ☐ **MM34 [3.3.3]** - Fraction of Periods

Section A: Advanced Trigonometric Algebra

Sub-Section: Recap of Particular & General Solutions

REMINDER: Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps:**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add and subtract the period to find all other solutions in the domain.

REMINDER: General Solutions

- Finding infinitely many solutions to a trigonometric equation.
- **Steps:**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add $period \cdot n$ where $n \in Z$.

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Question 1 Walkthrough.

Find the solutions to the following equation:

$$2\sin\left(2x+\frac{\pi}{3}\right)-1=0 \text{ for } x\in[0,2\pi]$$

Question 2

Find the solutions to the following equation:

$$\sqrt{2}\cos\left(2x-\frac{\pi}{4}\right)-1=0 \text{ for } x\in[0,2\pi]$$

Question 3 Walkthrough.

Find the general solutions to the following equation:

$$2\cos\left(3x-\frac{\pi}{3}\right)=2$$

Question 4

Find the general solutions to the following equation:

$$4\sin\left(2x+\frac{\pi}{6}\right)-2=0$$

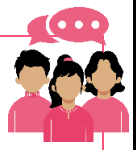
Question 5

Find the general solutions to the following equation:

$$2\tan\left(2x-\frac{\pi}{6}\right)-2\sqrt{3}=0$$

NOTE: The period of $\tan$ is $\frac{\pi}{n}$.

Discussion: What is the main difference between the general and particular solution questions?



Sub-Section: General Solutions with Domain Restrictions

Misconception

"When there is a domain restriction, we always get particular solutions"

TRUTH: If the domain restriction has either ∞ or $-\infty$, we can still have general solutions

Question 6 Walkthrough.

Solve for the following trigonometric equation.

$$\sin\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

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General Solution with Domain Restriction

$$\text{E.g., } \mathbf{trig}\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

- We can have infinite solutions for restricted domains.
- The value of n is also restricted.

Your Turn!

Question 7

Solve for the following trigonometric equation.

$$\cos\left(2x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2} \text{ for } x<0$$

NOTE: This was assessed in a VCAA exam!

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Sub-Section: Hidden Quadratics

Let's Have a Look at Hidden Quadratics for Circular Functions!

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

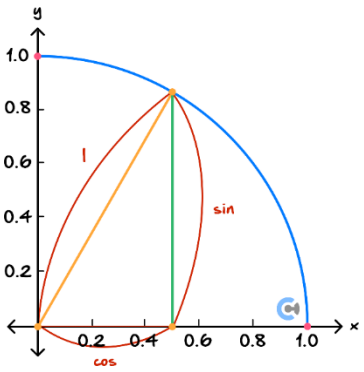
Question 8 Walkthrough.

Solve the following for the values of x.

$$\sin^2\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{\pi}{3}\right) = 2, 0 \leq x \leq 3\pi$$

NOTE: sin and cos are between -1 and 1.

REMINDER: Pythagorean Identity



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

- Can be used for finding one trigonometry function by using the other.

Active Recall: Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A =$_____

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Your Turn!

Question 9

Solve the following for the values of x.

$$-2\sin^2(2x) + 3\cos(2x) = 0$$

TIP: $\sin^2(\theta) = 1 - \cos^2(\theta)$

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Section B: Graphs of Sine and Cosine

Sub-Section: Understanding the Shape

What does a sine and cosine graph look like?

Exploration: Graph of Sine and Cosine

- Scan the following QR code on your device!

Sine	Cosine

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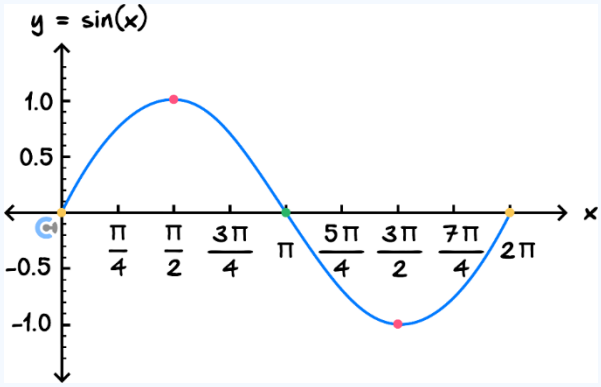
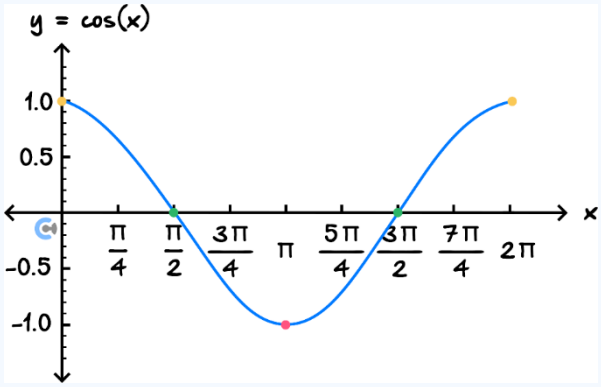
Sine and Cosine Graphs

Sine	Cosine
	

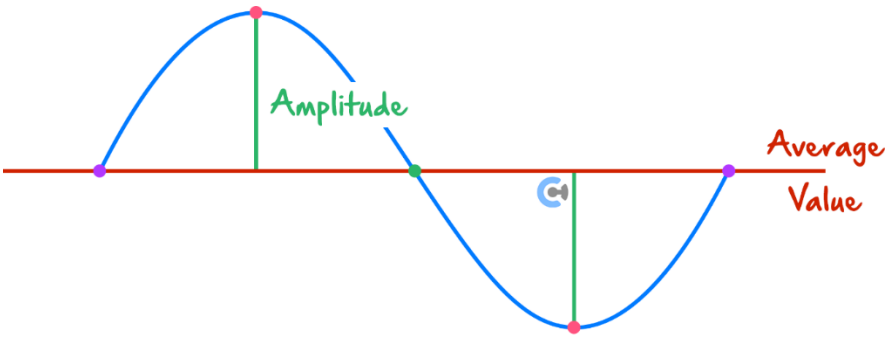
Discussion: Is $\cos(x)$ an even function or odd function. What about $\sin(x)$?

Discussion: What does $\sin\left(\frac{\pi}{2}+x\right)$ equal to? So how can we translate sin function to cosine function?

Sub-Section: Graphing Sine and Cosine Functions

Amplitude, Period and Average Value

For $y = A\sin/\cos(nx + b) + k$



Consider the sign of our graph

$$\text{Amplitude} = |A|$$

$$\text{Period} = \frac{2\pi}{n}$$

$$\text{Average Value} = k$$

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Question 10

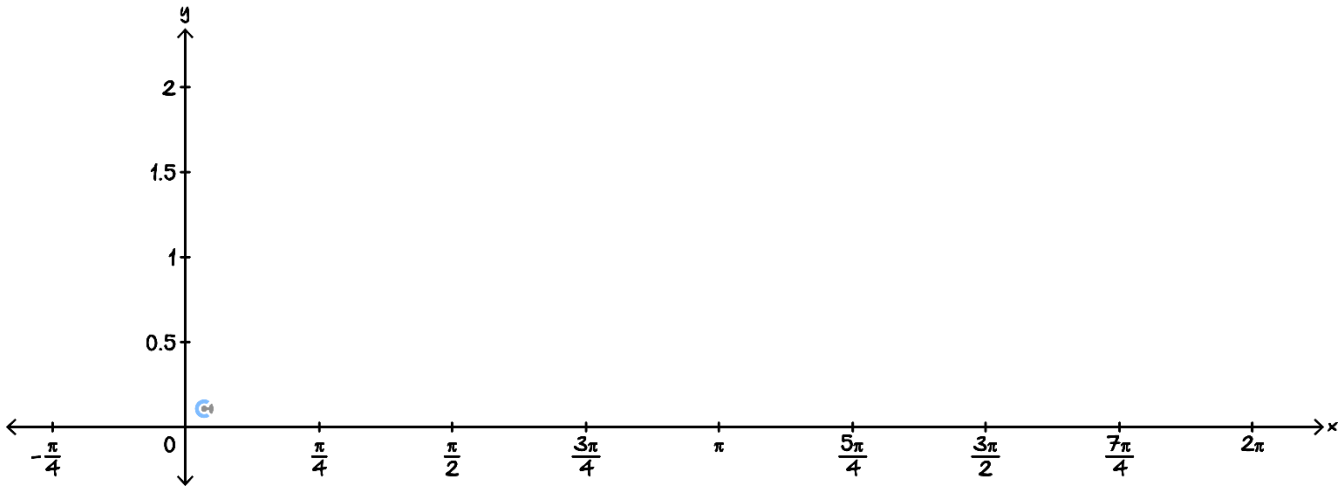
Identify the amplitude, period and average value of the following functions:

a. $f(x) = 2\sin\left(\frac{\pi}{3} - 3x\right) + 1.$

b. $g(x) = -3\cos(2x + 3) - 4.$

Exploration: Graphing of sin and cos Functions

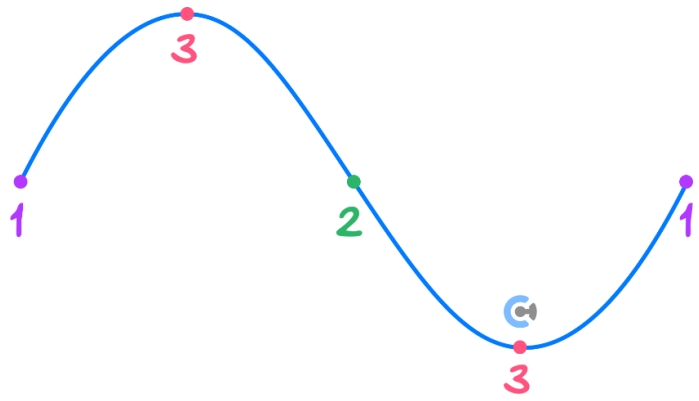
- Let's sketch $\sin(2x + \pi) + 1$ on the axes below!



1. Identify, Amplitude, Period, Mean Value and Positive/Negative Shape.
2. Create a "mini-version" of the graph you are about to draw.

3. Start plotting the function from when the angle $= 0$. **Why?**
   - **It allows us to always sketch the graph from the _____.**
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

Graphing of $\sin$ and $\cos$ Functions



1. Identify, Amplitude, Period, Mean Value and Positive/Negative Shape.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle$= 0$.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

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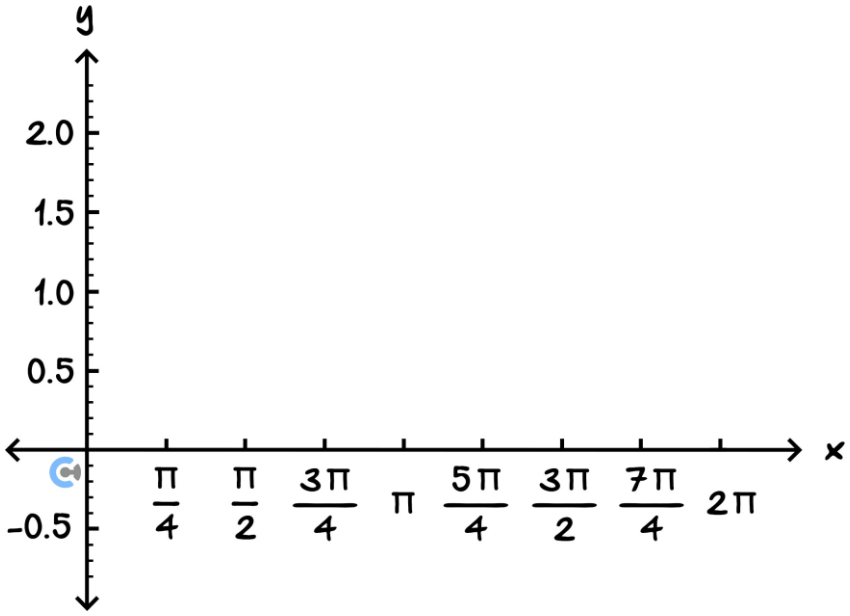
Question 11 Walkthrough.

Sketch the graph of $f(x) = -\sin(2x) + 1$ for $x \in [0, 2\pi]$ on the axes below, labelling all intercepts and endpoints with their coordinates.



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Active Recall: Graphing of $\sin$ and $\cos$ Functions

- Steps:

1. Identify:_____
   _____
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle = ___
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any_____.
6. Join all the points!

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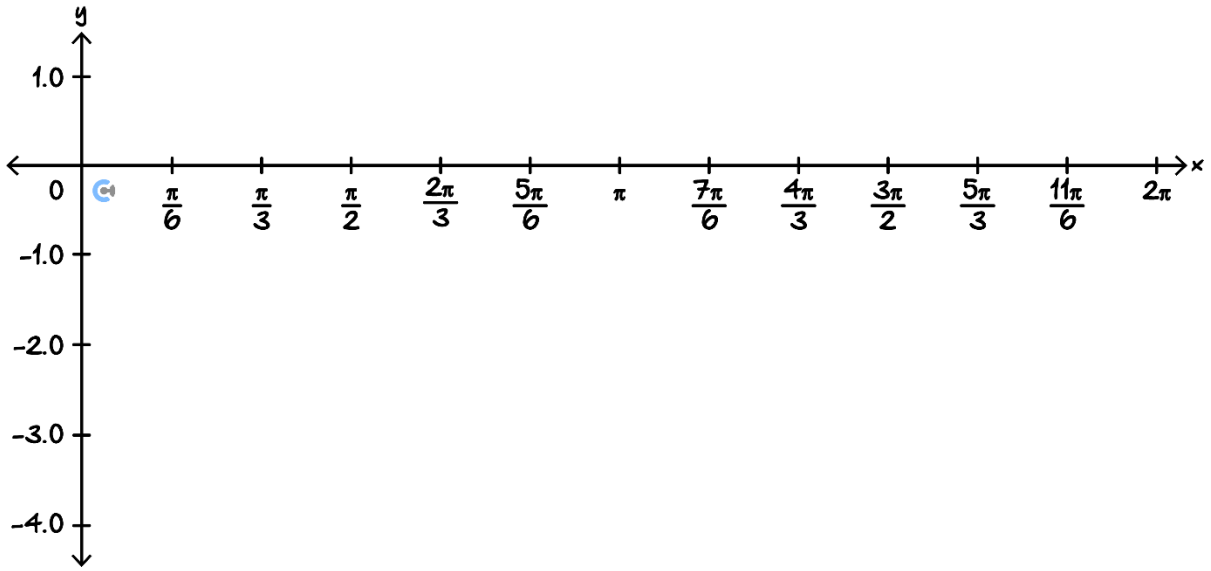
Question 12

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

$$y = 2\sin\left(2\left(x - \frac{\pi}{3}\right)\right) - \sqrt{3} \text{ for } x \in [0, 2\pi]$$



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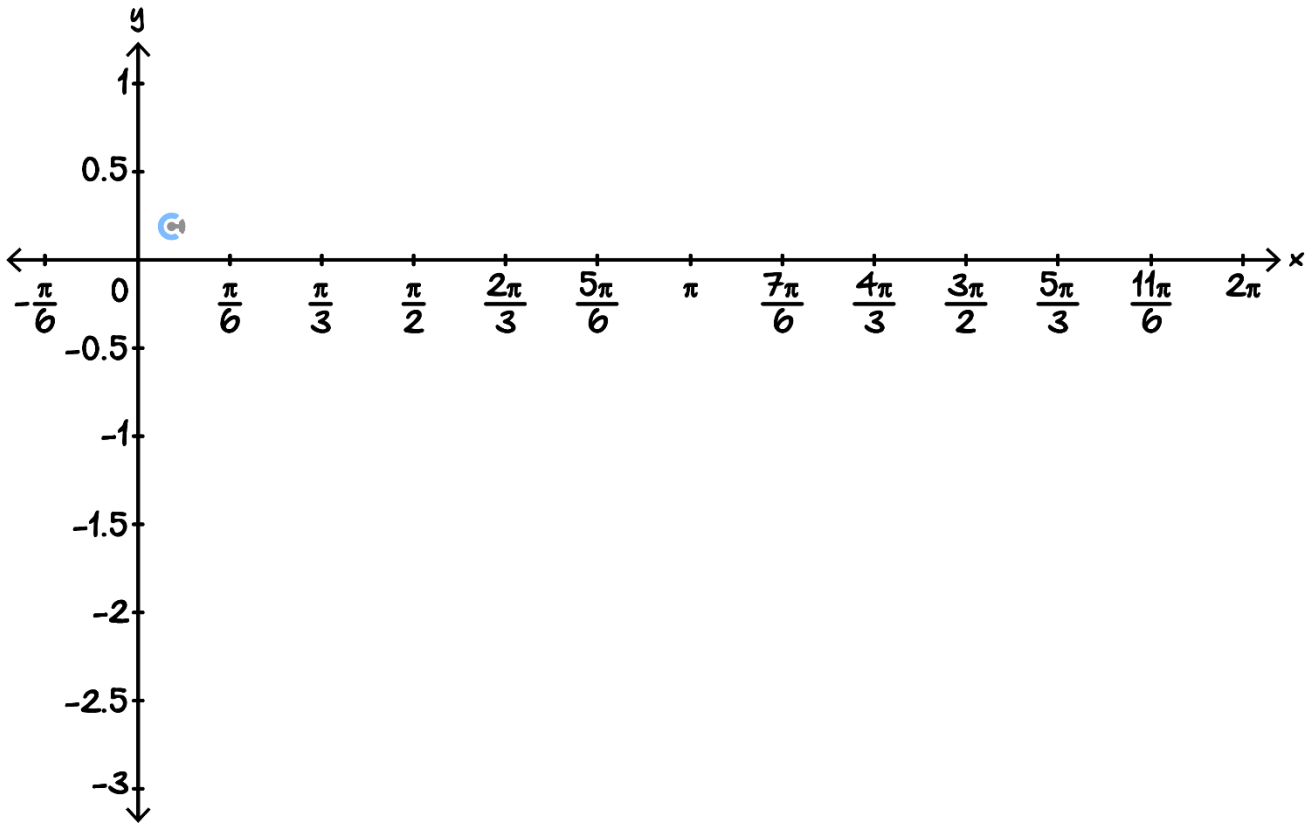
Question 13

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

$$y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1 \text{ for } x \in [0, 2\pi]$$



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Sub-Section: Finding the Rule

Finding the Rule

$$\text{Amplitude } (A) = \frac{max - min}{2}$$

$$\text{Average } (k) = \frac{max + min}{2}$$

Question 14 Walkthrough.

A function with rule $y = A\sin(nt) + b$ where $A > 0$ has a range $[-5,3]$ and period 4. Find A, n and b.

TIP: Graphing helps!

Active Recall: Finding the Rule

Amplitude (A) =_____

Average (k) =_____

Your Turn!

Question 15

A function with rule $y = A\cos(nt + \pi) + b$ where $A < 0$ has a range $[-5,7]$ and period 3. Find A, n and b.

Section C: Graphs of Tangent

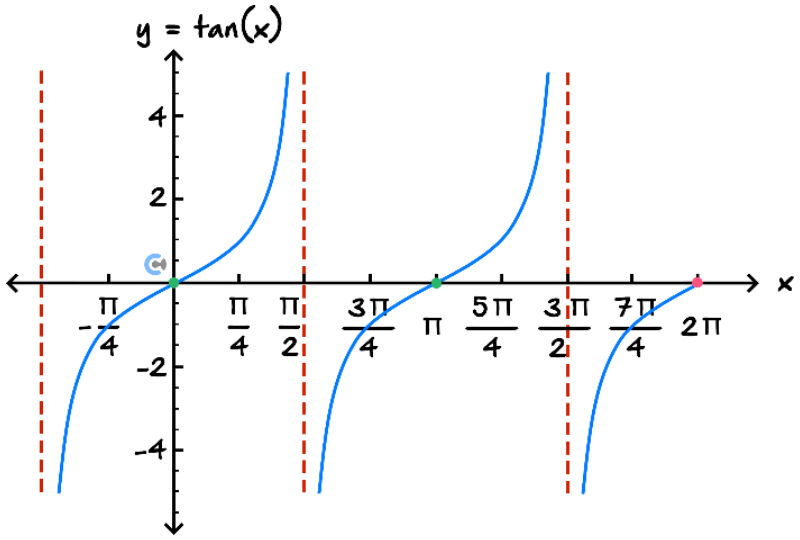
Sub-Section: Understanding Tangent Graphs

What does the tangent graph look like?

Exploration: Graph of Tangents

- Scan the QR code below on your device!

Graph of Tangent

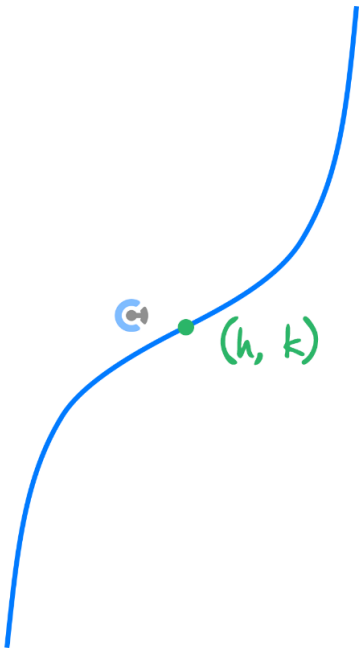


Sub-Section: Graphing Tangent Functions

Steps for Sketching tan Functions

1. Identify:
   - The period $= \frac{\pi}{n}$.
2. Find the vertical asymptotes by solving for angle $= \frac{\pi}{2}$.
   Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
   - For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x.
3. Plot the inflection point (h, k) (Midpoint of the two vertical asymptotes.).
   - x-value of inflection point $= x$-value which makes angle $= 0$.
   - y-value of inflection point $=$ vertical translation of the function.

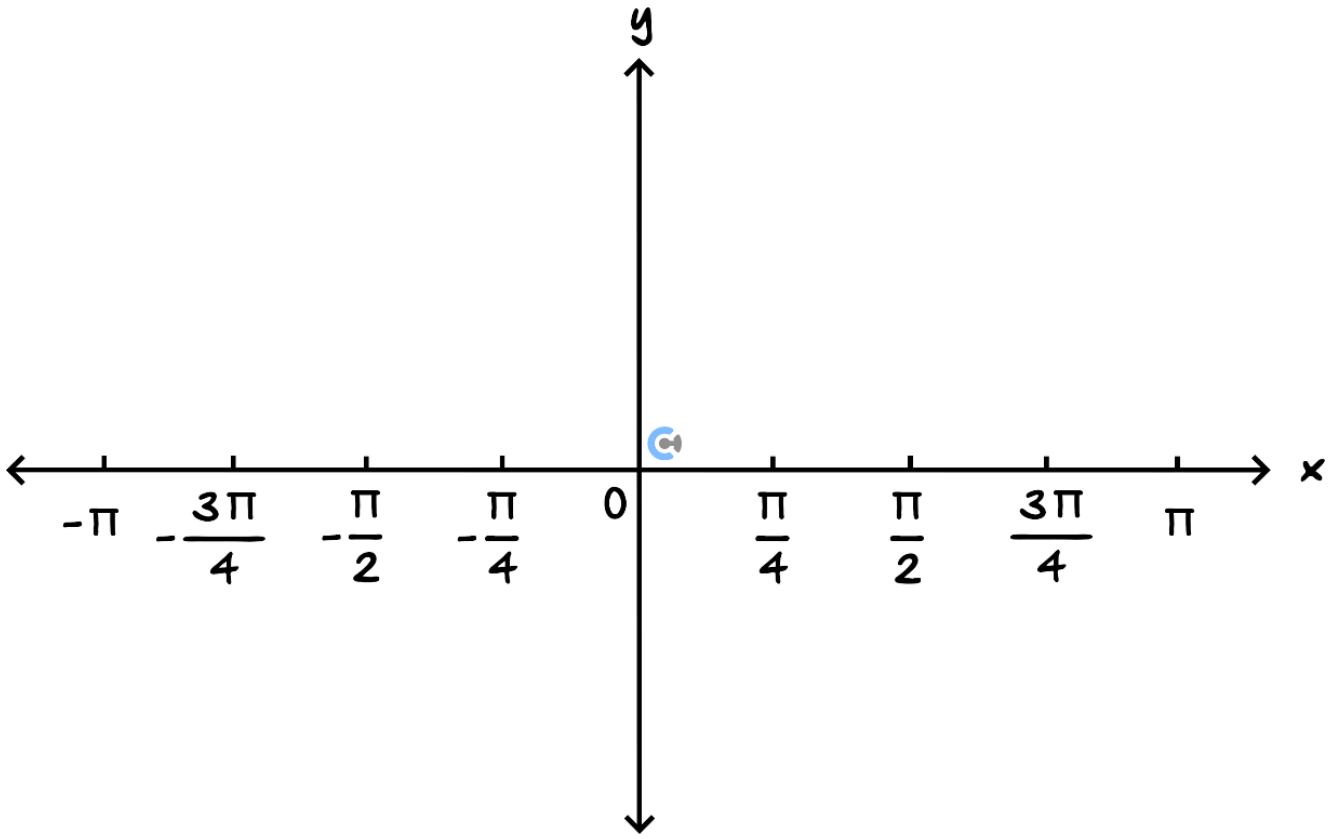
eg: tan(x−h)+k



4. Find any x -intercepts.
5. Sketch a "cubic-like" shape.

Question 16 Walkthrough.

Sketch the graph of $y = 3\tan(2x)$ for $x \in [-\pi, \pi]$.



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Active Recall: Steps for Sketching $\tan$ Functions

1. Identify:
   - The period =_____.
2. Find the vertical asymptotes by solving for angle =_____ .
   Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
3. Plot the inflection point (h, k) (Midpoint of the two _____.).
   - x-value of inflection point = x-value which makes angle = 0.
   - y-value of inflection point = vertical translation of the function.
4. Find any _____ .
5. Sketch a _____ shape.

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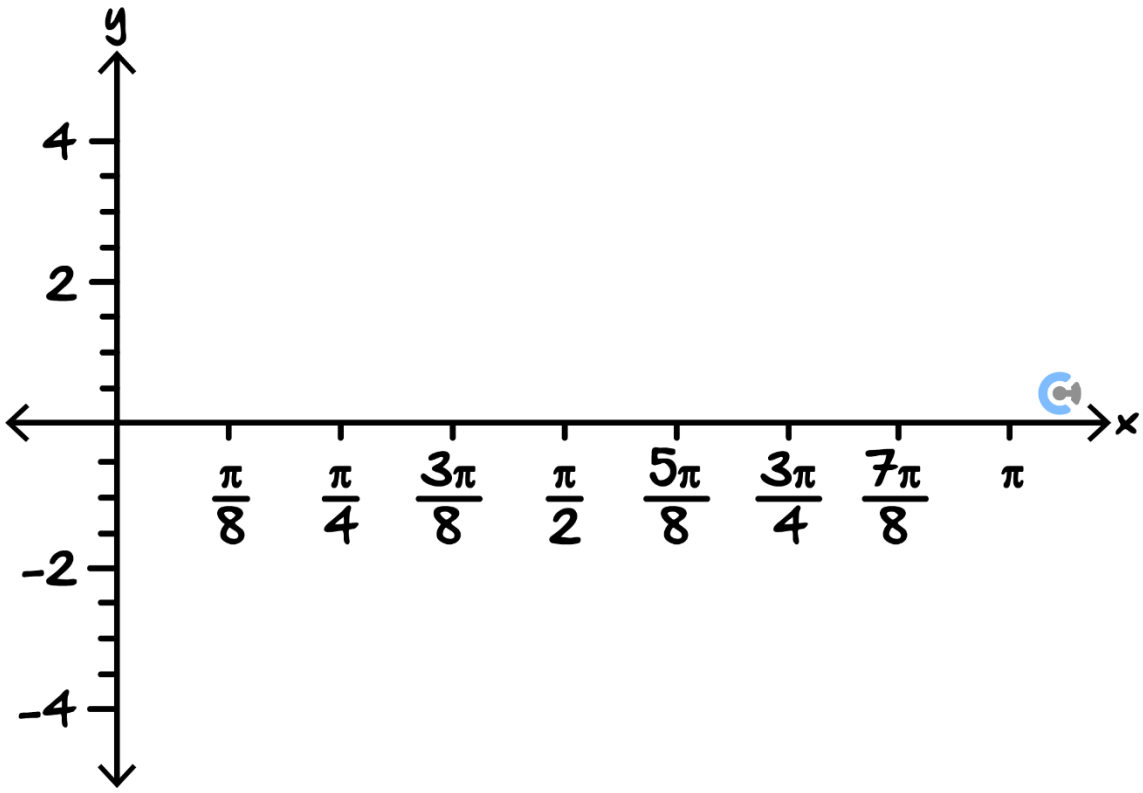
Your Turn!

Question 17

Sketch the following on the axes below, labelling all intercepts, points of inflection, and endpoints with their coordinates, and all asymptotes with their equations.

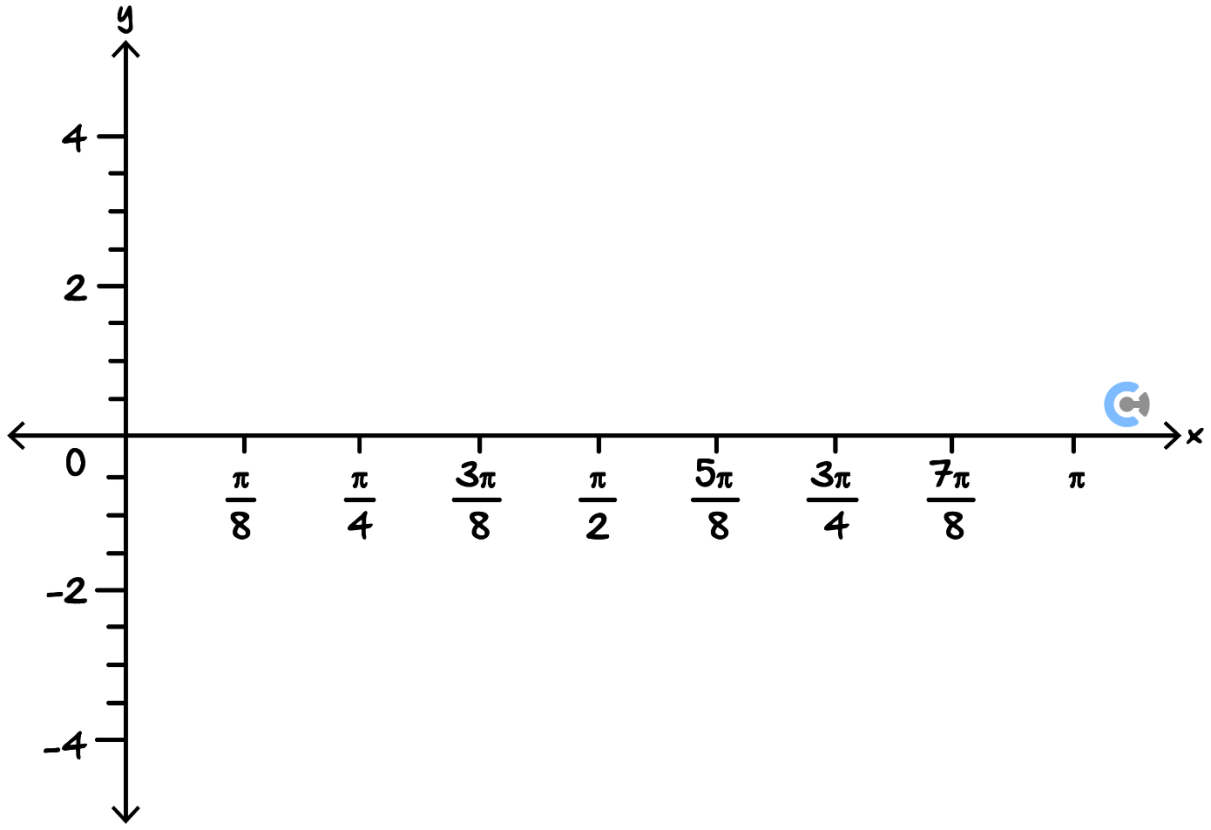
$$y = \tan\left(2x + \frac{\pi}{2}\right) + 1 \text{ for } x \in (0, \pi)$$



Question 18

Sketch the following on the axes below, labelling all intercepts, points of inflection, and endpoints with their coordinates, and all asymptotes with their equations.

$$f: [0, \pi] \to \mathbb{R}, f(x) = -3\tan(\pi + 4x) + \sqrt{3}$$



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Section D: Fraction of Period

Sub-Section: Fraction of Period

Definition: Fraction of Period

$$\textit{Fraction of Period} = \frac{\textit{Duration}}{\textit{Period}}$$

$$\%\ \textit{of Period} = \frac{\textit{Duration}}{\textit{Period}} \times 100\%$$

Question 19 Walkthrough.

The population of dogs in a certain household is modelled by $P(t)$.

$$P(t) = 5 - 2\cos\left(\frac{\pi}{4}t\right)$$

Where $P(t)$ is the number of dogs t years since 2024. Find the fraction of time where the population is above 4.

NOTE: Always sketch the function to find the duration!

Active Recall: Fraction of Period

$$\textit{Fraction of Period} = \_\_\_\_\_$$

$$\%\ \textit{of Period} = \_\_\_\_\_ \times 100\%$$

Question 20

The population of cats in a certain household is modelled by $P(t)$.

$$P(t) = 10 - 4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right)$$

Where $P(t)$ is the number of cats t years since 2024.

Find the fraction of time where the population is below 12.

Contour Check

Learning Objective: [3.3.1] – Solve Advanced trigonometric equations

Key Takeaways

General Solutions with domain restriction

- **Steps:**

1. Make the trigonometric function the subject.
2. Find the necessary _____ for one period.
3. Solve for x by equating the necessary angles to the _____ of the trigonometric functions.
4. Add $period \cdot n$ where the_____ of n is appropriately restricted.

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A =$_____

Learning Objective: [3.3.2] – Graph sine, cosine and tangent functions

Key Takeaways

Amplitude, Period and Average Value

For $y = A\sin/\cos(nx + b) + k$

$Amplitude =$_____

$Period =$_____

$Average\ Value =$___

Graphing of $\sin$ and $\cos$ Functions

- Steps:

1. Identify,
   _____
   _____
2. Create a "mini version" of the graph you are about to draw.
3. Start plotting the function from when the angle = ____.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any_____.
6. Join all the points!

Steps for Sketching $\tan$ Functions

1. Identify
   - The period =_____ .
2. Find the vertical asymptotes by solving for angle =_____ .
   Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
3. Plot the inflection point (h, k) (Midpoint of the two _____.).
   - x-value of inflection point = x-value which makes angle = 0.
   - y-value of inflection point = vertical translation of the function.
4. Find any _____ .
5. Sketch a _____ shape.

Learning Objective: [3.3.3] – Fraction of Periods

Key Takeaways

Fraction of Period

$$\textbf{\textit{Fraction of Period}} = \_\_\_\_\_$$

$$\%\ \textbf{\textit{of Period}} = \_\_\_\_\_ \times 100\%$$

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