Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾

Circular Functions II [3.3]

Test Solutions

30 Marks. 1 Minute Reading. 24 Minutes Writing.

Results:

Test Questions	_____ / 22
Extension Questions	_____ / 8

Section A: Test Questions (22 Marks)

Question 1 (3 marks)

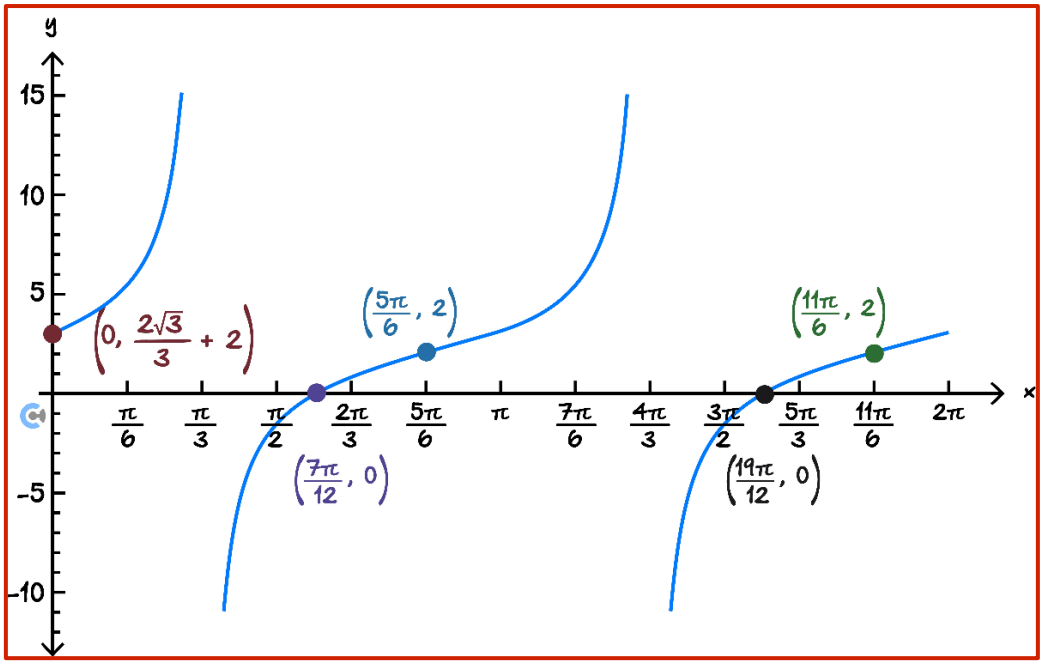
Tick whether the following statements are **true** or **false**.

Statement	True	False
a. For sin and cos functions, the amplitude is always the *coefficient* of sin and cos. False. It's the size of the coefficient.		✓
b. We should start sketching the function when the angle is equal to 0.	✓	
c. The y-value of the inflection of the tangent graph is always given by the vertical translation of the function.	✓	
d. To find the vertical asymptote of any tangent function, we simply let the angle equal to $\frac{\pi}{2}$.	✓	
e. For the sum of two trigonometric functions, the period of the overall sum is equal to the larger period.		✓
f. $\sin(x) \geq \frac{1}{2}$ for $\frac{1}{3}$ of its period.	✓	

Space for Personal Notes

Question 2 (3 marks)

Sketch the graph of $f(x) = 2\tan\left(x + \frac{\pi}{6}\right) + 2$ for $x \in [0, 2\pi]$ on the axes below, labelling all asymptotes, intercepts and endpoints with their coordinates.

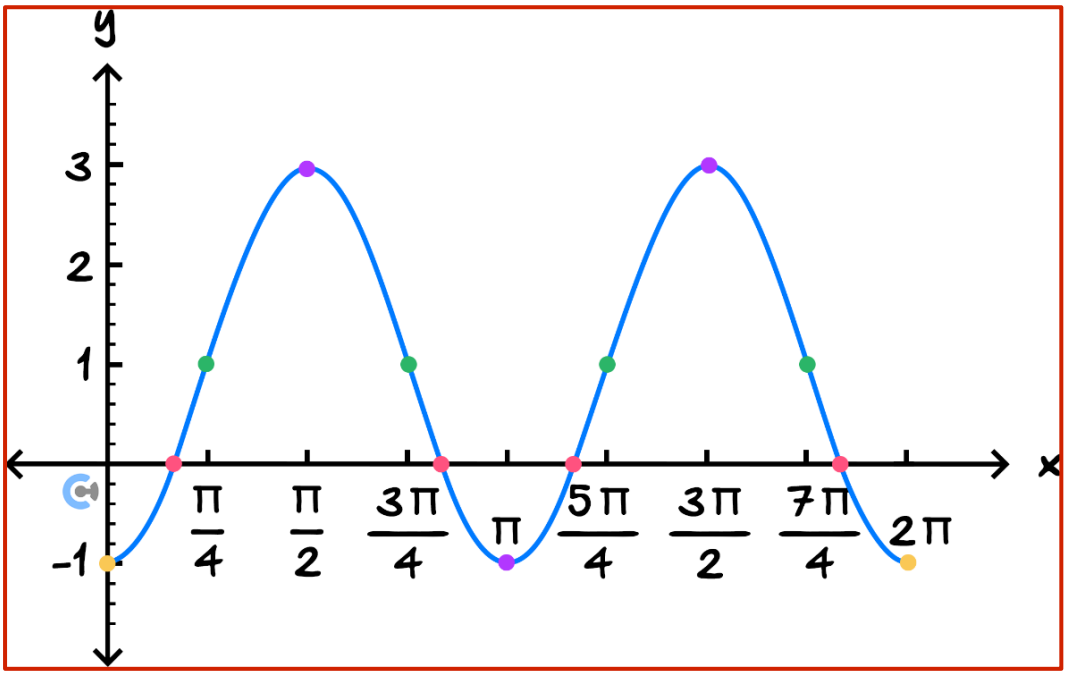


Asymptotes: $x = \frac{\pi}{3}, \frac{4\pi}{3}$

Space for Personal Notes

Question 3 (7 marks)

a. Sketch the graph of $f(x) = -2\cos(2x) + 1$ for $x \in [0, 2\pi]$ on the axes below, labelling all intercepts and endpoints with their coordinates. (3 marks)




```
In[5]:= Solve[-2 Cos[2 x] + 1 == 0 && 0 ≤ x ≤ 2 Pi]
Out[5]= {{x → π/6}, {x → 5 π/6}, {x → 7 π/6}, {x → 11 π/6}}
```

b. Solve $f(x) \geq 1$ for $x \in [0, 2\pi]$. (2 marks)

```
Reduce[-2 Cos[2 x] + 1 ≥ 1 && 0 ≤ x ≤ 2 π]
π/4 ≤ x ≤ 3 π/4 || 5 π/4 ≤ x ≤ 7 π/4
```

Consider a function $h(x) = f(x) + k$.

c. Solve for the value(s) of k such that the solutions of $h(x) = 0$ have a constant interval between them. (2 marks)

$k = 1, -1, -3$

Space for Personal Notes

Question 4 (9 marks)

The population of foxes in a certain forest varies according to the rule:

$$P(t) = 50 - 30\cos\left(\frac{\pi}{2}(t-2)\right)$$

Where $P(t)$ is the number of foxes t years after 2024.

a. Find the period and amplitude of this function. (2 marks)

Period $= 4$ years and amplitude $= 30$.

b. Find the maximum and minimum number of foxes in the forest. (2 marks)

Minimum $= 20$
Maximum $= 80$

c. After how many years is the population of foxes a minimum in the first 5 years? (2 marks)

Solve $P(t) = 20$ over $[0, 5]$
$t = 2$.
After 2 years.

Foxes are declared a vulnerable species if their population drops below 35.

d. Find the percentage of time when the foxes are declared as vulnerable species. (3 marks)

Fraction of time

$$= \frac{\frac{8}{3} - \frac{4}{3}}{4} = \frac{1}{3}$$

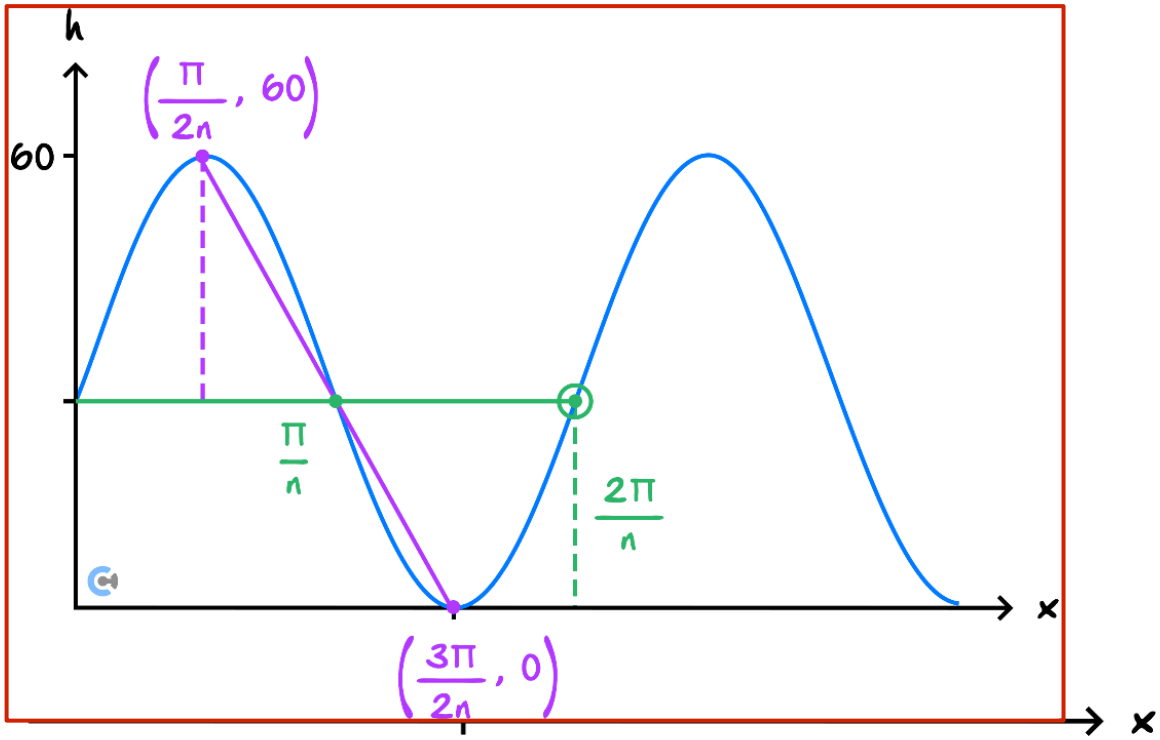
$$\frac{100}{3}\%$$

Space for Personal Notes

Section B: Extension Questions (8 Marks)

Question 5 (8 marks) **Tech-Active.**

The side view of a roller coaster is shown below, where h is the vertical height in m and x is the horizontal distance in m. The start of the roller coaster is at $(0,30)$ and its maximum height reaches $60\ m$.



The roller coaster can be modelled by the equation $h(x) = a\sin(nx) + b$ where $a, b, n \in \mathbb{R}$.

a. Find the values of A, B. (2 marks)

$A = 30$
$B = 30$

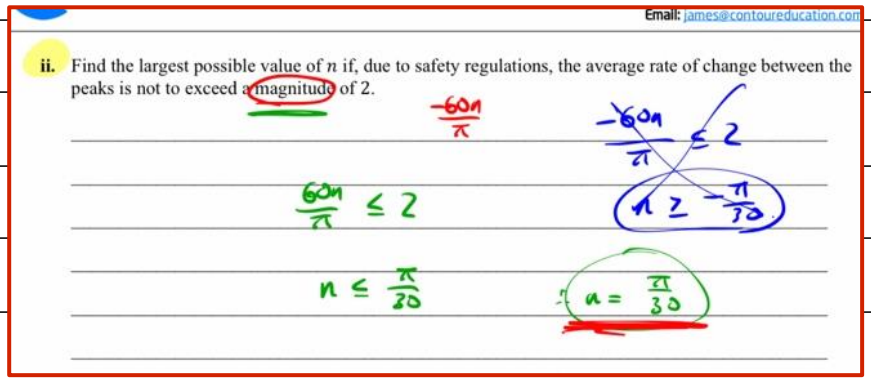
The average rate of change formula is given by $\frac{y_2-y_1}{x_2-x_1}$.

b.

i. Find the average rate of change between the first highest point and the first lowest point of the rollercoaster. (2 marks)

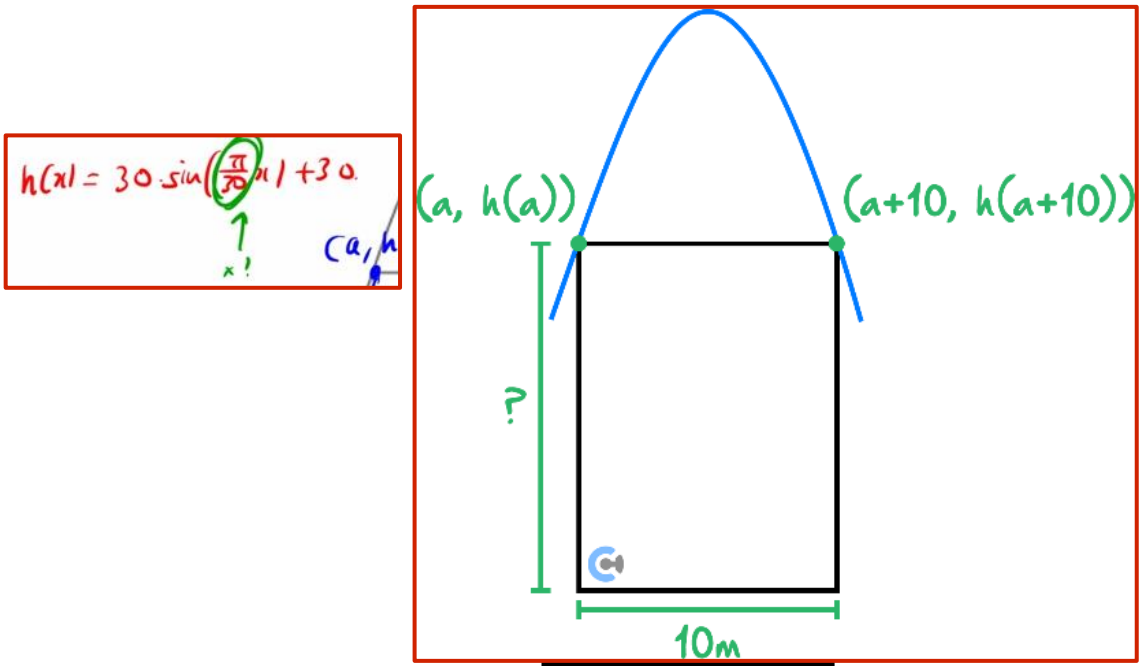
$$\frac{60-0}{\frac{\pi}{2\pi}-\frac{3\pi}{2\pi}}=\frac{60}{-\frac{\pi}{\pi}}=\frac{-60\pi}{\pi}$$

ii. Find the largest possible value of n if, due to safety regulations, the average rate of change between the peaks is not to exceed a magnitude of 2. (2 marks)

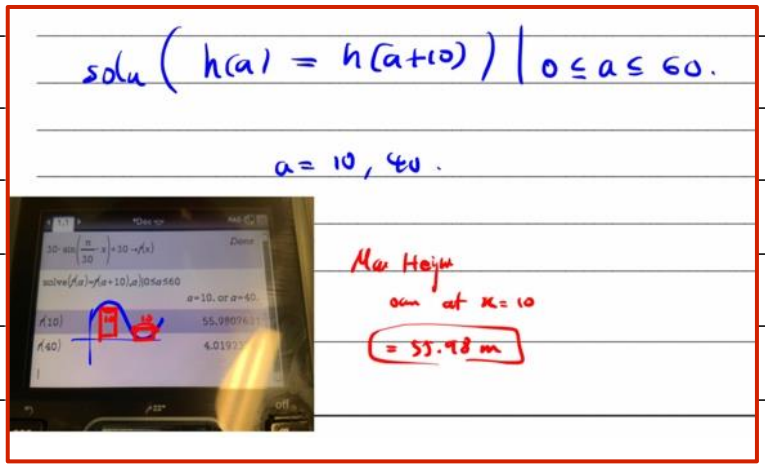


Use the largest value of n obtained from part **b. ii.** for the questions below.

It is found that the roller coaster needs support with a width of 10 m.



c. What is the maximum height of the support? Give your answer correct to 2 decimal places. (2 marks)



Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make the Most of These (Free) Services!

- Experienced Contour tutors (45 + raw scores, 99 + ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➢ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➢ One active booking at a time (must attend before booking the next).	➢ Message +61 440 138 726 with questions. ➢ Save the contact as "Contour Methods".

Booking Link for Consults

bit.ly/contour-methods-consult-2025

Number for Text-Based Support

+61 440 138 726