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VCE Mathematical Methods ¾
Circular Functions II [3.3]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 22
Supplementary Questions	Pg 23 - Pg 38

Section A: Compulsory Questions

Sub-Section [3.3.1]: Solve Advanced Trigonometric Equations

Question 1

Find the general solution to the following trigonometric equations over the specified domain.

a. $2\sin(x) - 1 = 0$, for $x \geq 0$.

b. $\tan(2x) = \sqrt{3}$, for $x < 0$.

c. $\sqrt{2}\cos\left(2\pi+\frac{\pi}{4}\right)=1$, for $x>0$.

Question 2

Solve the following trigonometric equations over the specified domain.

a. $2\cos^2(x)-3\cos(x)+1=0$, for $0\leq x\leq 2\pi$.

b. $\sin^2(2x) + \sin(2x) - 2 = 0$, for $x \in \mathbb{R}$.

c. $3\sin^2(x) - 6\sin(x) - \cos^2(x) + 3 = 0$, for $0 \leq x \leq 2\pi$.

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Question 3

Find the value(s) of k such that $4\sin^2(x) + k\cos(x) - 2 = 0$ has 2 solutions in the interval $[0, \pi]$.

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Question 4 Tech-Active.

a. Find the general solution to the equation $2\cos^2(x) + 3\cos(x) - 2 = 0$.

b. Hence, find the solutions to $2\cos^2(x) + 3\cos(x) - 2 = 0$ for $x \in [0, 2\pi]$.

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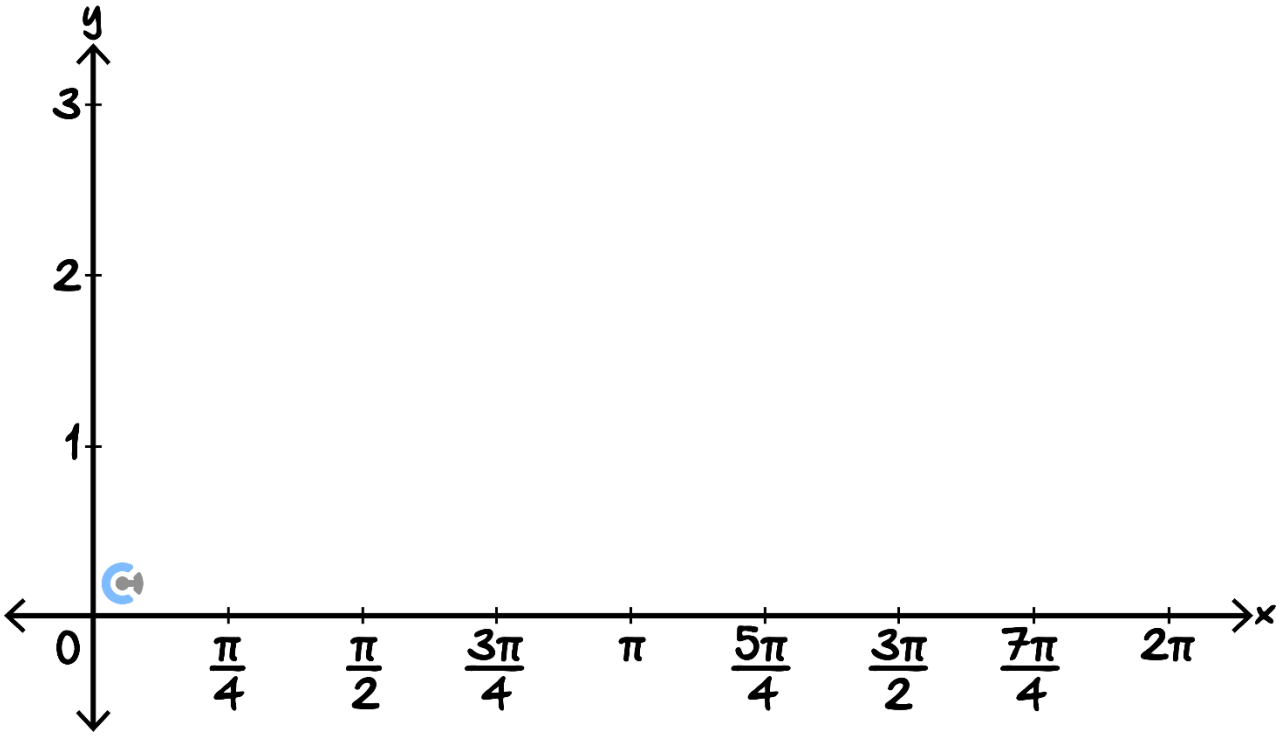
Sub-Section [3.3.2]: Graph Sine, Cosine, and Tangent Functions

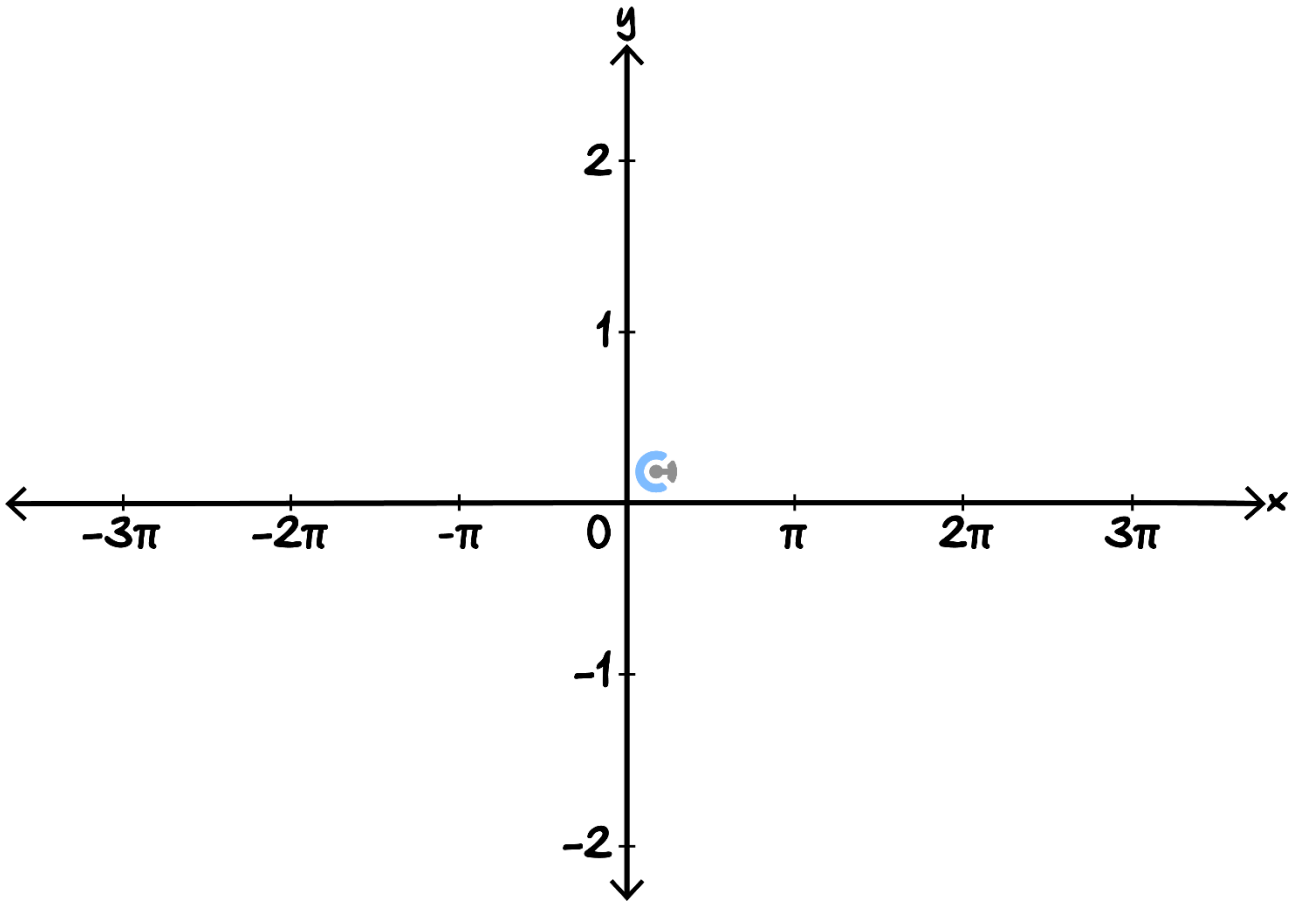
Question 5

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts, turning points and endpoints with their coordinates, and asymptotes with their equations.

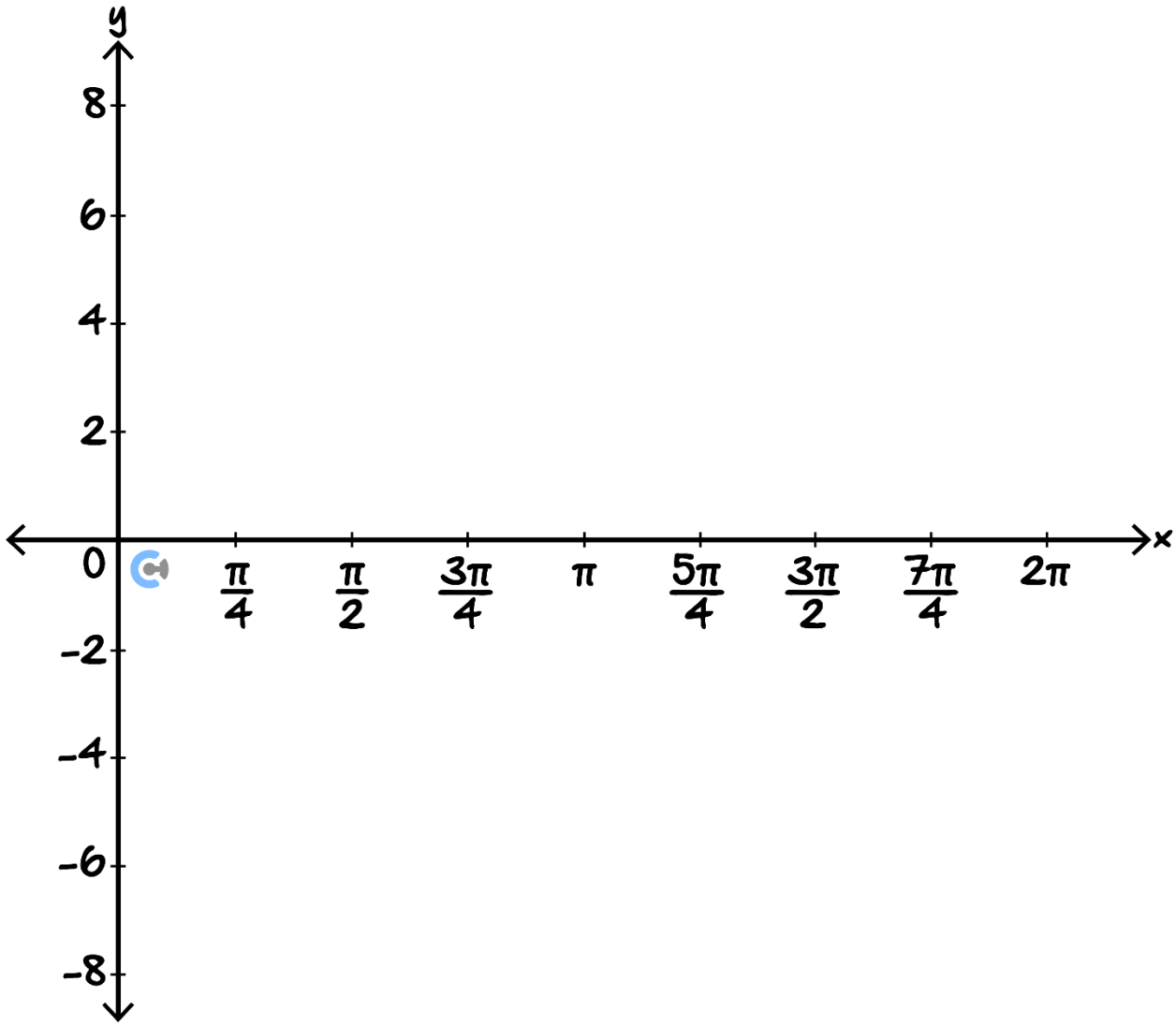
a. $y = \sin(2x) + 1$ for $0 \leq x \leq 2\pi$.



b. $y = \cos\left(\frac{x}{3}\right)$ for $-3\pi \leq x \leq 3\pi$.



c. $y = -\tan(x) + 1$ for $0 \leq x \leq 2\pi$.



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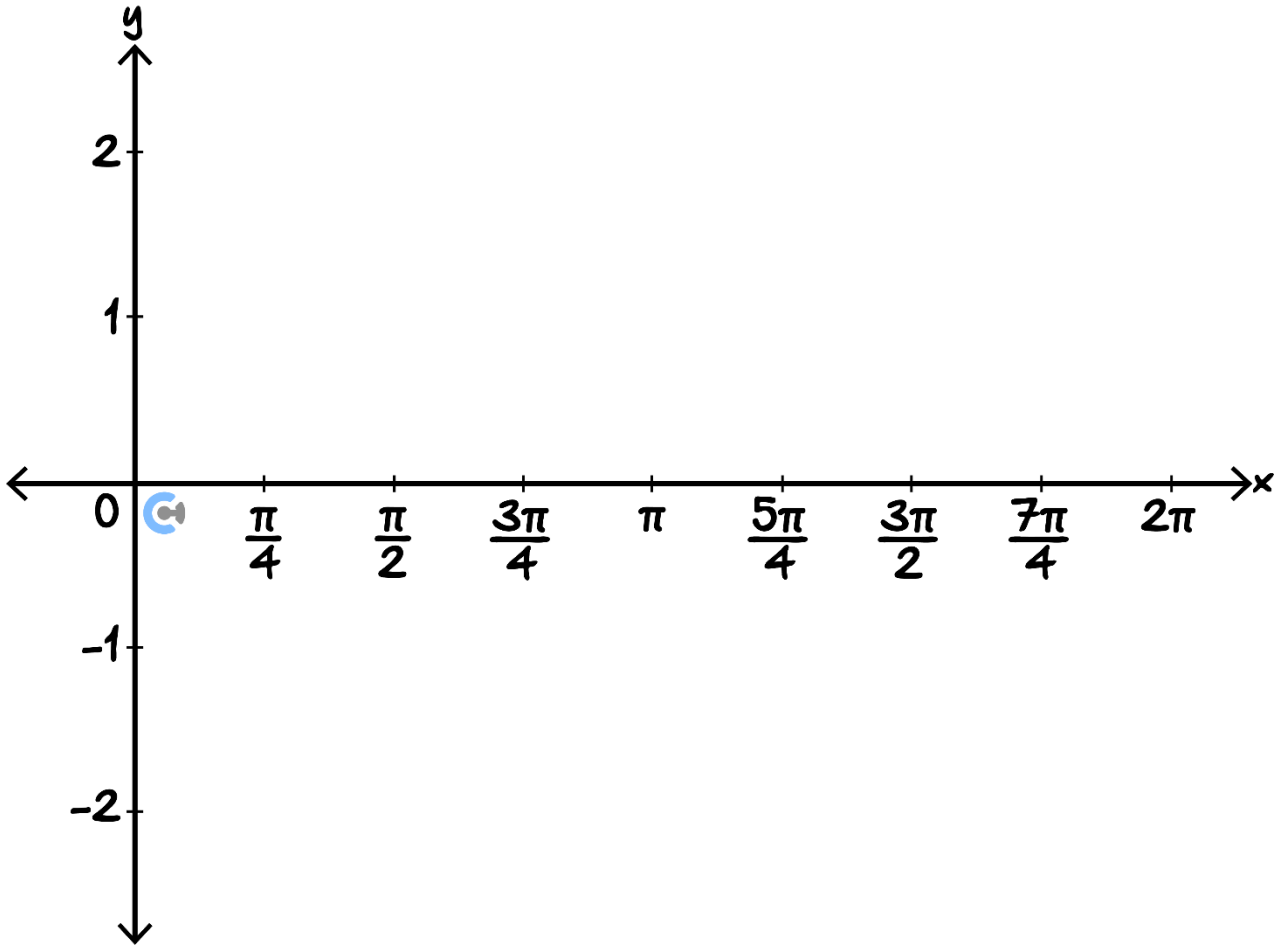
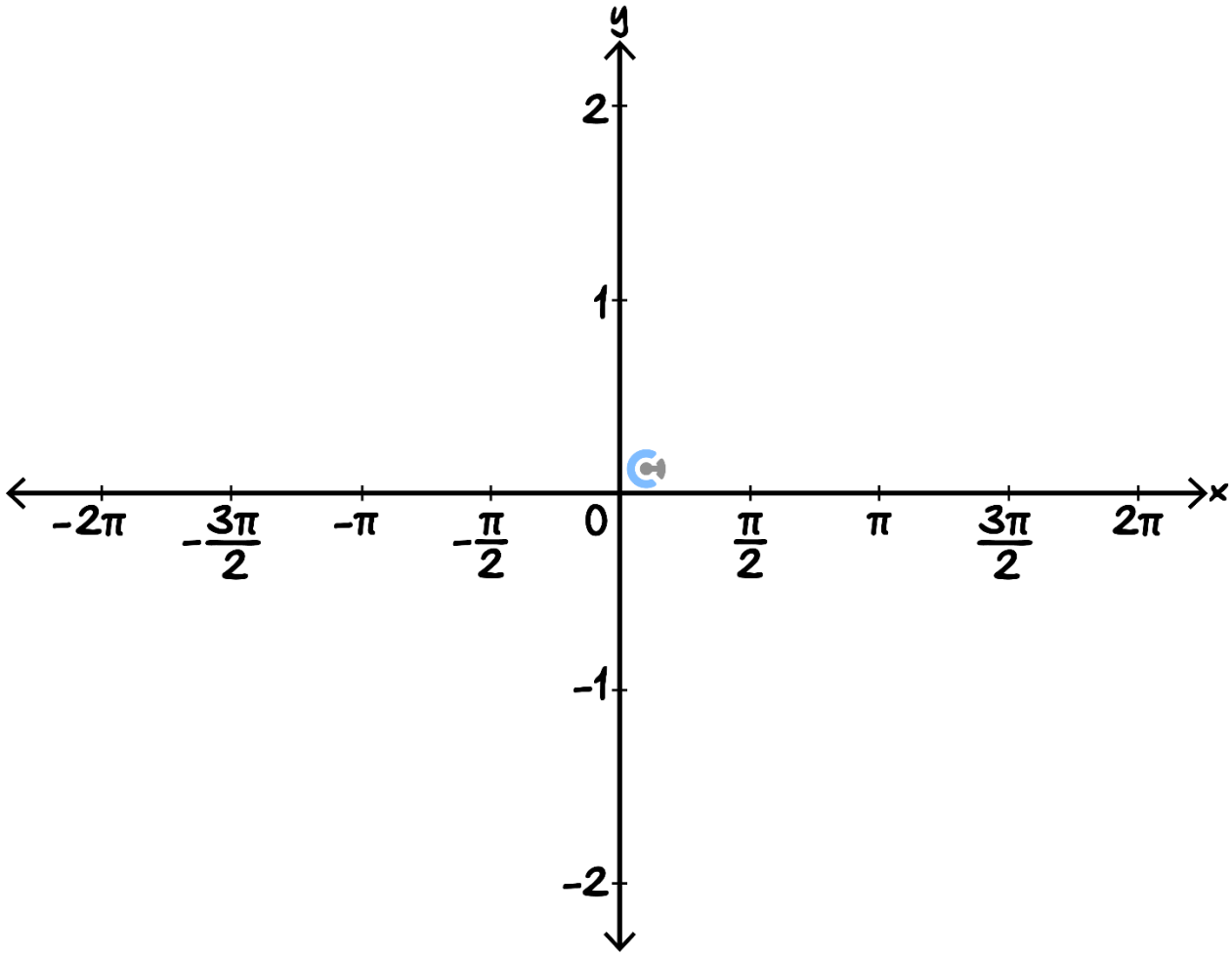
Question 6

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts, turning points and endpoints with their coordinates and asymptotes with their equations.

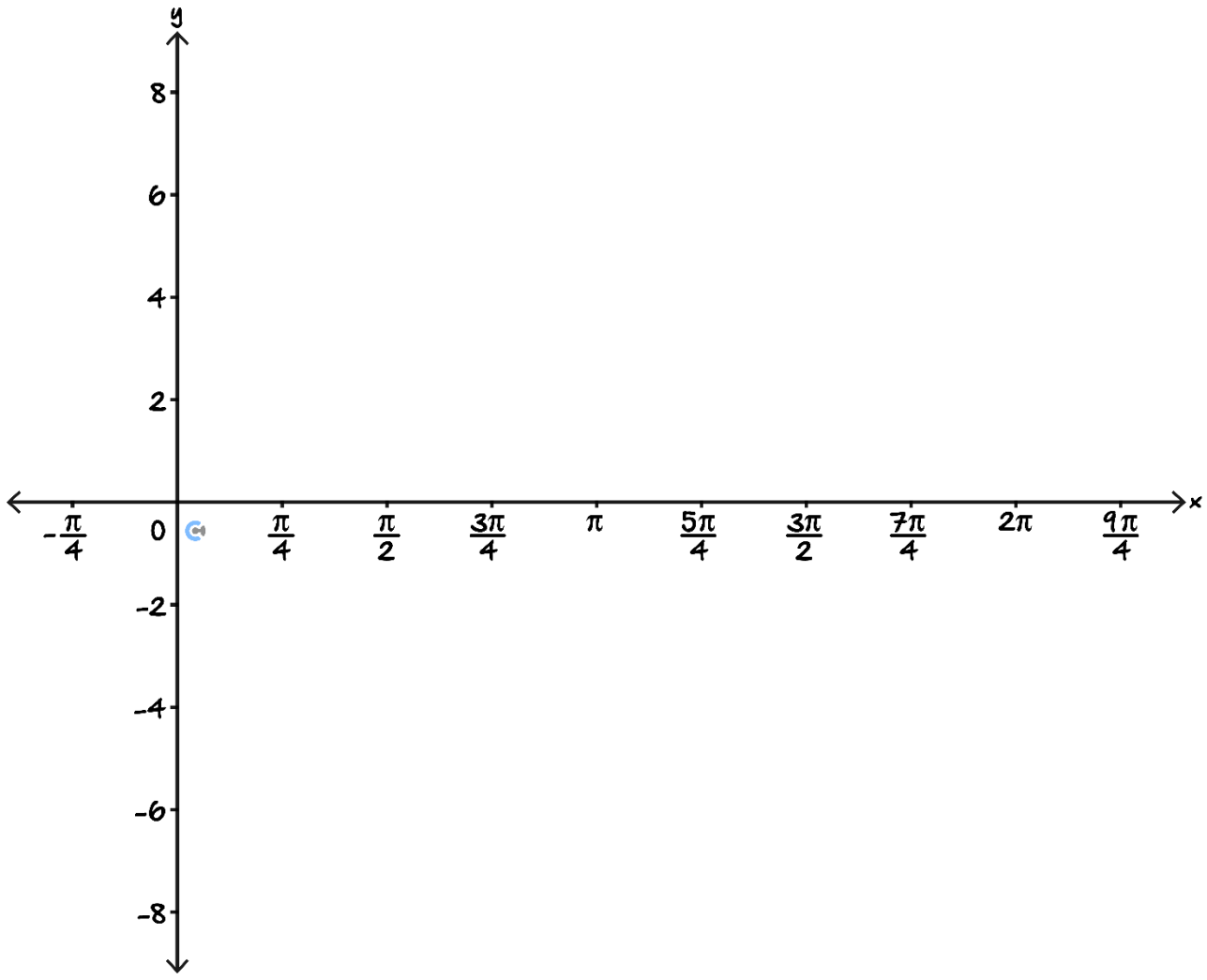
a. $y = 2\sin\left(2\left(x + \frac{\pi}{4}\right)\right)$ for $0 \leq x \leq 2\pi$.



b. $y = -\cos\left(x + \frac{\pi}{4}\right)$ for $-2\pi \leq x \leq 2\pi$.



c. $y = -2\tan\left(x + \frac{\pi}{4}\right)$ for $0 \leq x \leq 2\pi$.



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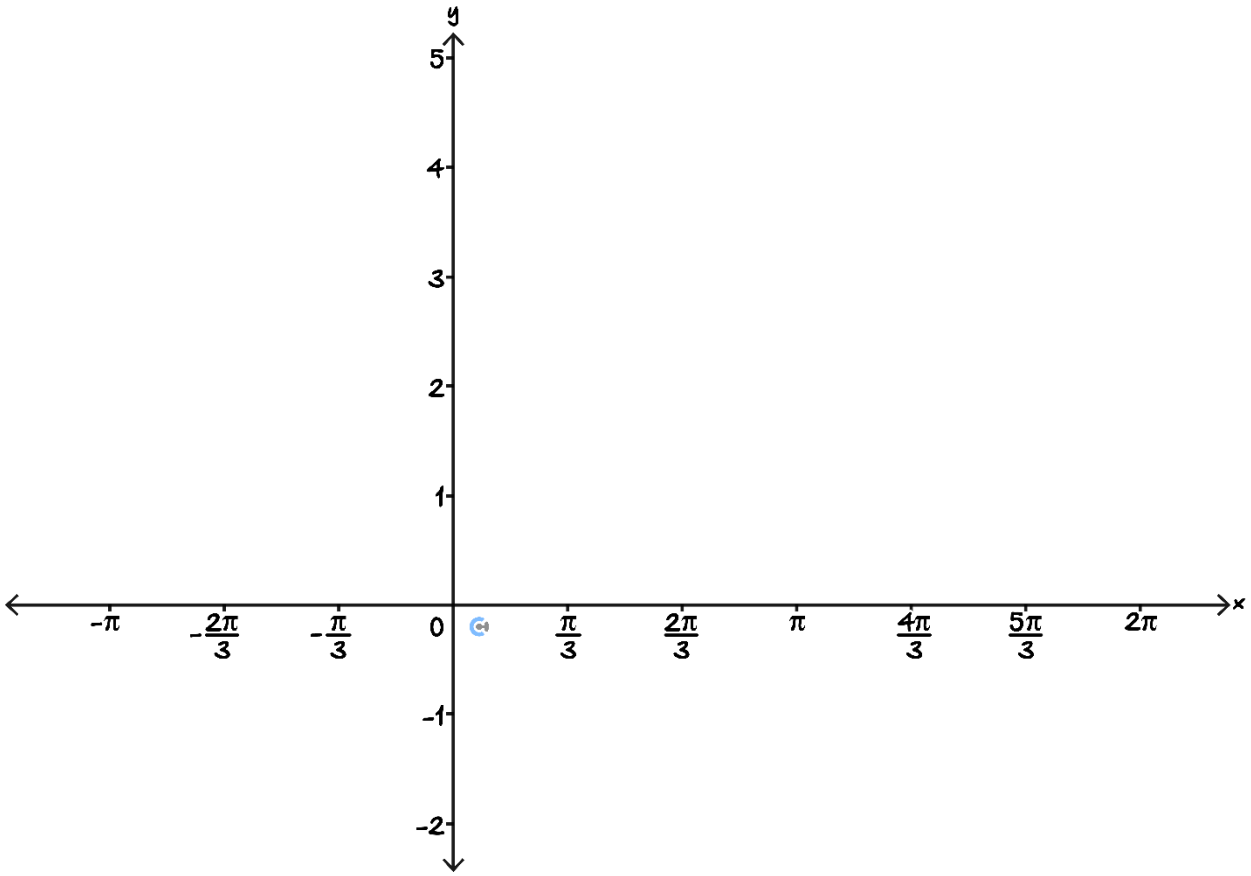
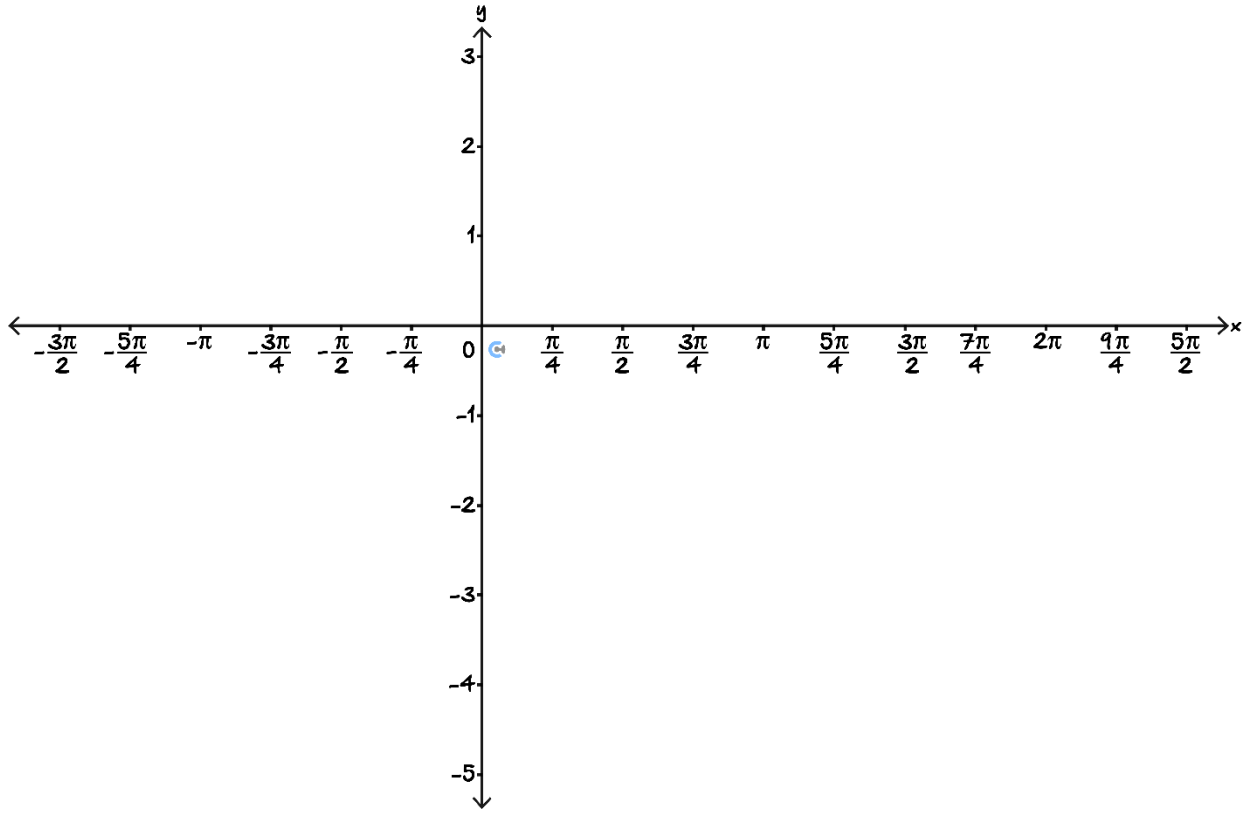
Question 7

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts and endpoints with their coordinates and asymptotes with their equation.

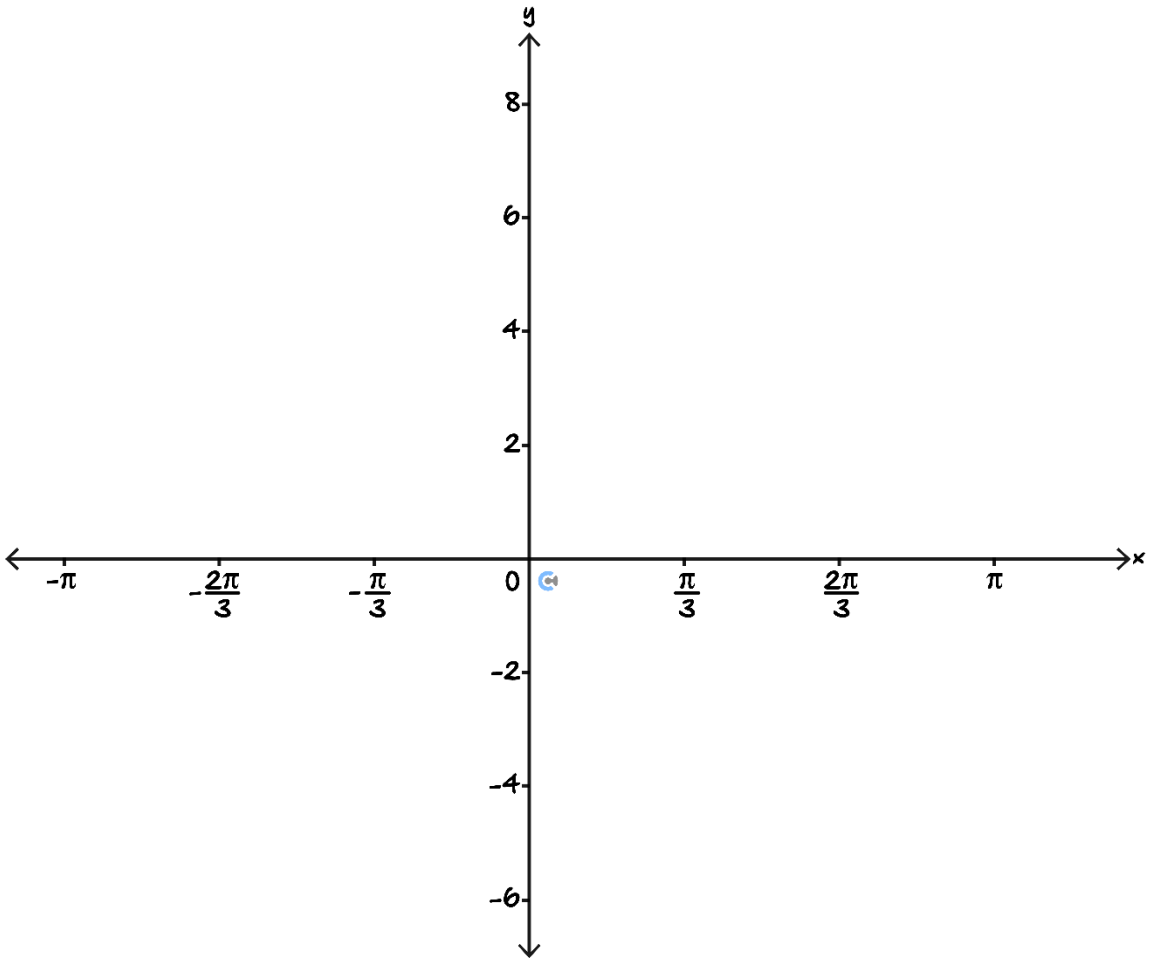
a. $y = -2\sin\left(2x + \frac{\pi}{6}\right) + 1$ for $-\pi \leq x \leq 2\pi$.



b. $y = 2\cos\left(2x - \frac{\pi}{4}\right) - 1, x \in [-\pi, 2\pi]$.



c. $y = -\tan\left(2x - \frac{\pi}{3}\right) + \sqrt{3}$ for $-\pi \leq x \leq \pi$.



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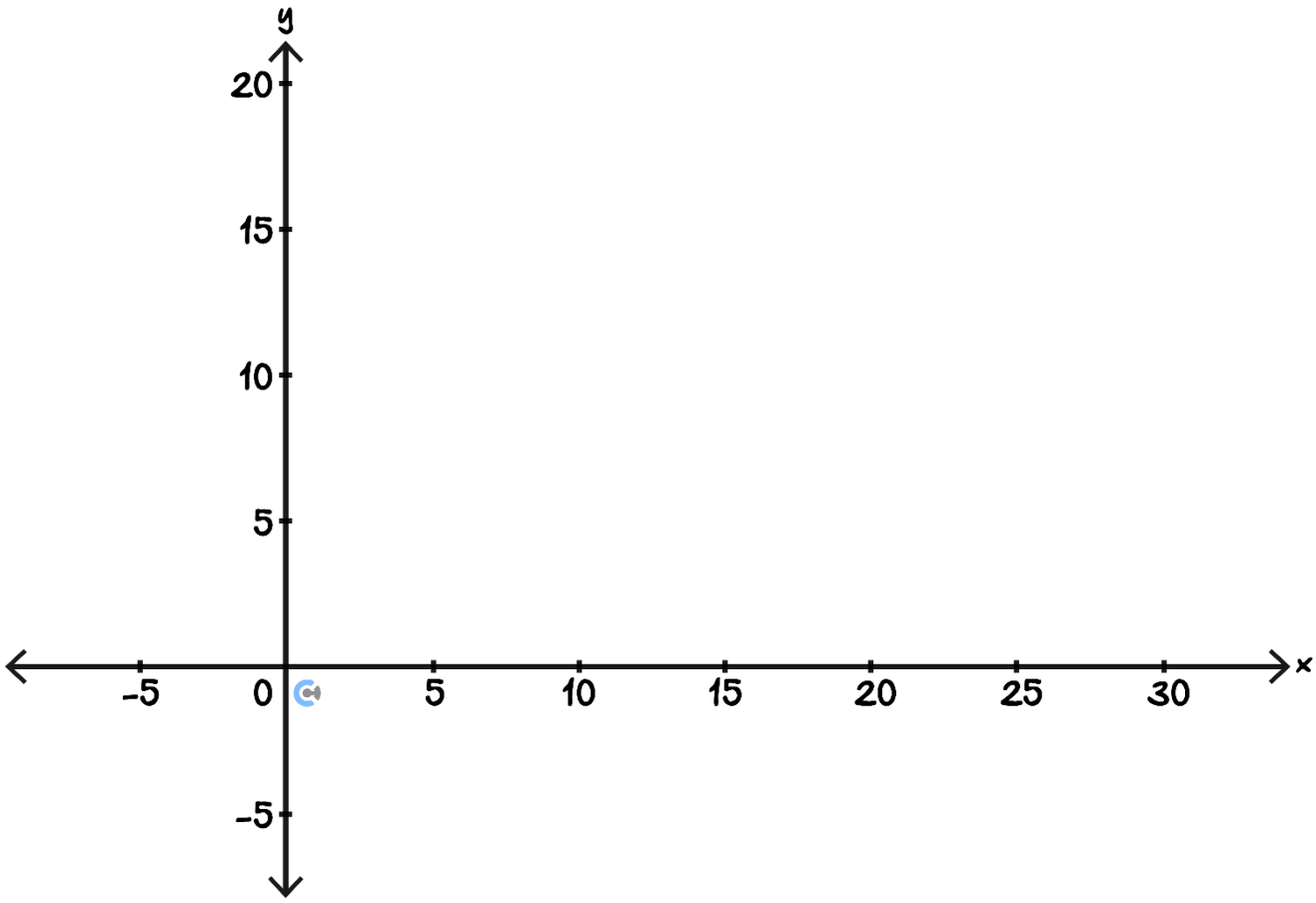
Question 8 Tech-Active.

Sketch the graph of $y = 12\sin\left(\frac{\pi x}{6}\right) + 6$ for $x \in [0, 24]$ on the axes below. Label all axial intercepts and turning points with coordinates.



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Sub-Section [3.3.3]: Fraction of Periods

Question 9

The temperature of a lake throughout the year is modelled by $T(t)$, where:

$$T(t) = 18 + 4\cos\left(\frac{\pi}{6}t\right)$$

Where $T(t)$ represents the temperature (in degrees Celsius) of the lake at t months since January.

Find the fraction of the year during which the temperature is above 20°C.

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Question 10 Tech-Active.

A research team is monitoring the depth of water in a tidal bay. The depth of the water, in metres, is modelled by the function:

$$D(t) = 8 + 3\cos\left(\frac{\pi}{6}t\right)$$

Where $D(t)$ represents the depth of the water t hours after midnight.

a. State the maximum and minimum depth of the water.

b. Determine the first two times after midnight when the water reaches a depth of 10 metres. Give your answers in hours after midnight, correct to two decimal places.

c. Find the percentage of a full tidal cycle during which the water depth is above 9 metres. Give your answer correct to two decimal places.

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Question 11 Tech-Active.

The temperature in a greenhouse fluctuates throughout the day and is modelled by the function:

$$T(t) = 22 + 5\sin\left(\frac{\pi}{12}t\right)$$

where $T(t)$ represents the temperature in degrees Celsius, and t is the time in hours after midnight.

a. Find the fraction of a full day during which the temperature exceeds 24°C. Give your answer as a decimal correct to three decimal places.

b. Find the value of k such that $T(t) + k$ exceeds 26°C for exactly 40% of the time. Give your answer correct to two decimal places.

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Sub-Section: Final Boss

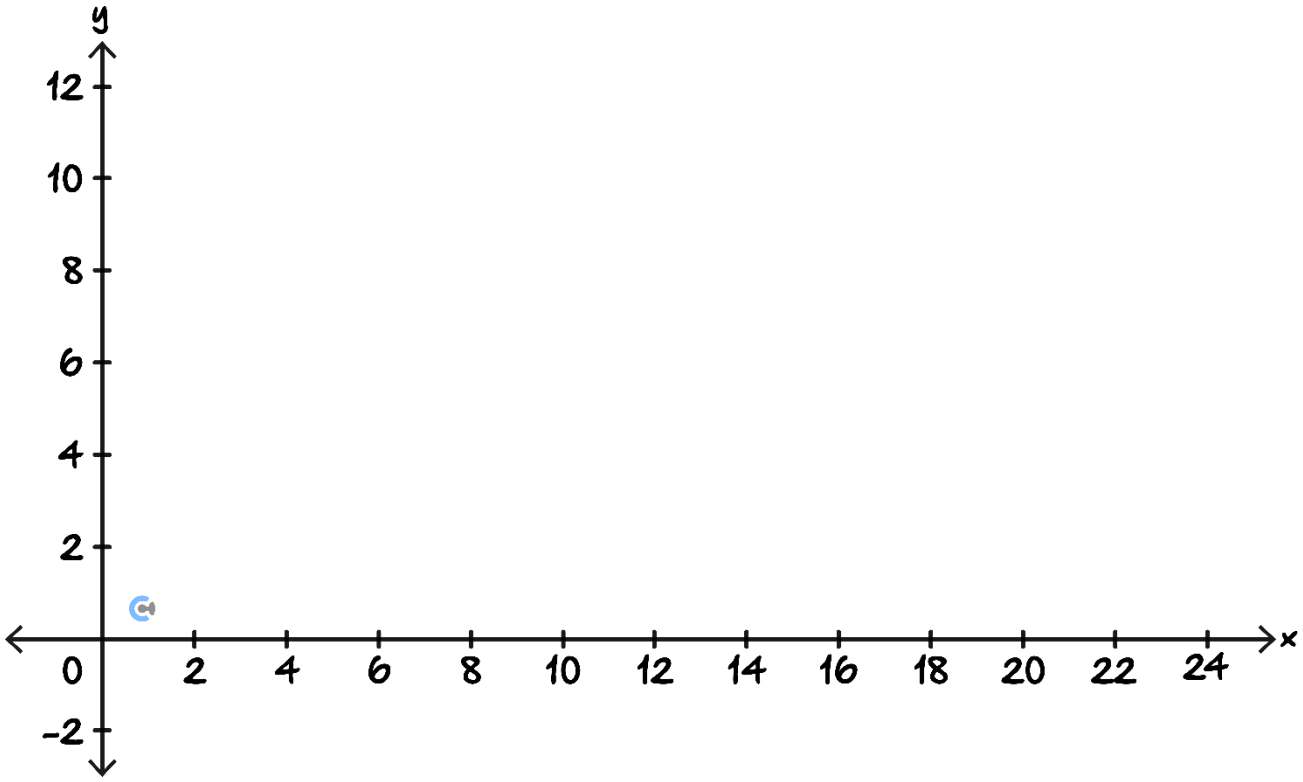
Question 12

The depth of water in a coastal bay varies throughout the day due to tidal motion. The depth of water, in metres, is modelled by the function:

$$D(t) = 6 + 3\cos\left(\frac{\pi}{6}t\right)$$

where t is the time in hours after midnight.

a. Sketch the graph of $D(t)$ for $0 \leq t \leq 24$.



b. Find the first two times after midnight when the water depth is exactly 7.5 metres.

c. Find the fraction of a full tidal cycle when the water depth is below 4.5 metres.

d. The harbour can only accommodate boats when the water depth is at least 5.8 metres. Find the smallest vertical translation k such that $D(t) + k$ ensures this condition is met for at least 75% of the tidal cycle.

Give an exact answer in the form $\frac{a\sqrt{b}-c}{d}$, for positive integers a, b, c, and d.

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Section B: Supplementary Questions

Sub-Section [3.3.1]: Solve Advanced Trigonometric Equations

Question 13

Find the general solution to the following trigonometric equations over the specified domain.

a. $\sin(2x) = \frac{\sqrt{3}}{2}$, for $x \geq 0$.

b. $\cos\left(x - \frac{\pi}{3}\right) = -\frac{1}{2}$, for $x < 0$.

c. $\tan\left(3x+\frac{\pi}{6}\right)=1$, for $x>0$.

Question 14

Solve the following equations for $x \in \mathbb{R}$. Note that some solutions will need to be expressed in terms of inverse trigonometric functions.

a. $4\sin^2(x)-4\sin(x)+1=0$, for $0 \leq x \leq 2\pi$.

b. $3\cos^2(2x) + 2\cos(2x) - 1 = 0$, for $x \in \mathbb{R}$.

c. $\tan^2(x) - \tan(x) - 2 = 0$, for $0 \leq x < 2\pi$.

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Question 15

Find the value(s) of k such that the equation:

$$3\cos^2(x) + k\sin(x) - 1 = 0$$

has exactly two solutions in the interval $[0, \pi]$.

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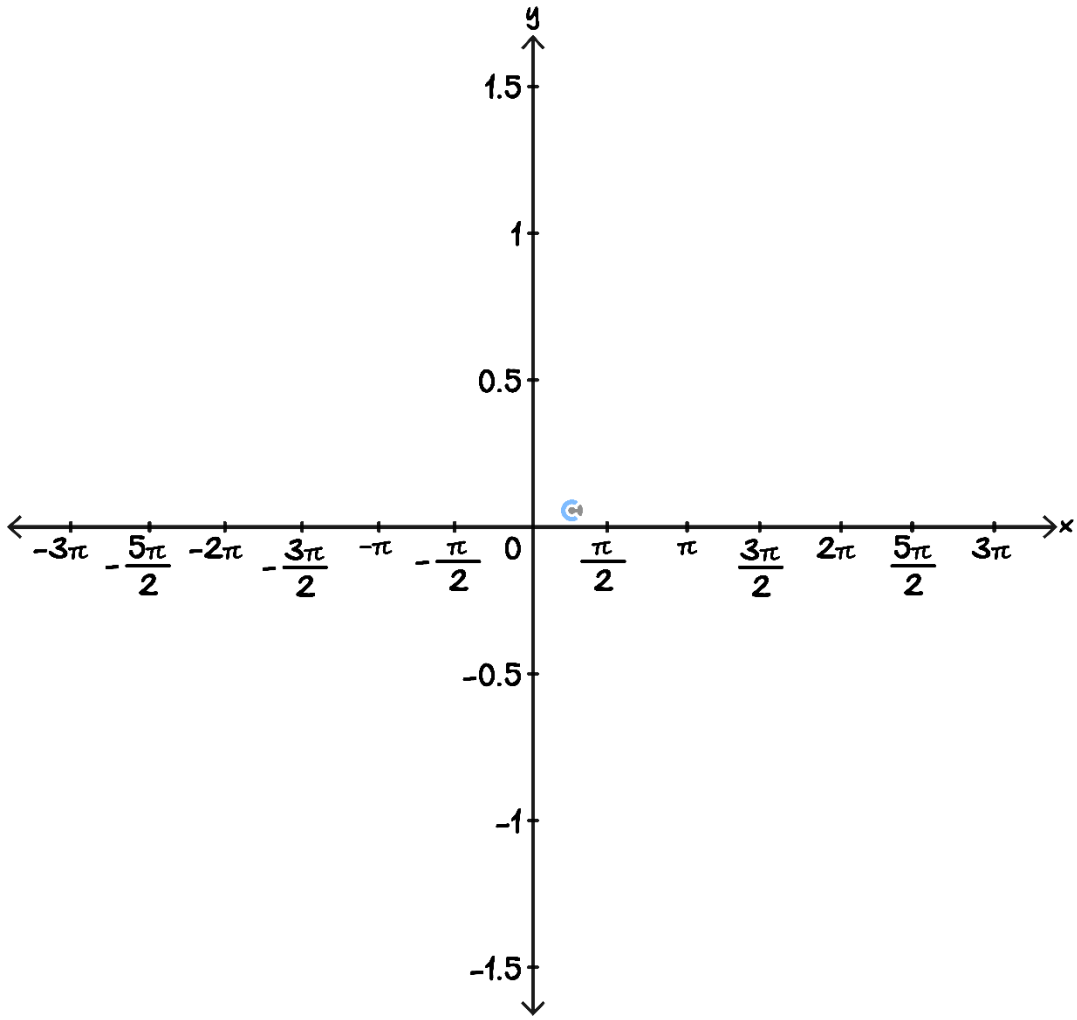
Sub-Section [3.3.2]: Graph Sine, Cosine, and Tangent Functions

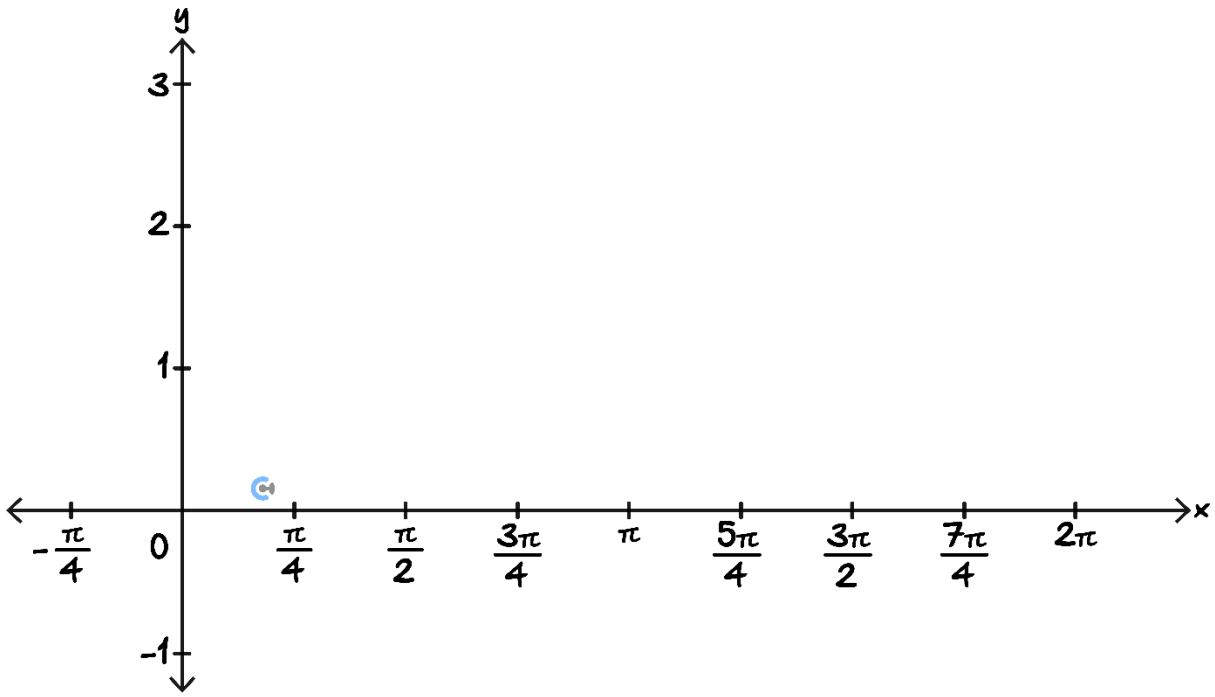
Question 16

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts, turning points and endpoints with their coordinates, and asymptotes with their equations.

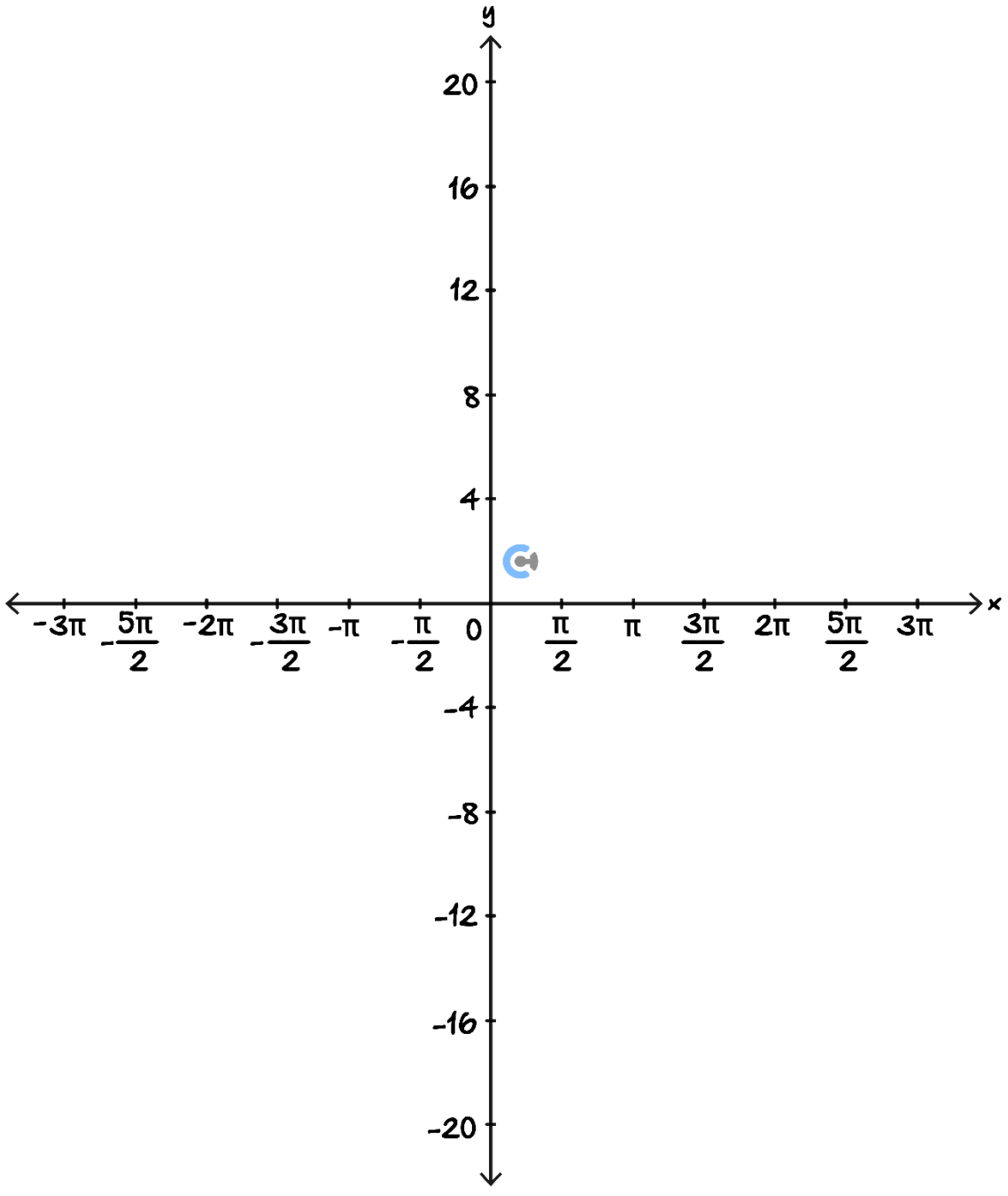
a. $y = \sin\left(\frac{2x}{3}\right)$ for $-3\pi \leq x \leq 3\pi$.



b. $y = -\cos(2x) + 1$ for $0 \leq x \leq 2\pi$.



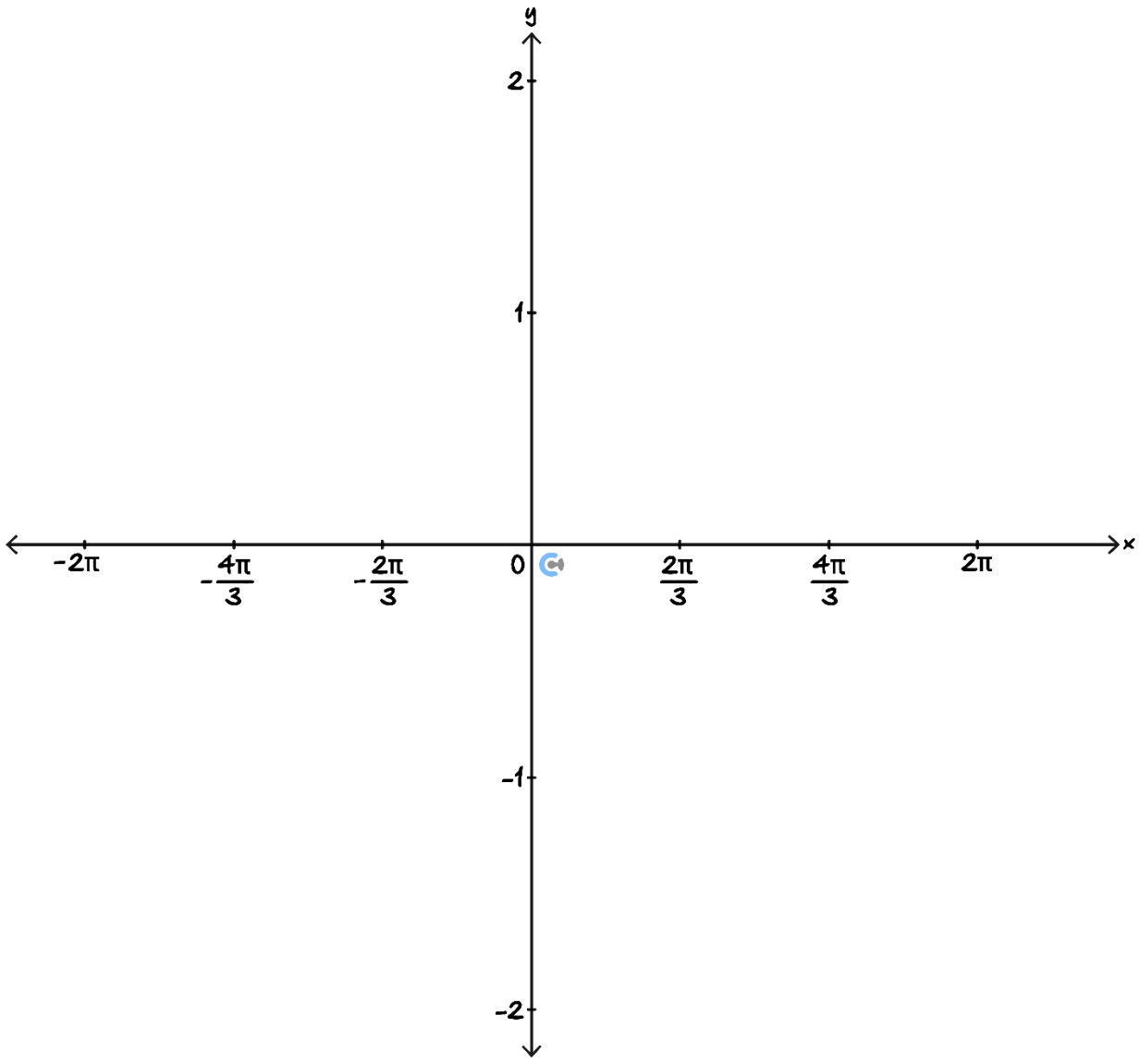
c. $y = \tan\left(\frac{x}{2}\right)$ for $-3\pi \leq x \leq 3\pi$.



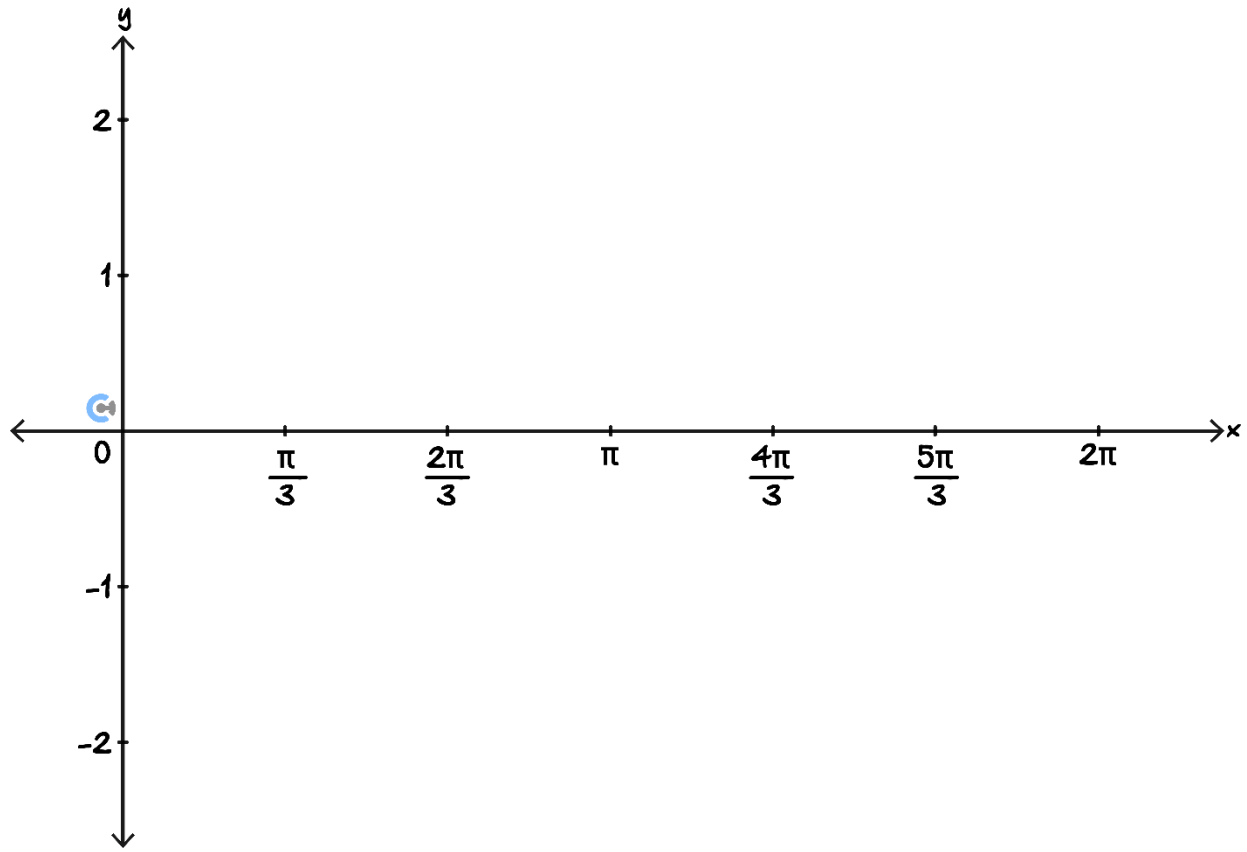
Question 17

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts, turning points and endpoints with their coordinates and asymptotes with their equations.

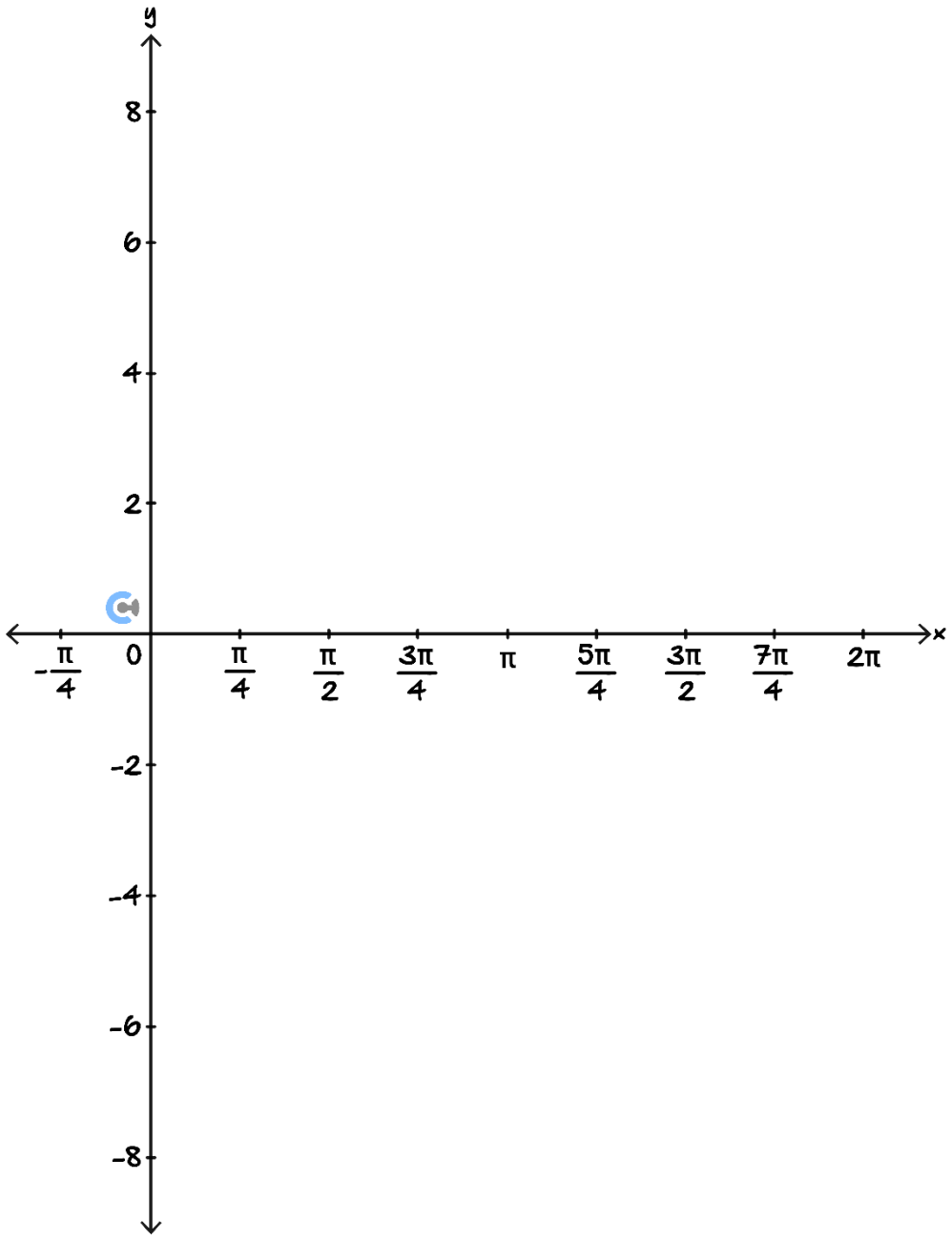
a. $y = -\sin\left(x + \frac{\pi}{3}\right)$ for $-2\pi \leq x \leq 2\pi$.



b. $y = 2\cos\left(2\left(x + \frac{\pi}{6}\right)\right)$ for $0 \leq x \leq 2\pi$.



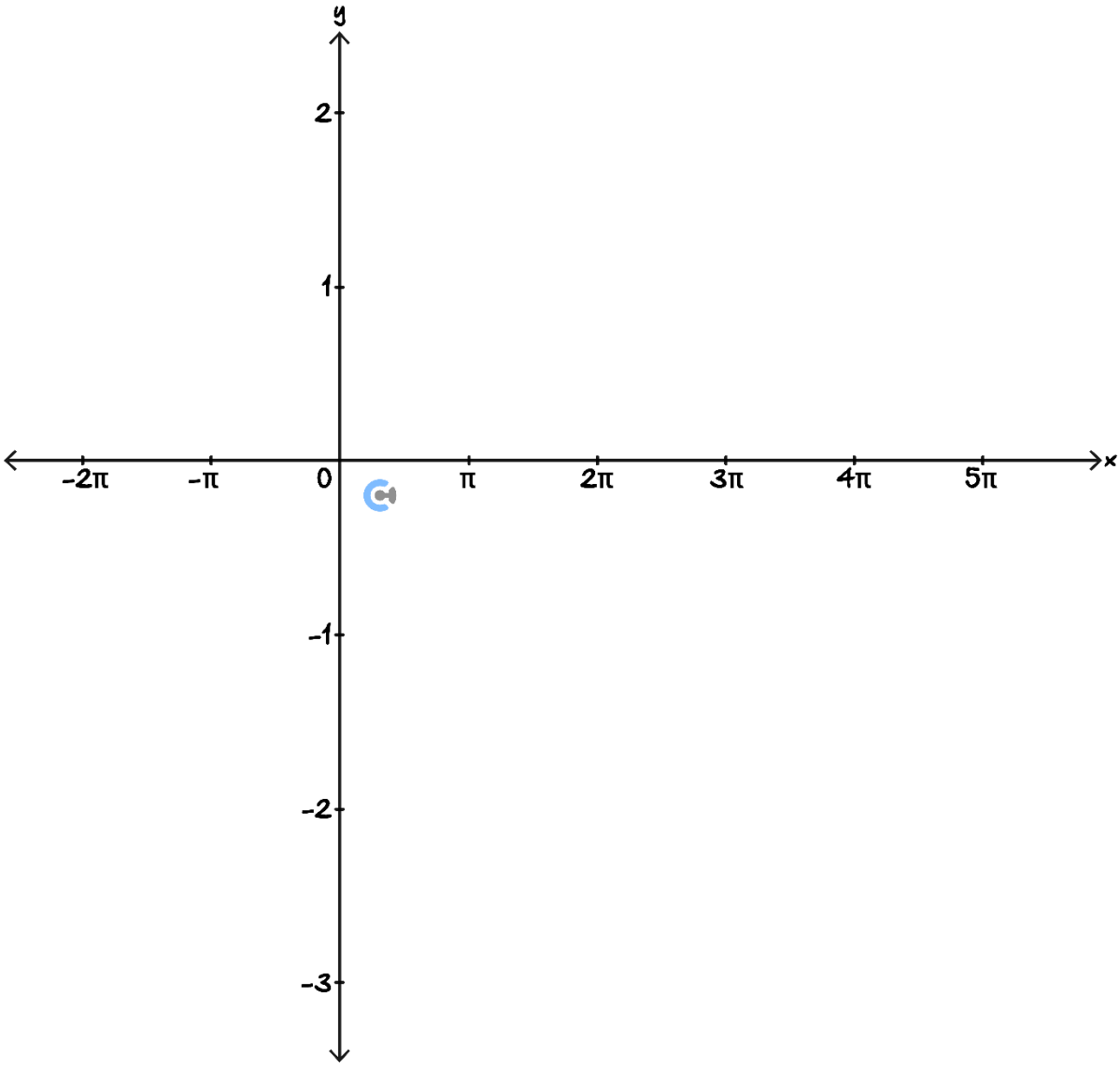
c. $y = \tan\left(2x - \frac{\pi}{2}\right)$ for $0 \leq x \leq 2\pi$.



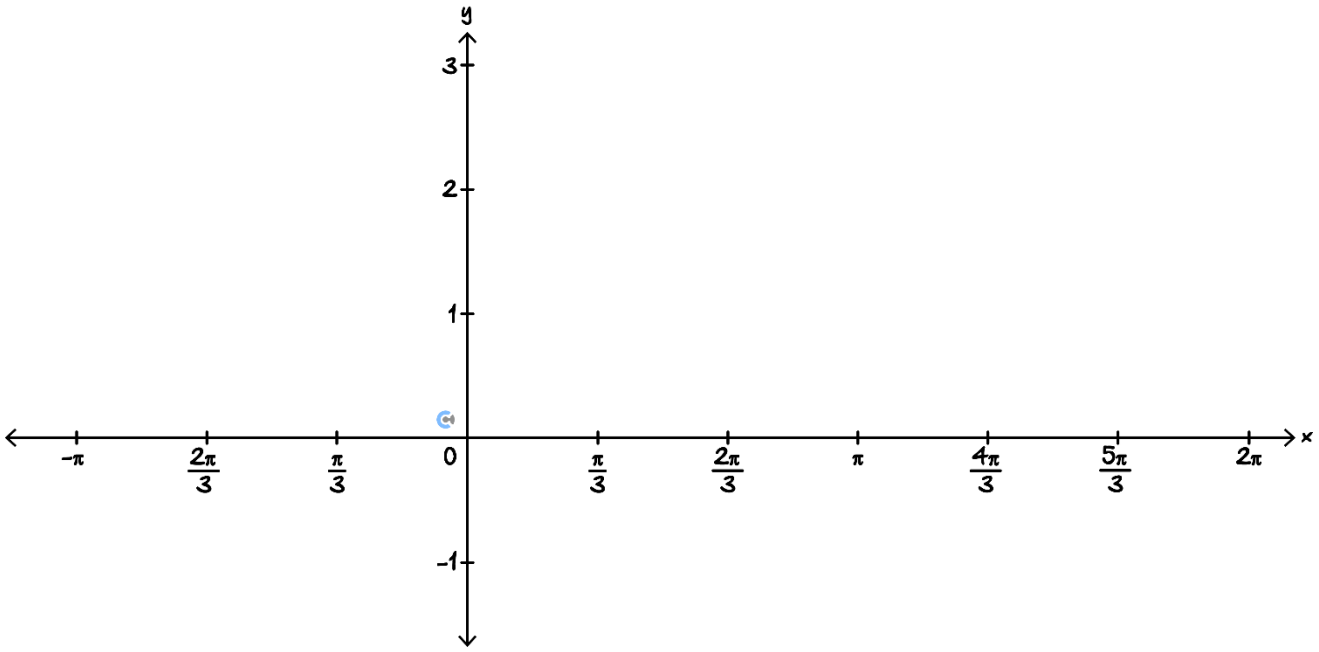
Question 18

Sketch the graphs of the functions over the specified domain on the given axes. Label all axes intercepts and endpoints with their coordinates and asymptotes with their equation.

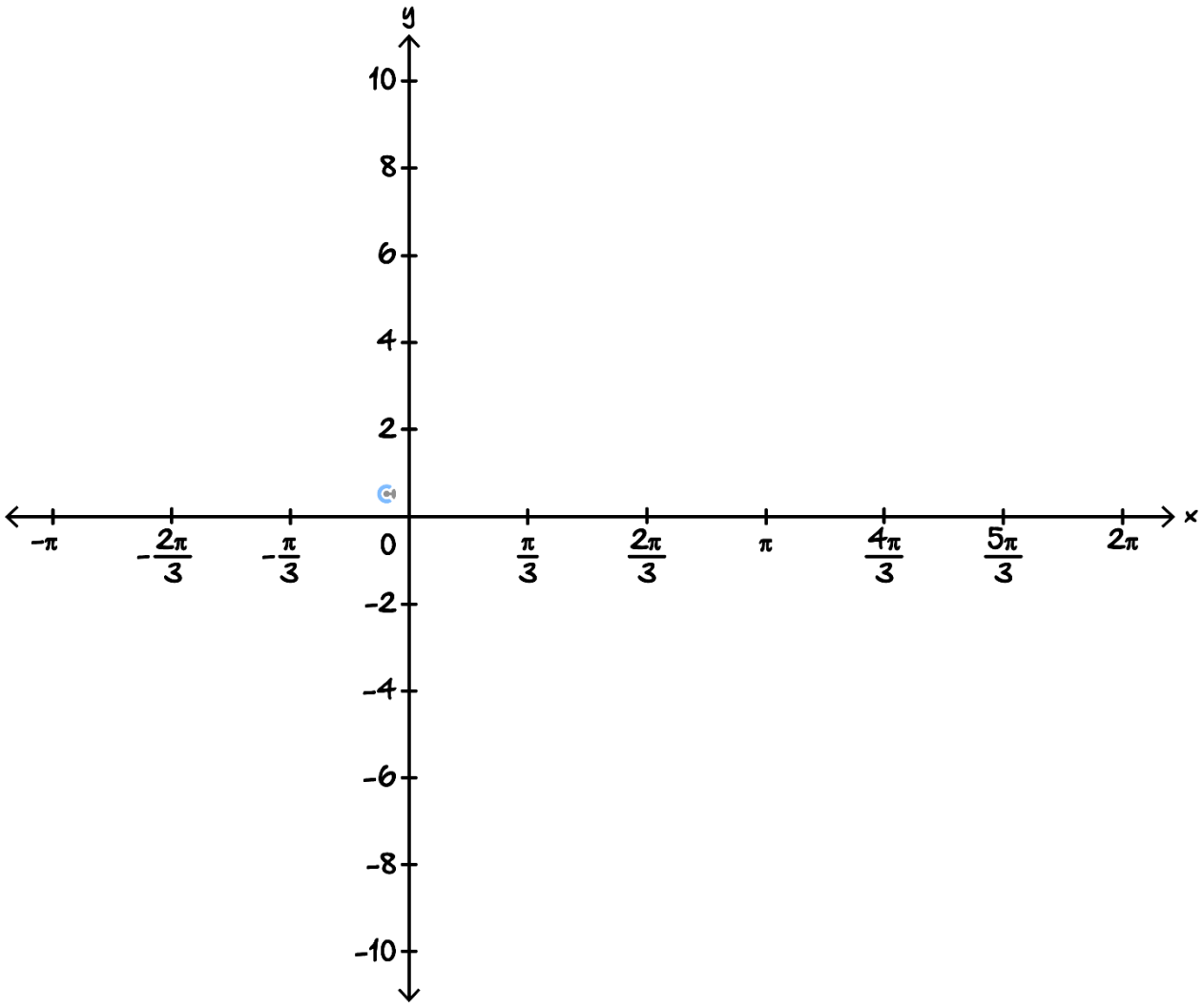
a. $y = \sqrt{2}\sin\left(\frac{x}{3} + \frac{\pi}{6}\right) - 1$ for $-2\pi \leq x \leq 5\pi$.



b. $y = -2\cos\left(-2x + \frac{\pi}{3}\right) + 1$ for $-\pi \leq x \leq 2\pi$.



c. $y = -\sqrt{3}\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)$ for $-\pi \leq x \leq 2\pi$.



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Sub-Section [3.3.3]: Fraction of Periods

Question 19

The temperature inside a storage container over a 24-hour period is modelled by

$$T(t) = 15 + 6\cos\left(\frac{\pi}{12}t\right),$$

Where $T(t)$ is in degrees Celsius and t is the number of hours since midnight.

Find the fraction of the day during which the temperature exceeds 18°C.

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Question 20 Tech-Active.

The depth of a river fluctuates due to tides and is modelled by:

$$D(t) = 10 + 2.5\sin\left(\frac{\pi}{6}t\right),$$

where $D(t)$ is the depth in metres and t is the time in hours since midnight.

a. State the maximum and minimum river depth.

b. Find the first two times after midnight the depth reaches exactly 11 metres. Give answers as hours after midnight correct to two decimal places.

c. Determine the percentage of a full tidal cycle during which the depth is greater than 12 metres.

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Question 21 Tech-Active.

The brightness inside a room with automated skylights varies during the day and is modelled by

$$B(t) = 300 + 100\sin\left(\frac{\pi}{12}t\right),$$

where $B(t)$ is the brightness in lumens and t is the number of hours since midnight.

a. Find the fraction of a full day during which the brightness exceeds 350 lumens.

b. Find the value of k such that $B(t) + k$ exceeds 400 lumens for exactly 30% of the time. Give your answer to two decimal places.

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