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VCE Mathematical Methods $\frac{3}{4}$
Circular Functions I [3.2]
Homework Solutions

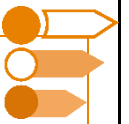
Admin Info & Homework Outline:



Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02 - Pg 13
Supplementary Questions	Pg 14 - Pg 21

Section A: Compulsory Questions

Sub-Section [3.3.1]: Finding Exact Values of Circular Functions



Question 1



Find the exact value of:

a. $\sin\left(\frac{\pi}{4}\right)$

$\frac{1}{\sqrt{2}}$

b. $\cos\left(\frac{\pi}{6}\right)$

$\frac{\sqrt{3}}{2}$

c. $\tan\left(\frac{\pi}{3}\right)$

$\sqrt{3}$

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Question 2


Find the exact value of:

a. $\sin\left(\frac{3\pi}{2}\right)$

-1

b. $\cos\left(\frac{5\pi}{6}\right)$

$-\frac{\sqrt{3}}{2}$

c. $\tan\left(\frac{5\pi}{3}\right)$

$-\sqrt{3}$

Question 3


Find the exact value of:

a. $\sin\left(\frac{16\pi}{3}\right)$

$-\frac{\sqrt{3}}{2}$

b. $\cos\left(\frac{26\pi}{3}\right)$

$-\frac{1}{2}$

c. $\tan\left(\frac{15\pi}{4}\right)$

-1

Question 4 Tech-Active.

Find the exact value of $\sin\left(\frac{\pi}{8}\right)$ in the form of $\frac{\sqrt{a-\sqrt{a}}}{2}$, for positive integer a .

$$\frac{\sqrt{2-\sqrt{2}}}{2}$$

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Sub-Section [3.3.2]: Applying the Pythagorean Identity and Symmetrical Properties

Question 5



Given that $\sin(x) = \frac{4}{5}$ and $0 < x < \frac{\pi}{2}$, find:

a. $\cos(x)$

$$\cos(x) = \frac{3}{5}$$

b. $\tan(x)$

$$\tan(x) = \frac{4}{3}$$

c. $\sin\left(x + \frac{\pi}{2}\right)$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x) = \frac{3}{5}$$

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Question 6

Given that $\sin(x) = \frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$, find:

a. $\cos(x)$

$$\cos(x) = -\frac{4}{5}$$

b. $\tan(x)$

$$\tan(x) = -\frac{3}{4}$$

c. $\cos\left(x - \frac{3\pi}{2}\right)$

$$\cos\left(x - \frac{3\pi}{2}\right) = -\sin(x) = -\frac{3}{5}$$

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Question 7

Given that $\tan(x) = \frac{12}{5}$ and $-\pi < x < -\frac{\pi}{2}$, find:

a. $\sin(x)$

$$\sin(x) = -\frac{12}{13}$$

b. $\cos(x)$

$$\cos(x) = -\frac{5}{13}$$

c. $\sin\left(x + \frac{7\pi}{2}\right)$

$$\sin\left(x + \frac{7\pi}{2}\right) = \sin\left(x + \frac{3\pi}{2}\right) = -\cos(x) = \frac{5}{13}$$

Question 8 Tech-Active.

Find $\sin(x) + \cos(y)$ if $\sin(y) = -\frac{8}{17}$ and $\cos(x) = \frac{12}{37}$, where $0 < x < \frac{\pi}{2}$ and $\frac{3\pi}{2} < y < 2\pi$.

$$\cos(y) = \frac{15}{17} \text{ and } \sin(x) = \frac{35}{37}. \text{ Therefore, } \sin(x) + \cos(y) = \frac{1150}{629}$$

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Sub-Section [3.3.3]: Finding Particular and General Solutions

Question 9



- a. Solve $2 \cos(x) = \sqrt{3}$ for $0 \leq x \leq 2\pi$.

$$\cos(x) = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

- b. Solve $\sqrt{2} \sin(x) = 1$ for $0 \leq x \leq 2\pi$.

$$\sin(x) = \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

- c. Solve $4 \tan(x) = 4$.

$$\tan(x) = 1$$

$$x = \frac{\pi}{4} + k\pi, \text{ where } k \in \mathbb{Z}.$$

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Question 10

- a. Solve $8 \cos(3x) = 4$ for x .

$$\cos(3x) = \frac{1}{2}$$

$$3x = 2k\pi \pm \frac{\pi}{3}$$

$$x = \frac{2k\pi}{3} \pm \frac{\pi}{9}, \text{ where } k \in \mathbb{Z}$$

- b. Solve $6 \sin\left(2x + \frac{\pi}{3}\right) = 3$ for $-2\pi \leq x \leq 2\pi$.

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$2x + \frac{\pi}{3} = x = -\frac{19\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}$$

$$x = -\frac{7\pi}{4}, -\frac{13\pi}{12}, -\frac{3\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$$

- c. Solve $2 \sin(3(x + \pi)) = 1$ for x .

$$3x + 3\pi = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

$$3x = -\frac{17\pi}{6} + 2k\pi, -\frac{13\pi}{6} + 2k\pi$$

$$x = -\frac{17\pi}{18} + \frac{2k\pi}{3}, -\frac{13\pi}{18} + \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z}.$$

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Question 11

- a. Find the general solution to $2 \cos\left(3x - \frac{\pi}{3}\right) = 1$, where $x \geq 0$.

$$3x - \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi, -\frac{\pi}{3} + 3k\pi$$

$$x = \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z}_{\geq 0} \text{ or } x = \frac{2\pi}{9} + \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z}_{\geq 0}.$$

- b. Find the general solution to $\sqrt{3} \tan\left(4x - \frac{\pi}{2}\right) = 3$, where $x \geq 0$.

$$4x - \frac{\pi}{2} = -\frac{2\pi}{3} = k\pi$$

$$x = -\frac{\pi}{24} + \frac{k\pi}{4}, \text{ where } k \in \mathbb{Z}_{\geq 1}.$$

- c. Find the general solution to $3 \sin\left(2x - \frac{\pi}{4}\right) = \frac{3}{2}$, where $x \leq 0$.

$$2x - \frac{\pi}{4} = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{24} + k\pi, \frac{13\pi}{24} + k\pi$$

$$x = -\frac{11\pi}{24} - k\pi, \text{ where } k \in \mathbb{Z}_{\geq 0} \text{ or } x = \frac{5\pi}{24} - k\pi, \text{ where } k \in \mathbb{Z}_{\geq 1}.$$

Question 12 Tech-Active.

Find the general solution to $2 \cos\left(3x + \frac{\pi}{4}\right) = \sqrt{2}$.

$$x = -\frac{\pi}{6} + \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z} \text{ or } x = \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z}$$

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Sub-Section: The 'Final Boss'

Question 13

Let $f(x) = \sin(2x)$.

- a. Write the function $g(x) = f\left(x - \frac{\pi}{4}\right) + f\left(x + \frac{3\pi}{4}\right) + f\left(x + \frac{9\pi}{4}\right)$ in terms of $\cos(2x)$ only.

$$\begin{aligned} g(x) &= \sin\left(2x - \frac{\pi}{2}\right) + \sin\left(2x + \frac{3\pi}{2}\right) + \sin\left(2x + \frac{9\pi}{2}\right) \\ &= -\cos(2x) - \cos(2x) + \cos(2x) \\ &= -\cos(2x) \end{aligned}$$

- b. Find all values of a such that:

i. $f(x - a) = f(x)$

$$\sin(2x - 2a) = \sin(2x) \rightarrow 2a = 2k\pi \Rightarrow a = k\pi \text{ for } k \in \mathbb{Z}$$

ii. $f(x + a) = \cos(2x)$

$$f(x + a) = \sin(2x + 2a) = \cos(2x) \Rightarrow 2a = \frac{\pi}{2} + 2k\pi \Rightarrow a = \frac{\pi}{4} + k\pi \text{ for } k \in \mathbb{Z}$$

- c. Find the general solution to $2f\left(x - \frac{\pi}{8}\right) = \sqrt{2}$.

$$\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$2x - \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi$$

$$2x = \frac{\pi}{2} + 2k\pi, \pi + 2k\pi$$

$$x = \frac{\pi}{4} + k\pi \quad \text{or} \quad x = \frac{\pi}{2} + k\pi, \text{ where } k \in \mathbb{Z}.$$

- d. Solve the equation $2\sin^2(2x) + \sin(2x) = 1$ for $x \in [-\pi, \pi]$.

$$2\sin^2(2x) + \sin(2x) - 1 = 0$$

$$\text{Let } a = \sin(2x)$$

$$(2a - 1)(a + 1) = 0$$

$$a = -1, \frac{1}{2}$$

$$\sin(2x) = -1 \Rightarrow 2x = -\frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = -\frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sin(2x) = \frac{1}{2} \Rightarrow 2x = -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12}.$$

Therefore,

$$x = -\frac{11\pi}{12}, -\frac{7\pi}{12}, -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$$

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Section B: Supplementary Questions

Sub-Section [3.3.1]: Finding Exact Values of Circular Functions



Question 14



Find the exact value of:

a. $\sin\left(\frac{\pi}{6}\right)$

$$\frac{1}{2}$$

b. $\cos\left(\frac{\pi}{4}\right)$

$$\frac{1}{\sqrt{2}}$$

c. $\tan\left(\frac{\pi}{6}\right)$

$$\frac{1}{\sqrt{3}}$$

Question 15



Find the exact value of:

a. $\sin\left(\frac{7\pi}{6}\right)$

$$-\frac{1}{2}$$

b. $\cos\left(\frac{4\pi}{3}\right)$

$$-\frac{1}{2}$$

c. $\tan\left(\frac{7\pi}{4}\right)$

$$-1$$


Question 16

Find the exact value of:

a. $\sin\left(\frac{19\pi}{6}\right)$

$$-\frac{1}{2}$$

b. $\cos\left(\frac{25\pi}{6}\right)$

$$\frac{\sqrt{3}}{2}$$

c. $\tan\left(\frac{17\pi}{4}\right)$

$$1$$

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Sub-Section [3.3.2]: Applying the Pythagorean Identity and Symmetrical Properties

Question 17



Given that $\sin(x) = \frac{5}{13}$ and $0 < x < \frac{\pi}{2}$, find:

a. $\cos(x)$

$$\cos(x) = \frac{12}{13}$$

b. $\tan(x)$

$$\tan(x) = \frac{5}{12}$$

c. $\sin\left(x + \frac{\pi}{2}\right)$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x) = \frac{12}{13}$$

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Question 18

Given that $\sin(x) = \frac{7}{25}$ and $\frac{\pi}{2} < x < \pi$, find:

a. $\cos(x)$

$$\cos(x) = -\frac{24}{25}$$

b. $\tan(x)$

$$\tan(x) = -\frac{7}{24}$$

c. $\cos\left(x - \frac{3\pi}{2}\right)$

$$\cos\left(x - \frac{3\pi}{2}\right) = -\sin(x) = -\frac{7}{25}$$

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Question 19

Given that $\tan(x) = \frac{8}{15}$ and $-\pi < x < -\frac{\pi}{2}$, find:

a. $\sin(x)$

$$\sin(x) = -\frac{8}{17}$$

b. $\cos(x)$

$$\cos(x) = -\frac{15}{17}$$

c. $\sin\left(x + \frac{7\pi}{2}\right)$

$$\sin\left(x + \frac{7\pi}{2}\right) = \sin\left(x + \frac{3\pi}{2}\right) = -\cos(x) = \frac{15}{17}$$

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Sub-Section [3.3.3]: Finding Particular and General Solutions

Question 20



- a. Solve $3 \cos(x) = \frac{3}{2}$ for $0 \leq x \leq 2\pi$.

$$\cos(x) = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

- b. Solve $2 \sin(x) = \sqrt{2}$ for $0 \leq x \leq 2\pi$.

$$\sin(x) = \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

- c. Solve $5 \tan(x) = 5$.

$$\tan(x) = 1$$

$$x = \frac{\pi}{4} + k\pi, \text{ where } k \in \mathbb{Z}.$$

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Question 21

- a. Solve $6 \cos(2x) = 3$ for x .

$$\cos(2x) = \frac{1}{2}$$

$$2x = 2k\pi \pm \frac{\pi}{3}$$

$$x = k\pi \pm \frac{\pi}{6}, \text{ where } k \in \mathbb{Z}$$

- b. Solve $4 \sin\left(3x + \frac{\pi}{4}\right) = 2$ for $-2\pi \leq x \leq \pi$.

$$\sin\left(3x + \frac{\pi}{4}\right) = \frac{1}{2}$$

$$3x + \frac{\pi}{4} = -\frac{31\pi}{6}, -\frac{23\pi}{6}, -\frac{19\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$3x = -\frac{65\pi}{12}, -\frac{49\pi}{12}, -\frac{41\pi}{12}, -\frac{25\pi}{12}, -\frac{17\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}$$

$$x = -\frac{65\pi}{36}, -\frac{49\pi}{36}, -\frac{41\pi}{36}, -\frac{25\pi}{36}, -\frac{17\pi}{36}, -\frac{\pi}{36}, \frac{7\pi}{36}, \frac{23\pi}{36}, \frac{31\pi}{36}$$

- c. Solve $2 \sin(4(x + \pi)) = 1$ for x .

$$4x + 4\pi = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

$$4x = -\frac{23\pi}{6} + 2k\pi, -\frac{19\pi}{6} + 2k\pi$$

$$x = -\frac{23\pi}{24} + \frac{k\pi}{2}, -\frac{19\pi}{24} + \frac{k\pi}{2}, \text{ where } k \in \mathbb{Z}.$$

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Question 22

- a. Find the general solution to $2 \cos\left(2x - \frac{\pi}{4}\right) = 1$, where $x \geq 0$.

$$2x - \frac{\pi}{4} = \frac{\pi}{3} + 2k\pi, -\frac{\pi}{3} + 2k\pi$$

$$2x = \frac{7\pi}{12} + 2k\pi, -\frac{\pi}{12} + 2k\pi$$

$$x = \frac{7\pi}{24} + k\pi, \text{ where } k \in \mathbb{Z}_{\geq 0} \text{ or } x = \frac{11\pi}{24} + k\pi, \text{ where } k \in \mathbb{Z}_{\geq 0}.$$

- b. Find the general solution to $\sqrt{3}\tan\left(5x - \frac{\pi}{3}\right) = 3$, where $x \geq 0$.

$$5x - \frac{\pi}{3} = -\frac{2\pi}{3} + k\pi$$

$$x = -\frac{\pi}{15} + \frac{k\pi}{5}, \text{ where } k \in \mathbb{Z}_{\geq 1}.$$

- c. Find the general solution to $4 \sin\left(3x - \frac{\pi}{6}\right) = 2$, where $x \leq 0$.

$$3x - \frac{\pi}{6} = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{9} + \frac{2k\pi}{3}, \frac{\pi}{3} + \frac{2k\pi}{3}$$

$$x = \frac{\pi}{9} - \frac{2k\pi}{3}, \text{ or } x = \frac{\pi}{3} - \frac{2k\pi}{3}, \text{ where } k \in \mathbb{Z}_{\geq 1}$$



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