Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Exponentials, Logarithms & Its Exam Skills [3.1]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 36
Supplementary Questions	Pg 37 - Pg 52

Section A: Compulsory Questions

Sub-Section [3.1.1]: Apply Index Law and Log Law to Simplify Expressions

Question 1

Apply exponential and log laws to simplify the following expressions.

a. $\log_2(7) + \log_2(5)$

b. $\log_2\left(\frac{5}{7}\right) - \log_2\left(\frac{1}{7}\right)$

c. $x^3 \times x^2$

Space for Personal Notes

Question 2

Apply exponential and log laws to simplify the following expressions.

a. $\frac{a^{5x} \times a^{2x+1}}{a^{3x}}$

b. $\frac{8a^2b^3 \times 4a^4b^2}{6a^3b^2}$

c. $\log_4(32) - \log_9(27) + \log_4\left(\frac{1}{64}\right)$

Space for Personal Notes

Question 3

Apply exponential and log laws to simplify the following expressions.

a. $\log_e\left(\frac{a^3b^5}{c^2}\right) - \frac{1}{3}\log_e(a^6b^3) + \log_e(\mathrm{c})$

b. $\log_e\left(\frac{xy^2}{z}\right) + \log_e\left(\frac{z^2}{xy}\right) - \frac{1}{2}\log_e(y^2z^2)$

c. $\frac{16^{\frac{x}{2}} \times 2^{x-3}}{8^{x+4} \times 4^{3x}}$

Space for Personal Notes

Question 4 Tech-Active.

a. $\log_5\left(\frac{1}{25}\right) + \log_2(1024) + \log_3(27)$

b. $\frac{3^5 \times 2^{8x}}{3^2 \times 4^{3x}}$

Space for Personal Notes

Sub-Section [3.1.2]: Solve Basic Exponential and Logarithmic Equations and Inequalities

Question 5

Solve for x in each of the following equations.

a. $4^{2x-4} = 1$

b. $\frac{3^{x-2}}{3^{4-x}} = 9$

c. $\log_2(8) + 3\log_2(4) = x - 3$

Space for Personal Notes

Question 6

Solve for x in each of the following equations.

a. $\log_x\left(\frac{1}{32}\right) = 5$

b. $2^{2x+5} > 16$

c. $8^{x+2} - 34 \cdot 8^x - 64 < 0$

Space for Personal Notes

Question 7

Solve for x in each of the following equations.

a. $2\log_{10}(x+1) - \log_{10}(2x+1) = \log_{10}(5x+8) - 1$

b. $6^{x+1} - 14 \cdot 6^x + 32 > 0$

c. $(27 \cdot 3^x)^x \cdot 3^{4x} = 27^x \cdot 3^{\frac{1}{4}}$

Space for Personal Notes

Question 8 Tech-Active.

Solve $(8 \cdot 2^x)^x \cdot 4^{4x} = 2^x \cdot 2^{\frac{1}{4}}$ for x.

Question 9 Tech-Active.

Solve $2\log_e(x+1) - \log_e(x+2) = \log_e(x-3)$ for x.

Space for Personal Notes

Sub-Section [3.1.3]: Solve Hidden Quadratics Within Exponential Equations

Question 10

Solve $2^{2x} - 5(2^x) = -6$ for x.

Question 11

Solve $7^{2x+1} = 8 \cdot 7^x - 1$ for x.

Space for Personal Notes

Question 12

Solve $4^x + \left(\frac{1}{2}\right)^{-x} = 5(2^x) + 2$ for x.

Question 13 Tech-Active.

Solve $4^x + \left(\frac{1}{2}\right)^{-2x} = 3(2^x) + 4$ for x.

Space for Personal Notes

Sub-Section [3.1.4]: Solving Logarithmic Equations using Log Laws

Question 14

Solve $\log_2(x-1) + \log_2(x) = \log_2(6)$ for x.

Question 15

Solve $2\log_e(x+2) - \log_e(x) = \log_e(2x+2)$ for x.

Space for Personal Notes

Question 16

Solve $2\log_e(1-x) + \log_e(2) = \log_e(3-x)$ for x.

Question 17 Tech-Active.

Solve $2\log_e(x+3) - 3\log_e(x+1) = \log_e(2x)$ for x.

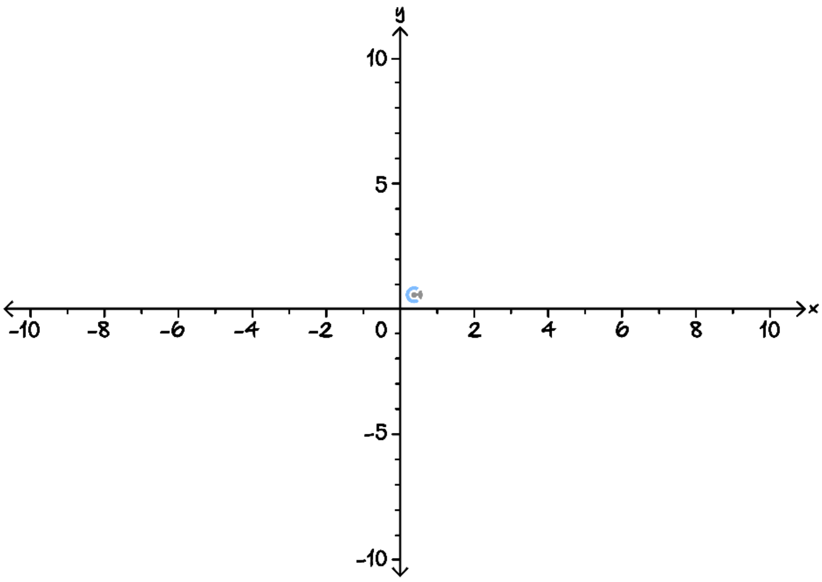
Space for Personal Notes

Sub-Section [3.1.5]: Graph Exponentials and Logarithms

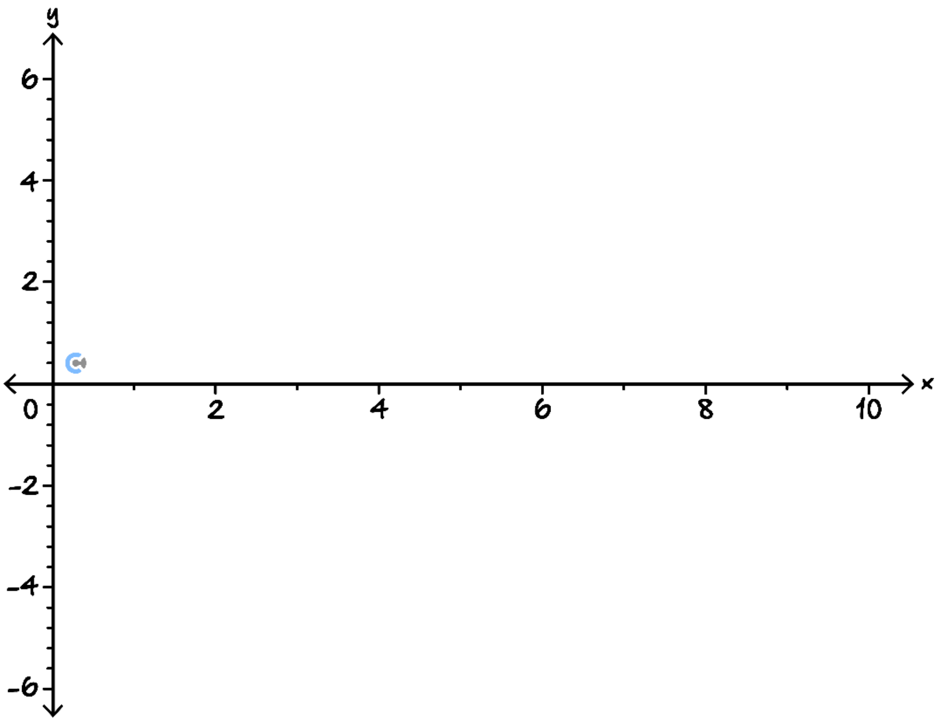
Question 18

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and asymptotes.

a. $y = 2^x - 3$



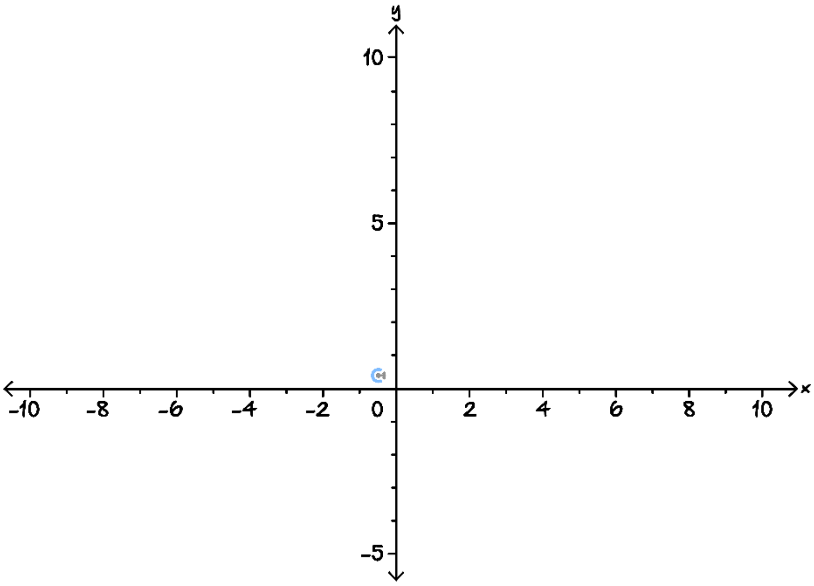
b. $y = \log_2(x-1) - 1$



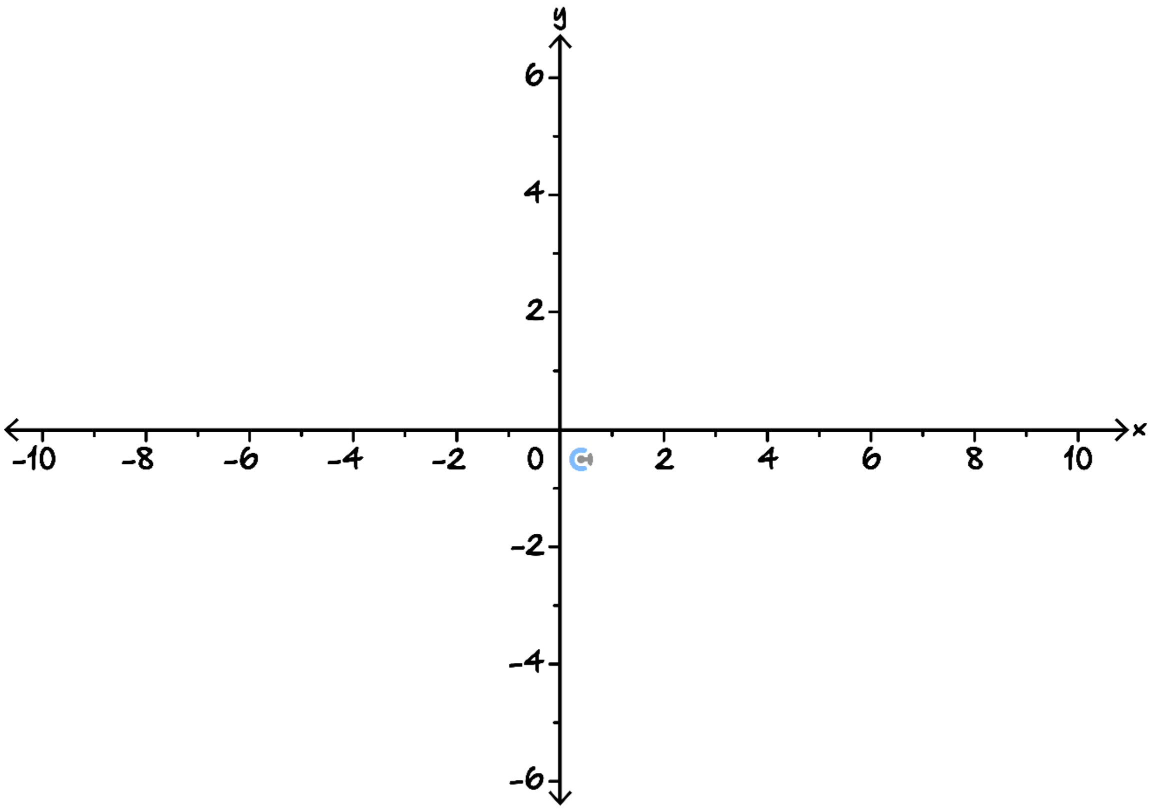
Question 19

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and asymptotes.

a. $y = \frac{1}{2} \cdot 2^{x+1} - 2$



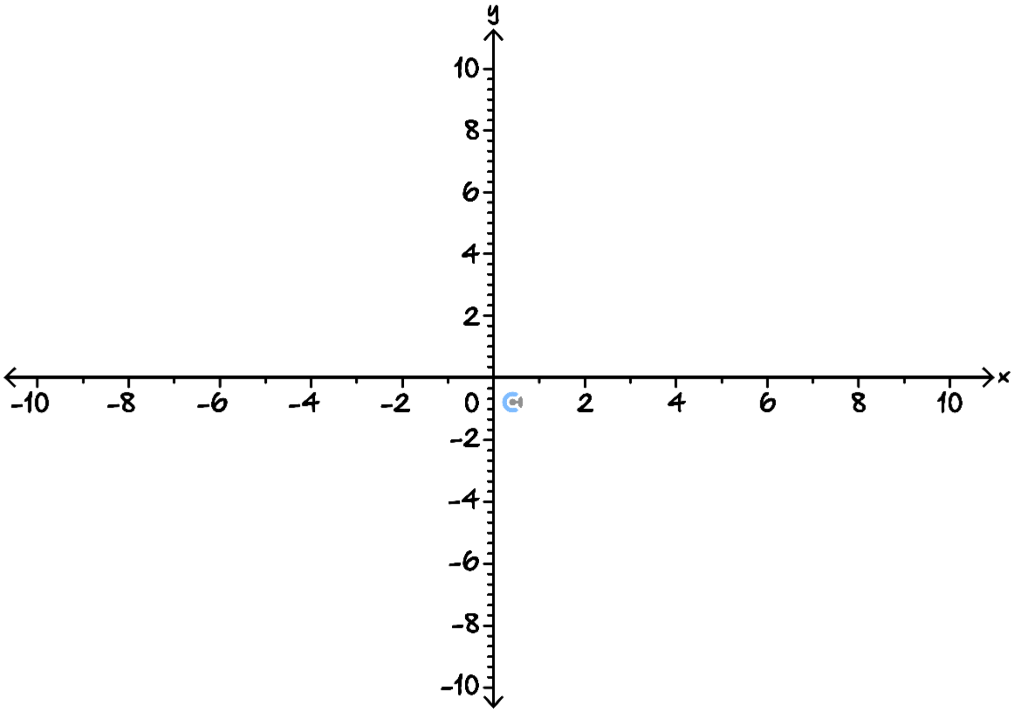
b. $y = 2\log_e\left(\frac{x}{4} + 1\right) + 2$



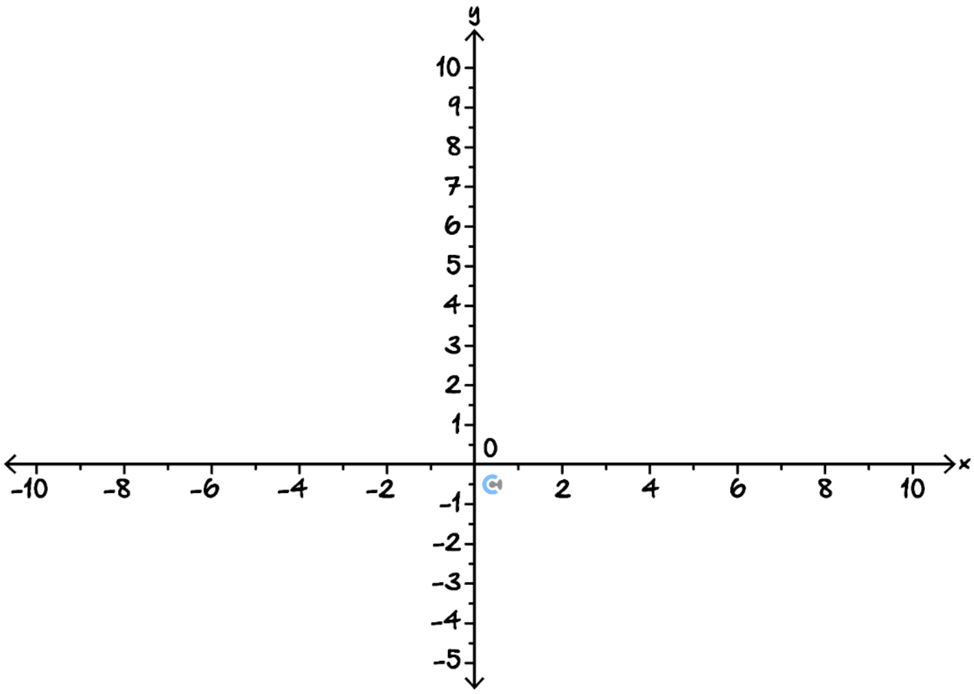
Question 20

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and asymptotes.

a. $y = -\frac{1}{3} \cdot \left(\frac{1}{3}\right)^{\frac{1}{2}x} + 8$



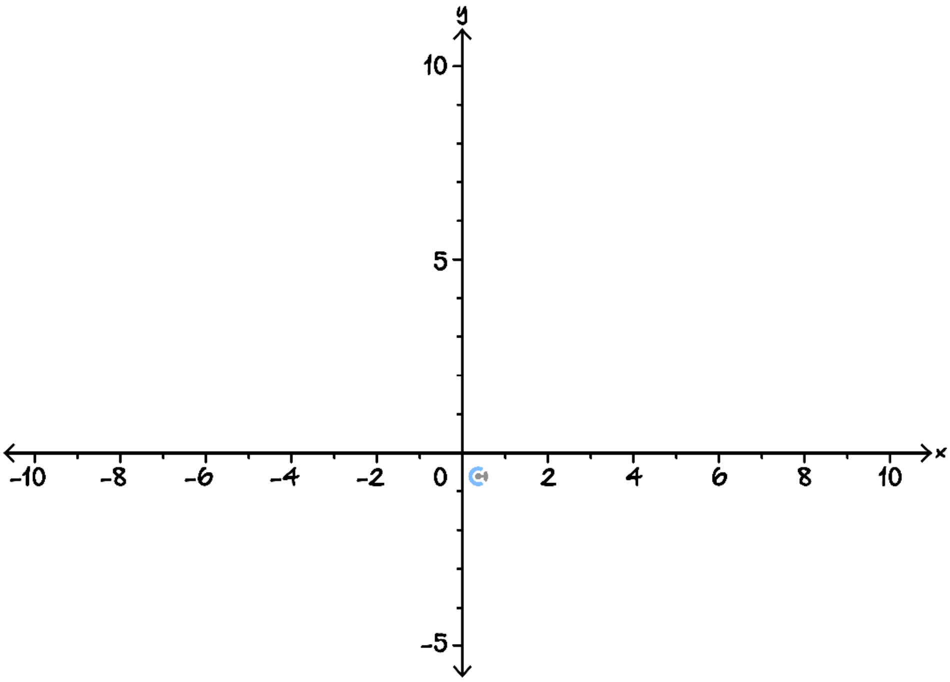
b. $y = -2\log_e(3 - 2x) + 4$



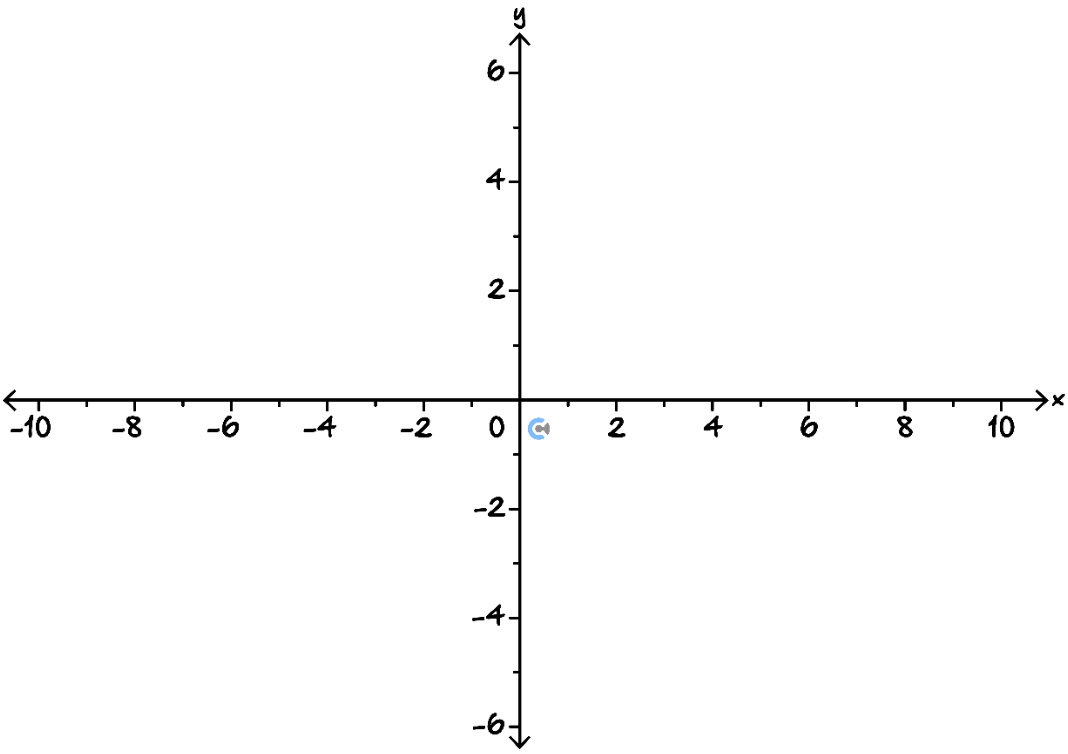
Question 21 Tech-Active.

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and asymptotes.

a. $y = 2^{x-1} + 1$



b. $y = 2\log_e\left(1 - \frac{x}{4}\right) + 1$



Sub-Section [3.1.6]: Apply Exponential Functions to Modelling

Question 22 Tech-Active.

A scientist is studying the growth of a bacteria population in a controlled environment. The population of bacteria, $P(t)$, at time t (in hours) is modelled by the equation:

$$P(t) = 500e^{0.3t}$$

Where $P(t)$ represents the number of bacteria present at time t.

a. Find the initial number of bacteria in the environment.

b. Determine the number of bacteria present after 5 hours. Give your answer correct to the nearest whole number.

c. Find the time, to the nearest minute, when the bacterial population reaches 5000.

Space for Personal Notes

Question 23 Tech-Active.

A certain radioactive substance decays over time according to the function:

$$M(t) = M_0 e^{-0.02t}$$

Where M_0 is the initial mass of the substance, and $M(t)$ is the remaining mass (in grams) after t years.

a. If the initial mass of the substance is 200 grams, determine the amount of the substance remaining after 10 years. Round your answer to two decimal places.

b. Find the half-life of the substance, correct to two decimal places (half-life is the time it takes for the mass to decay to half its original value).

c. Determine the time required for the mass of the substance to decay to 20 grams. Give your answer correct to two decimal places.

Question 24 Tech-Active.

The population of a newly discovered fish species in a lake is modelled by:

$$P(t) = P_0 e^{kt}$$

Where P_0 is the initial population, and k is a constant.

a. After two years the population of fish in the lake was 1000 and after 5 years, the population had grown to 1500. Find the values of P_0 and k, correct to four decimal places.

b. After how many years will the population reach 5000? Give your answer correct to one decimal place.

c. If the lake has a maximum sustainable population of $10{,}000$ fish, find the percentage of the lake's capacity that will be occupied after 8 years. Give your answer to one decimal place.

Space for Personal Notes

Sub-Section: Final Boss

Question 25

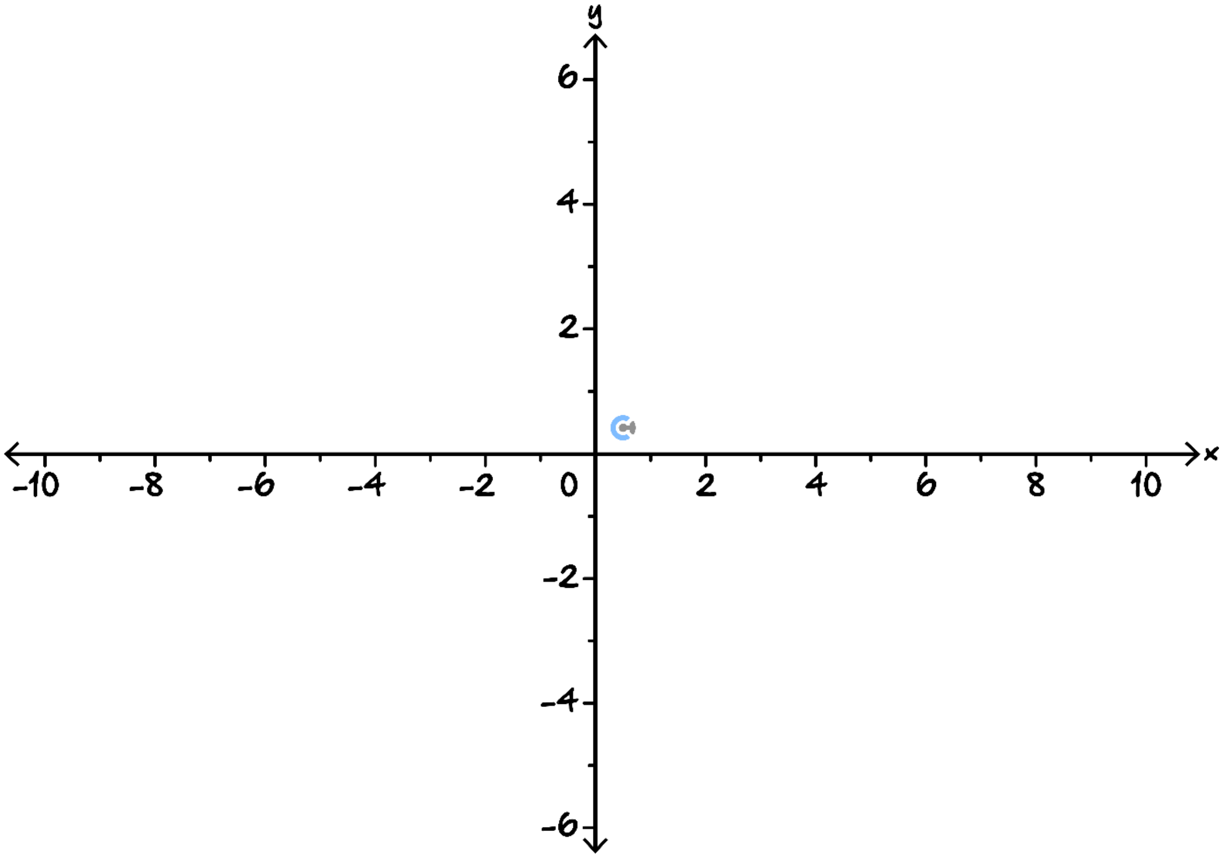
Consider the function $f: (a, \infty) \to \mathbb{R}, f(x) = \log_2(2x + 4) - 1$.

a. Find the smallest value of a.

b. Solve $f(x) = 2$.

c. Solve $2f(x) = \log_2(x + 2) + 3$.

d. Sketch the function f on the axis below. Label all key features, including axes, intercepts, and asymptotes.



e. Define f^{-1}, the inverse function of f, where f is defined on its maximal domain. Give the rule $f^{-1}(x)$ in the form $f^{-1}(x) = a^x - b$, for positive integers a and b.

f. Find an integer solution to the equation $f^{-1}(x) = x$.

g. f and f^{-1} have a point of intersection at approximately $(-1.69, -1.69)$. Sketch f^{-1} on the same axes as the sketch of f. Label all axes, intercepts and asymptotes.

h. Solve $f^{-1}(x) > 3$.

Space for Personal Notes

Sub-Section: Exam 1

Question 26 (3 marks)

Solve the following equation for x.

$$3^{2x-1} - 4 \times 3^{x} + 9 = 0$$

Space for Personal Notes

Question 27 (3 marks)

Solve the following equation for x.

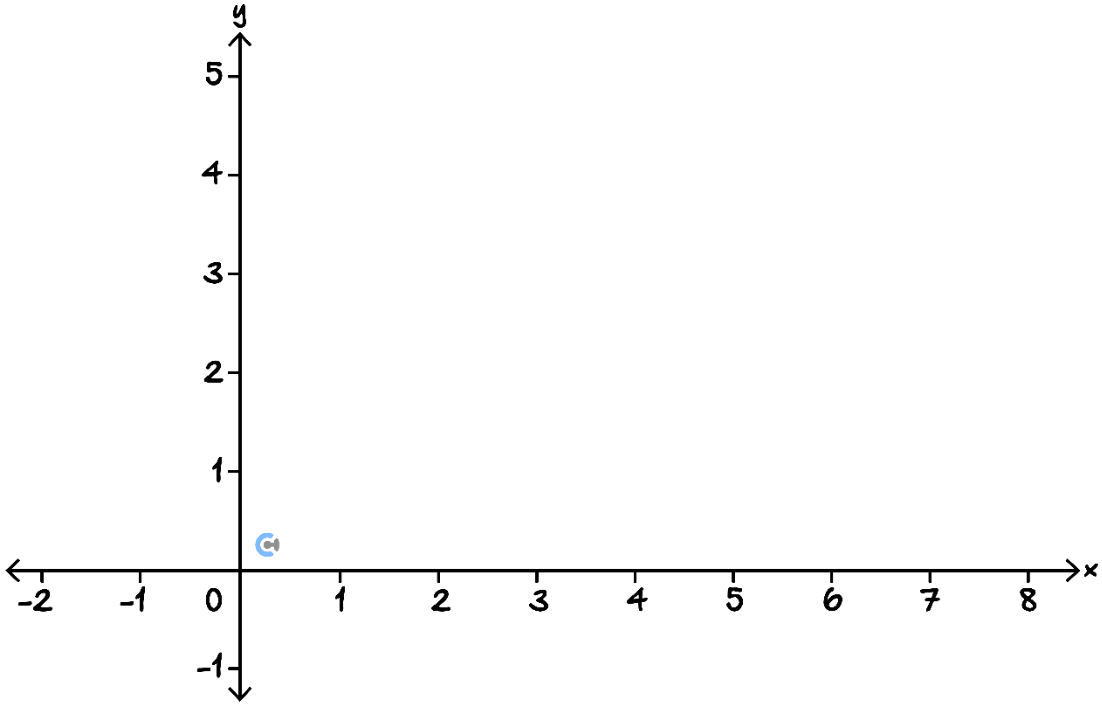
$$\log_3(x-3)+\log_3(x+5)=2$$

Space for Personal Notes

Question 28 (4 marks)

A function f is defined by the rule $f(x) = 4 - 3e^{-x}$ for $x \geq 0$.

a. Sketch the graph of f on the axes below, clearly labelling any axes intersects with their coordinates, and any asymptotes with their equations. (2 marks)



b. Define the inverse function f^{-1}. (2 marks)

Space for Personal Notes

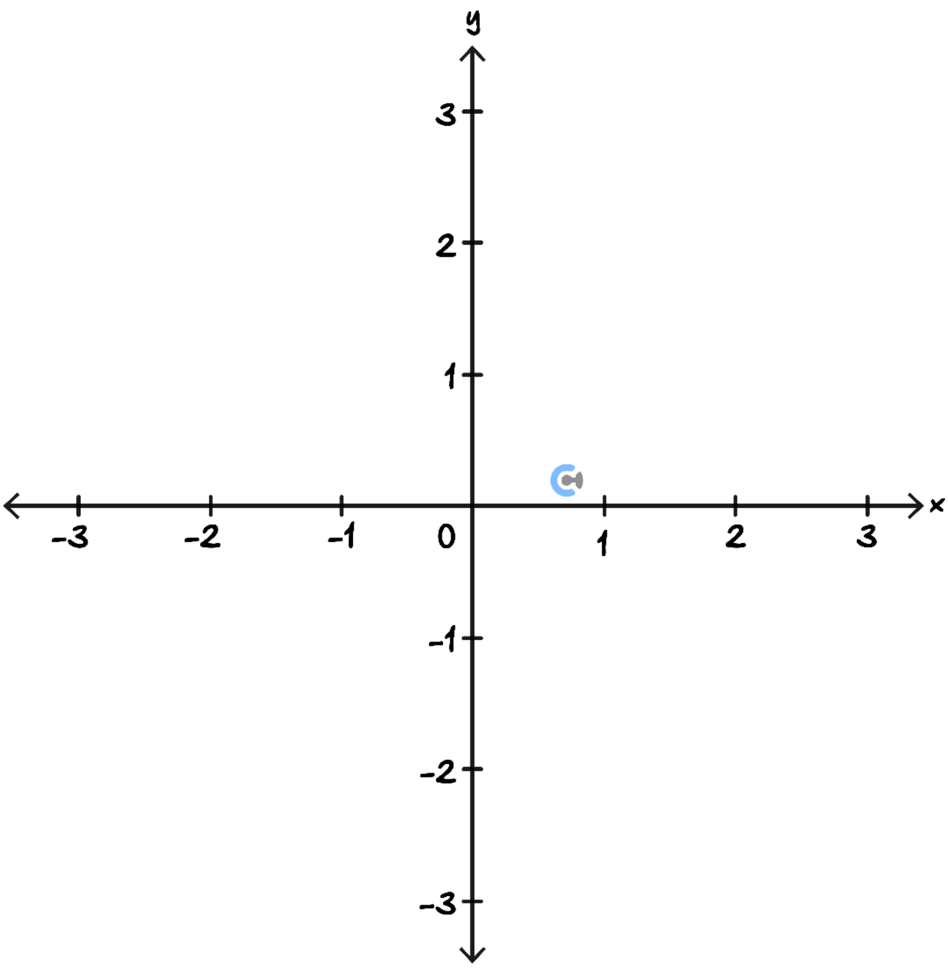
Question 29 (9 marks)

Consider the following function:

$$f: (a, \infty) \to \mathbb{R}, f(x) = \log_2(x + 2) - 1$$

a. State the smallest value of a. (1 mark)

b. Sketch the function on the axis below, labelling all key features including axes, intercepts and asymptotes. (2 marks)



c. Define the rule for the function $f^{-1}(x)$. (2 marks)

d. Sketch $f^{-1}(x)$ on the same axis above, labelling all key features including the axes, intercepts and asymptotes. (2 marks)

e. Define the function $f^{-1}(f(x))$. (2 marks)

Space for Personal Notes

Sub-Section: Exam 2

Question 30 (1 mark)

If $x = 4$ is a solution of the equation $\log_e(ax + 2) = 3$, then the exact value of a is:

A. $\frac{\log_e(3)-2}{4}$

B. $\frac{e}{4}$

C. $\frac{e^3}{4} - 2$

D. $\frac{e^3-2}{4}$

Question 31 (1 mark)

Evaluate the expression: $e^{2\log_e(x)-\log_e(2x)}$

A. $2\log_e\left(\frac{1}{2}\right)$

B. 0

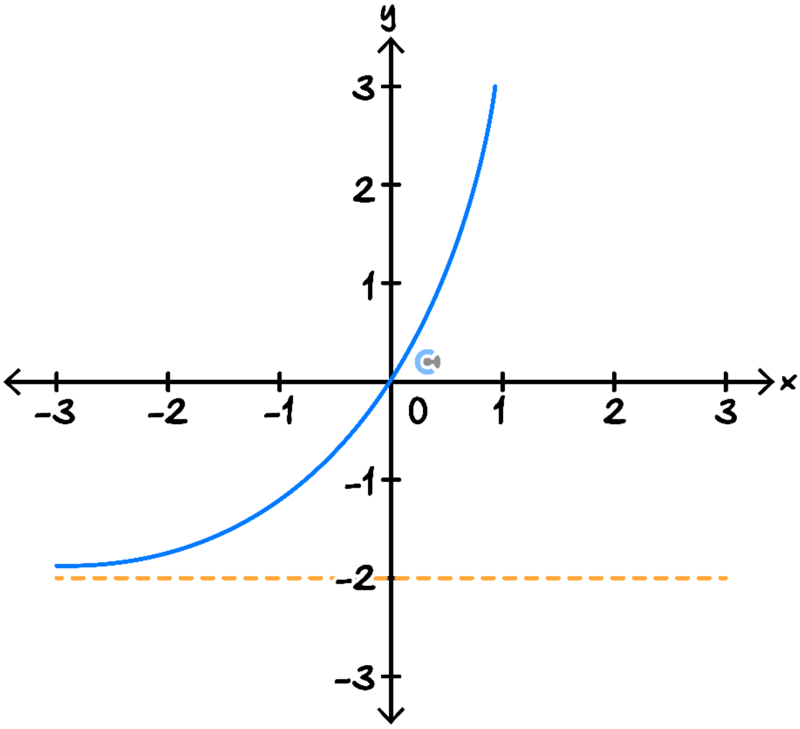
C. $\frac{x}{2}$

D. $x^2 - 2x$

Space for Personal Notes

Question 32 (1 mark)

The graph of the function with equation $y = Ae^x + B$, where A and B are constants, is shown below.



The values of A and B, respectively, are:

A. $A = -2, B = 0$

B. $A = 0, B = -2$

C. $A = 2, B = -2$

D. $A = -2, B = 2$

Question 33 (1 mark)

If $5e^{ax} = 2$, then x is equal to:

A. $0.4 \log_e(a)$

B. $a \log_e(0.4)$

C. $\frac{\log_e(0.4)}{a}$

D. $\frac{\log_e(2)}{a \log_e(5)}$

Question 34 (1 mark)

If $2\log_e(x) - \log_e(x+2) = 1 + \log_e(y)$, then y is equal to:

A. $\frac{2x}{x+2} - 1$

B. $\frac{x^2}{x+2}$

C. $\frac{2x}{x+2}$

D. $\frac{x^2}{e(x+2)}$

Space for Personal Notes

Question 35 (8 marks)

A colony of viruses is grown in a laboratory by putting a number of the viruses in a dish of nutrient. At first, the viruses multiply quickly, but later the numbers decline, approaching a long-term stable population. The number of viruses in the dish, N million, at time t days may be modelled by the formula:

$$N(t) = \frac{1}{2(t-1)^2} - e^{-6t} + 1, t \geq 0$$

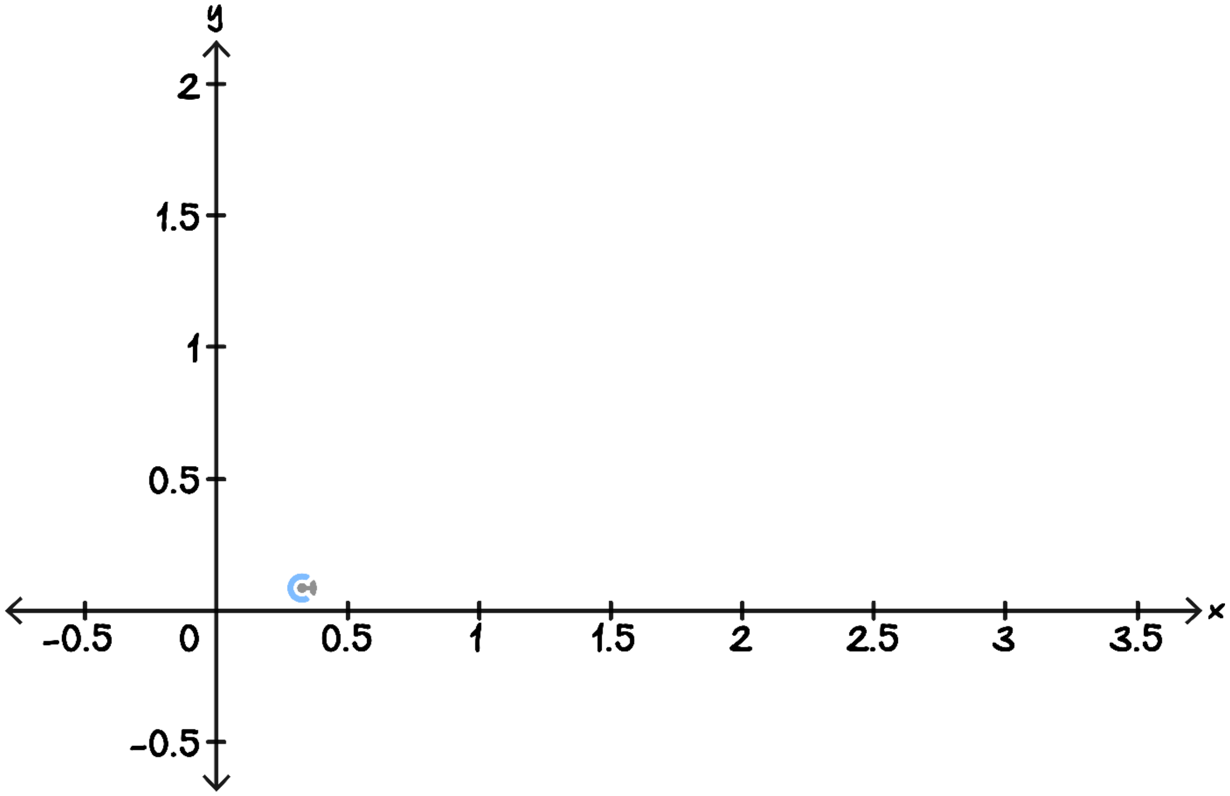
Use this model to answer the following questions.

a.

i. How many viruses were put in the dish initially? (1 mark)

ii. What value does $N(t)$ approach as time increases? (1 mark)

iii. On the axes below, sketch the graph of $y = N(t)$. Clearly label any t and y intercepts. Label any asymptote with its equation and the turning point with coordinates correct to two decimal places. (3 marks)



b.

i. Find $\frac{dN}{dt}$. (1 mark)

ii. Find the number of days after the experiment started, correct to two decimal places, when the maximum number of viruses is present in the dish. Also, find this maximum number, correct to the nearest thousand. (2 marks)

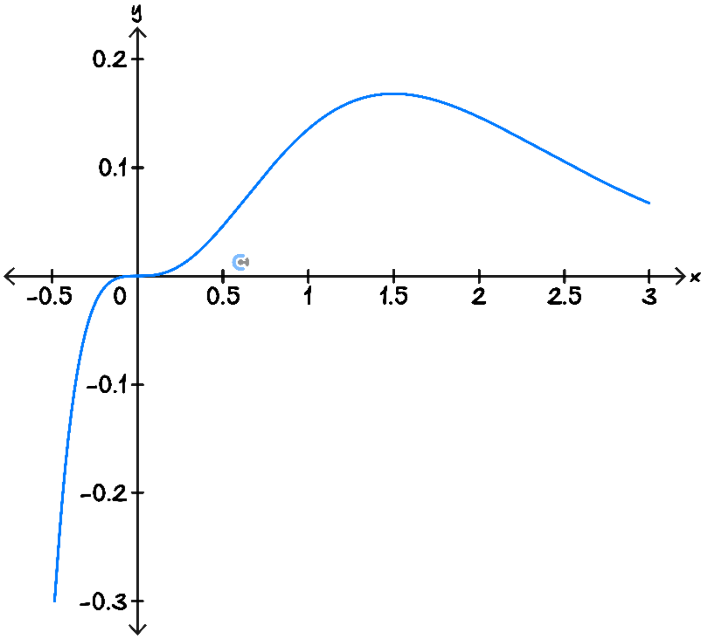
Space for Personal Notes

Question 36 (8 marks)

Consider the function $f: \mathbb{R} \to \mathbb{R}, f(x) = x^3 e^{-2x}$.

a. $f'(x)$ may be written as $f'(x) = e^{-2x}(ax^3 + bx^2)$ where a and b are real constants. Find the exact values of a and b. (1 mark)

b. The graph of $y = f(x)$ is as shown below.



Find the exact coordinates of the two stationary points and state their nature. (2 marks)

c.

i. Show that, at the point on the graph where $x = 1$, an equation of the tangent is $y = e^{-2}x$. (1 mark)

ii. Write down an equation of the tangent to the curve at the point $(0, 0)$. (1 mark)

iii. Show that the tangents of **parts i.** and **ii.** are the only two tangents to the curve which pass through the origin. (3 marks)

Space for Personal Notes

Section B: Supplementary Questions

Sub-Section [3.1.1]: Apply Index Law and Log Law to Simplify Expressions

Question 37

Apply exponential and log laws to simplify the following expressions.

a. $\log_3(4) + \log_3(9)$

b. $\log_5\left(\frac{10}{3}\right) - \log_5\left(\frac{1}{3}\right)$

c. $a^4 \times a^6$

Space for Personal Notes

Question 38

Apply exponential and log laws to simplify the following expressions.

a. $\frac{b^{6y+2} \times b^{3y-1}}{b^{4y+3}}$

b. $\frac{12x^3y^4 \times 6x^2y^5}{9x^4y^6}$

c. $\log_6(216) - \log_{10}(1000) + \log_6\left(\frac{1}{36}\right)$

Space for Personal Notes

Question 39

Apply exponential and log laws to simplify the following expressions.

a. $\log_e\left(\frac{x^4}{y^2z}\right) + 2\log_e(yz) - \log_e\left(\frac{x^2}{z^3}\right)$

b. $\log_e\left(\frac{ab^3}{c}\right) + \log_e\left(\frac{c^2}{ab}\right) - \frac{1}{2}\log_e(b^2c^2)$

c. $\frac{25^{\frac{y}{2}} \times 5^{y-4}}{125^{y+3} \times 25^{2y}}$

Space for Personal Notes

Sub-Section [3.1.2]: Solve Basic Exponential and Logarithmic Equations and Inequalities

Question 40

Solve for x in each of the following equations.

a. $5^{3x-6} = 1$

b. $\frac{4^{x+1}}{4^{5-x}} = 16$

c. $\log_3(9) + 2\log_3(3) = x - 4$

Space for Personal Notes

Question 41

Solve for x in each of the following equations.

a. $\log_y\left(\frac{1}{49}\right) = 2$

b. $3^{x+4} > 81$

c. $5^{x+1} - 24 \cdot 5^x - 25 < 0$

Space for Personal Notes

Question 42

Solve for x in each of the following equations.

a. $2\log_{10}(x+2) - \log_{10}(2x+1) = \log_{10}(2x+10) + 1$

b. $7^{x+2} - 20 \cdot 7^{x} + 49 > 50$

c. $(16 \cdot 2^{x})^{x} \cdot 2^{3x} = 16^{x} \cdot 2^{\frac{1}{3}}$

Sub-Section [3.1.3]: Solve Hidden Quadratics Within Exponential Equations

Question 43

Solve $3^{2x} - 7(3^x) = -12$ for x.

Question 44

Solve $5^{2x+1} = 6 \cdot 5^x - 1$ for x.

Space for Personal Notes

Question 45

Solve $9^x + \left(\frac{1}{3}\right)^{-x} = 7(3^x) + 2$ for x.

Space for Personal Notes

Sub-Section [3.1.4]: Solving Logarithmic Equations using Log Laws

Question 46

Solve $\log_3(x - 1) + \log_3(x) = \log_3(12)$ for x.

Question 47

Solve $2\log_e(x + 3) - \log_e(x) = \log_e(4x + 6)$ for x.

Space for Personal Notes

Question 48

Solve $2\log_e(2-x)+\log_e(3)=\log_e(5-x)$ for x.

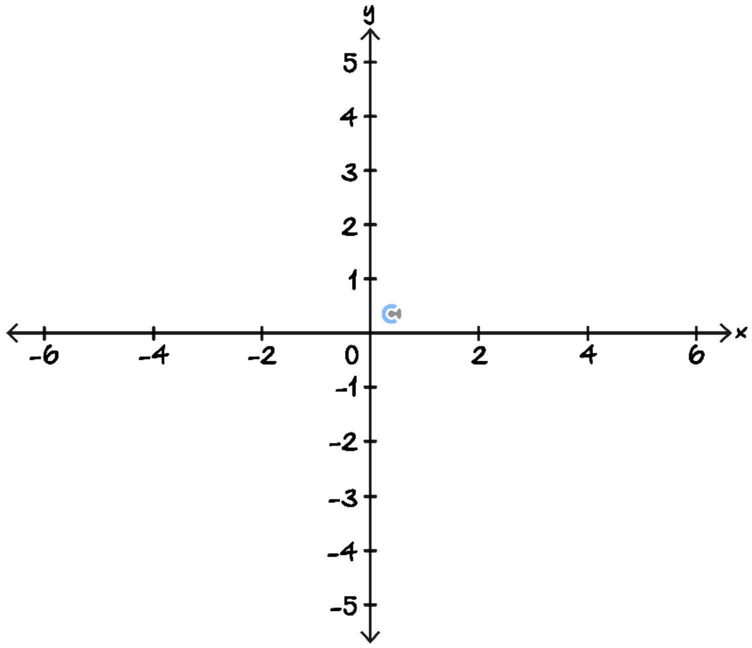
Space for Personal Notes

Sub-Section [3.1.5]: Graph Exponentials and Logarithms

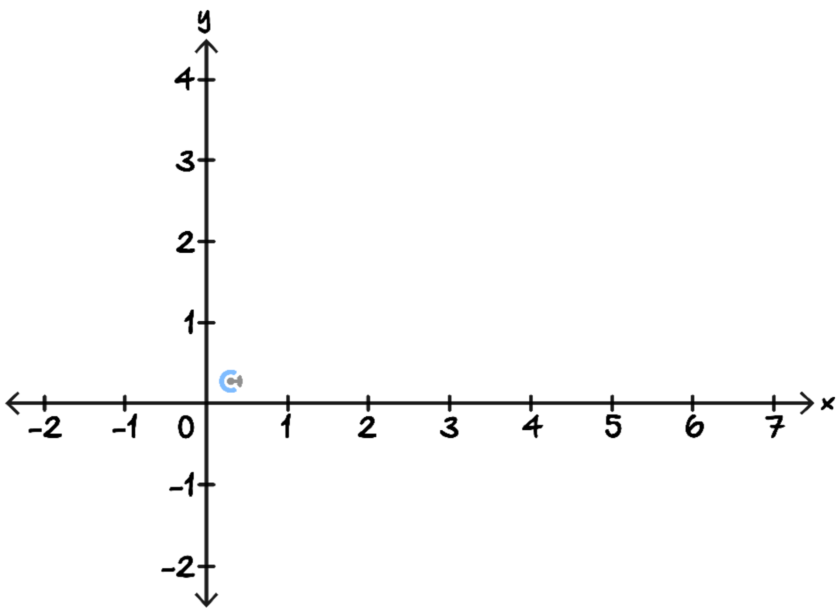
Question 49

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and equations of asymptotes.

a. $y = 2^x - 4$



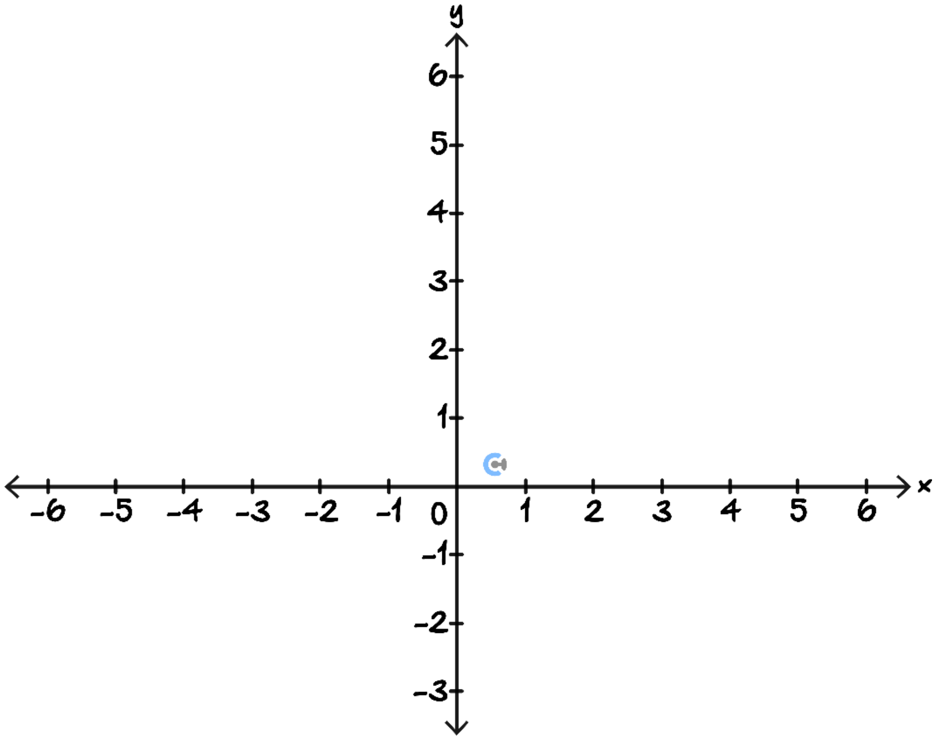
b. $y = \log_2(x-1) + 1$



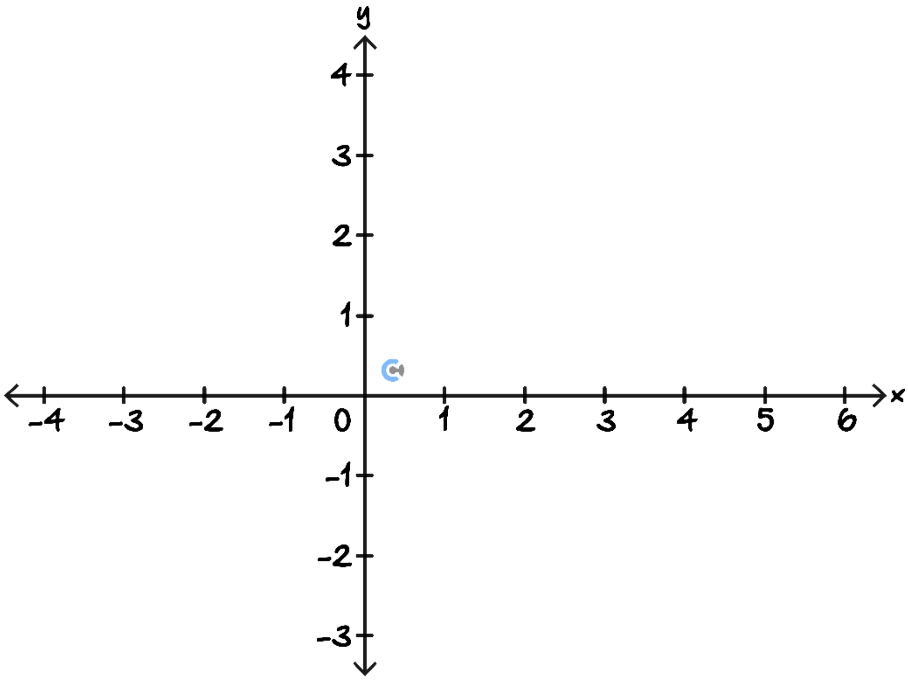
Question 50

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and equations of asymptotes.

a. $y = \frac{1}{3} \cdot 3^{x+2} - 3$



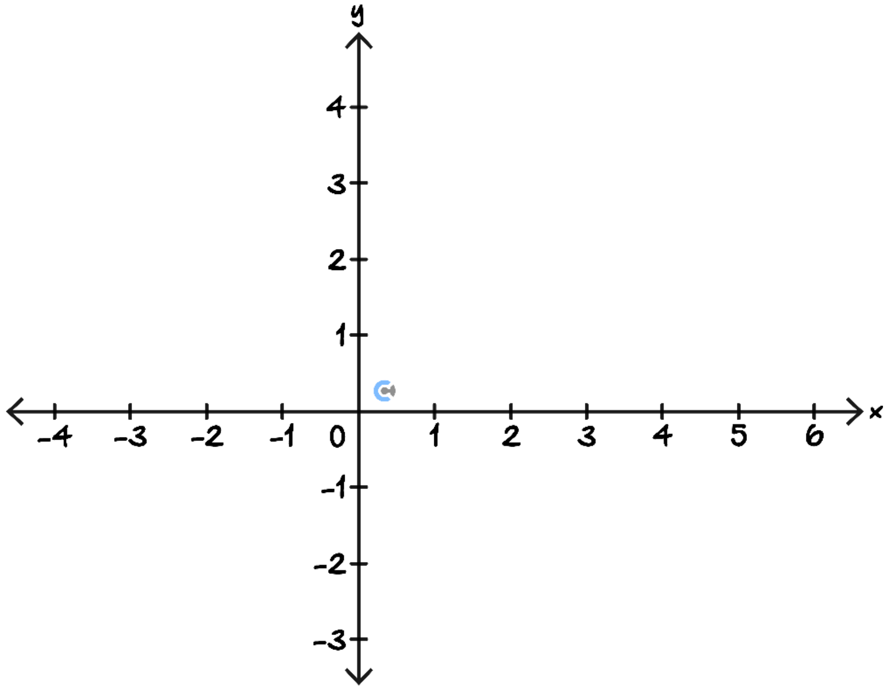
b. $y = 2\log_2(2x + 4) - 2$



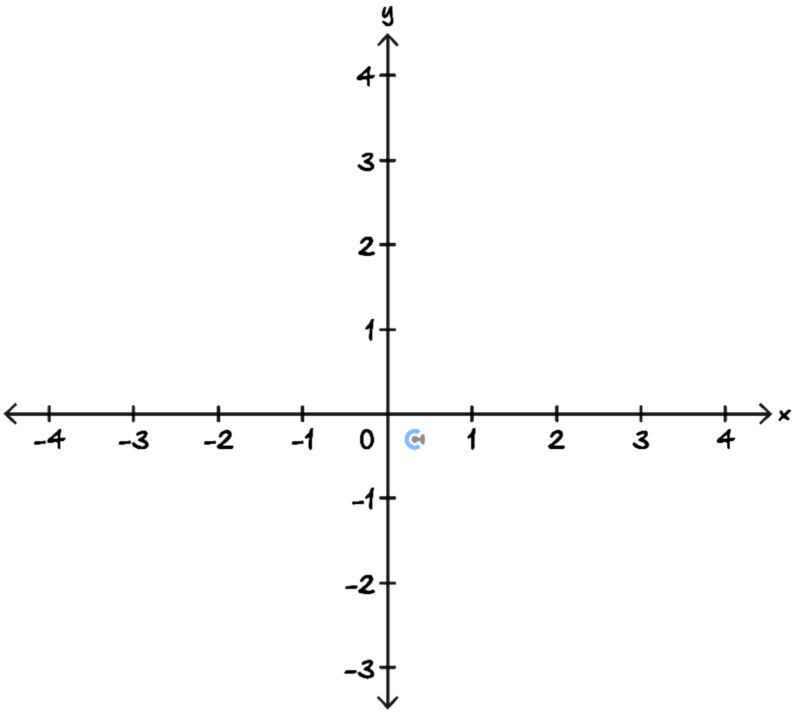
Question 51

Sketch the graph for the following functions. Label all key features, including axes, intercepts, and equations of asymptotes.

a. $y = -2\left(\frac{1}{2}\right)^{2x} + 4$



b. $y = -\frac{1}{2}\log_e(4 - 2x) + 1$



Sub-Section [3.1.6]: Apply Exponential Functions to Modelling

Question 52 Tech-Active.

A biologist is studying the growth of a certain tree species in a rainforest. The height of the tree, $H(t)$, in metres at time t (in years) is modelled by the equation:

$$H(t) = 2e^{0.25t}$$

Where $H(t)$ represents the height of the tree, in metres, at time t.

a. Find the initial height of the tree.

b. Determine the height of the tree after 6 years. Give your answer correct to two decimal places.

c. Find the time, to the nearest month, when the tree reaches a height of 20 metres.

Space for Personal Notes

Question 53 Tech-Active.

A scientist is investigating the decay of a certain chemical compound over time, which follows the function:

$$C(t) = C_0 e^{-0.05t}$$

Where C_0 is the initial concentration of the compound, and $C(t)$ is the concentration (in mg/L) after t hours.

a. If the initial concentration of the compound is $500\ mg/L$, determine the concentration remaining after 8 hours. Give your answer correct to two decimal places.

b. Find the half-life of the compound, correct to two decimal places (half-life is the time it takes for the concentration to decay to half its original value).

c. Determine the time required for the concentration to decay to $50\ mg/L$. Give your answer correct to two decimal places.

Space for Personal Notes

Question 54 Tech-Active.

The population of a rare bird species in a national park is modelled by:

$$P(t) = P_0 e^{rt}$$

Where P_0 is the initial population, and r is a growth rate constant.

a. After 3 years, the bird population is 850. After 7 years, the population grows to 1200. Find the values of P_0 and r, correct to four decimal places.

b. After how many years will the population reach 3000? Give your answer correct to one decimal place.

c. If the park has a maximum carrying capacity of 5000 birds, find the percentage of the park's capacity occupied after 10 years. Give your answer to one decimal place.

Space for Personal Notes