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VCE Mathematical Methods ¾

AOS 3 Revision [3.0]

SAC 3 Solutions

59 Marks. 15 Minutes Reading. 75 Minutes Writing.

Section A: SAC Questions (59 Marks)

Question 1 (24 marks)

Consider the following function:

$$h(x) = (e^{-x} + 1)(x^2 - 4)$$

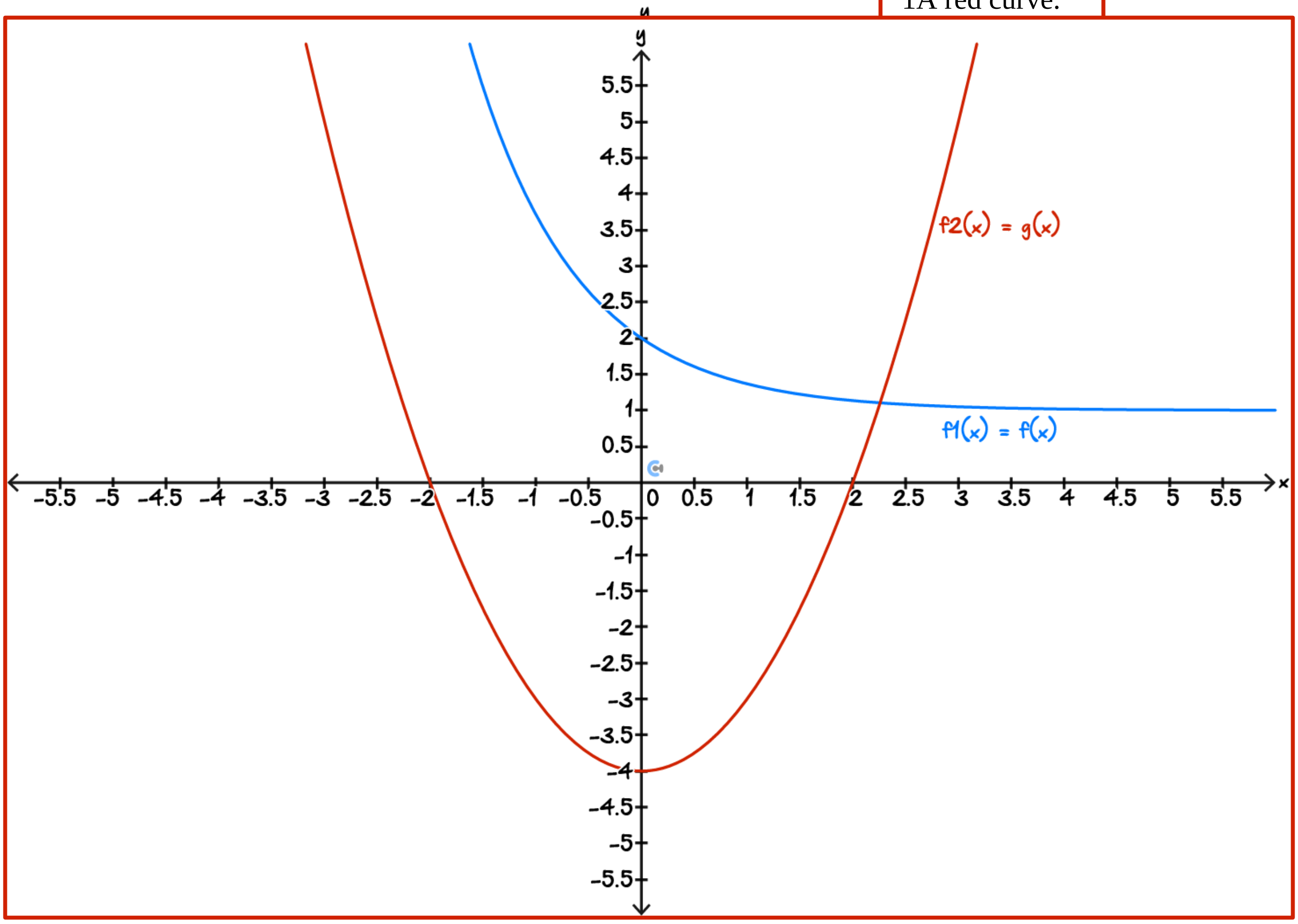
a. Express $h(x)$ as a product of two functions $f(x)$ and $g(x)$, where $g(x)$ is a non-linear polynomial. (1 mark)

$$f(x) = e^{-x} + 1$$
$$g(x) = x^2 - 4$$

1A
Must be exactly this.
Can't say $g(x) = 1$ or $g(x) = x + 2$ etc. (must be non-linear).

b. On the axes below, sketch the graphs of the 2 functions found in **part a.** No labels are required; however, the graphs must still be to scale. (2 marks)

1A blue curve.
1A red curve.



c.

i. Solve $f(x) = 0$, expressing your answer using set notation. (1 mark)

$\{x \in \mathbb{R} \mid f(x) = 0\} = \emptyset$ **1A**

ii. Solve $g(x) = 0$, expressing your answer using set notation. (1 mark)

$\{x \in \mathbb{R} \mid g(x) = 0\} = \{-2,2\}$ **1A**

iii. What does this imply for the set of values for x, such that $f(x) \times g(x) = 0$? Justify. (2 marks)

For a product of two real functions to be zero, at least one of them must be zero by null factor law. **1A**
Since $f(x) > 0$ for all x, $h(x) = 0$ only when $g(x) = 0$.
So, $h(x) = 0$ "at $x = -2,2$. **1A**

iv. On the axes above, label all the x-intercepts of $\boldsymbol{h(x)}$. (2 marks)

$(-2,0), (2,0)$

d.

i. Suppose one of the functions equals ± 1, what is the value of $h(x)$ in terms of $f(x)$ or $g(x)$? (1 mark)

If $f(x) = \pm 1$, then $h(x) = \pm g(x)$.
If $g(x) = \pm 1$, then $h(x) = \pm f(x)$.

ii. Solve $f(x) = 1$, expressing your answer using set notation. (1 mark)

$\{x \in \mathbb{R} \mid f(x) = 1\} = \emptyset$ **1A**

iii. Solve $g(x) = 1$, expressing your answer using set notation. (1 mark)

$\{x \in \mathbb{R} \mid g(x) = 1\} = \{-\sqrt{5}, \sqrt{5}\}$ **1A**

e.

i. Show that $h(x) \to \infty$ as $x \to \pm\infty$. Justify your reasoning using the end behaviour of each part of the function. (3 marks)

As $x \to \infty$:

$$e^{-x} \to 0 \Rightarrow e^{-x} + 1 \to 1 \text{ } \mathbf{1M}$$

$$x^2 - 4 \to \infty$$

$$h(x) \to (1)(\infty) = \infty$$

As $x \to -\infty$

$$e^{-x} \to \infty \Rightarrow e^{-x} + 1 \to \infty \text{ } \mathbf{1M}$$

$$x^2 - 4 \to \infty$$

$$h(x) \to \infty \cdot \infty = \infty$$

1M considering long term behaviour of parabola and product.

ii. Determine the coordinates of any stationary point correct to two decimal places. Classify and justify its type, referring to your answer in **part e. i.** (4 marks)

$$\text{solve}\left(\frac{d}{dx}(h(x))=0 \text{ and } y=h(x), x, y\right)$$

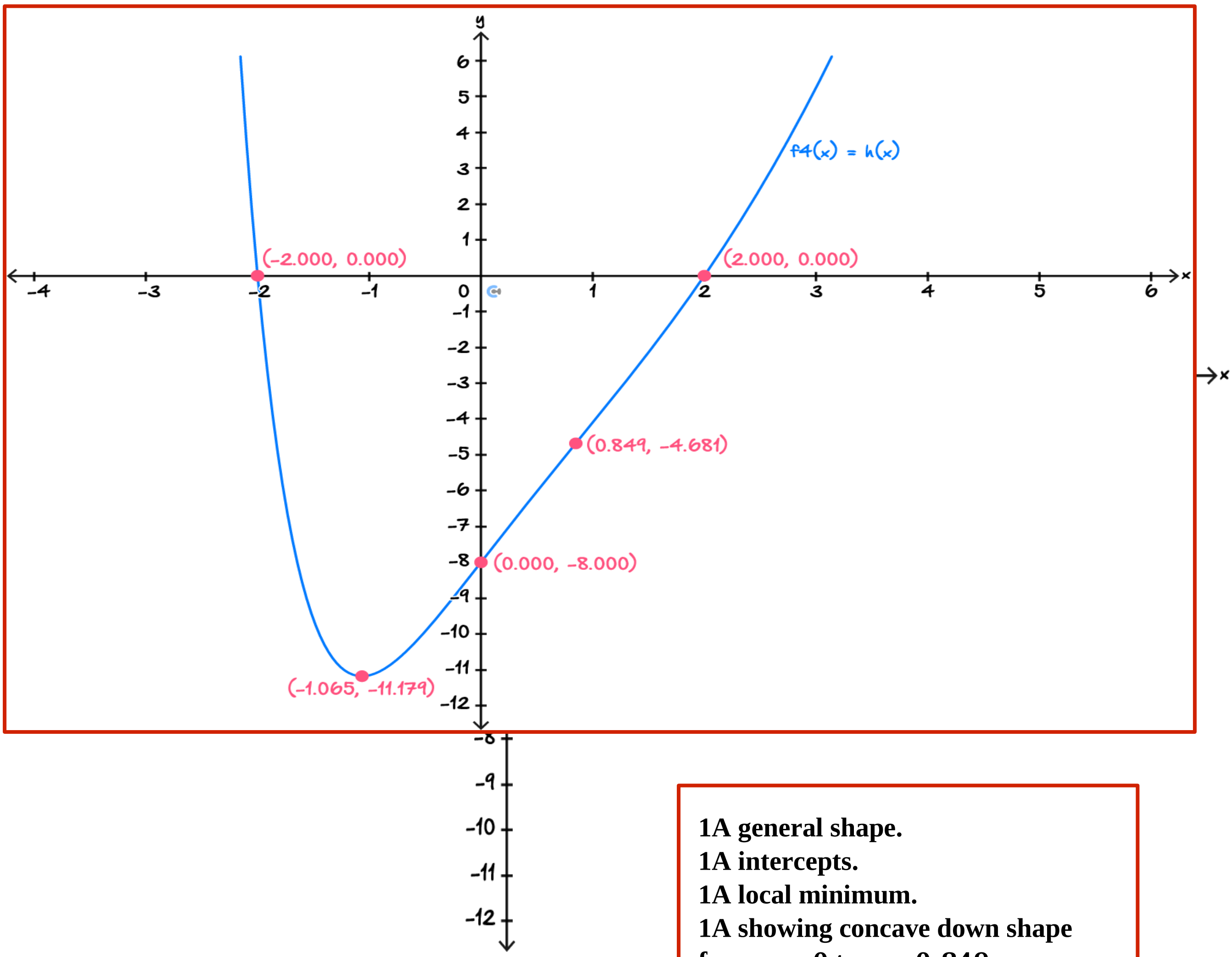
$$x=-1.065368 \text{ and } y=-11.$$

1M

$(-1.07, -11.18)$ **1A**

Local minimum **1A** because the curve must be concave up at the stationary point in order to approach infinity for both positive and negative ends **1M**.

iii. Hence, sketch the graph of the function $h(x)$. Label all axial intercepts, the turning point and point of inflections as coordinates correct to three decimal places. (5 marks)



1A general shape.
1A intercepts.
1A local minimum.
1A showing concave down shape from $x = 0$ to $x = 0.849$.
1A second POI at $x = 0.849$ labelled.

Space for Personal Notes

Question 2 (18 marks)

A kinetic sculpture is first designed using computer graphics that graph functions. For this specific kinetic sculpture, it uses the following function:

$$f(x) = (x - t)(x + t) \cdot e^{-x^2}, x \in [-\pi, \pi]$$

However, any single value for t only represents a snapshot in time. The parameter, t would need to be varied to see the whole motion of the sculpture. Assume $t \geq 0$.

a. Explain how the shape of the sculpture changes as t increases. In particular, describe what happens at the centre of the sculpture $x = 0$, and at the outer edges. Provide a justification. (3 marks)

$$f(0) = -t^2$$

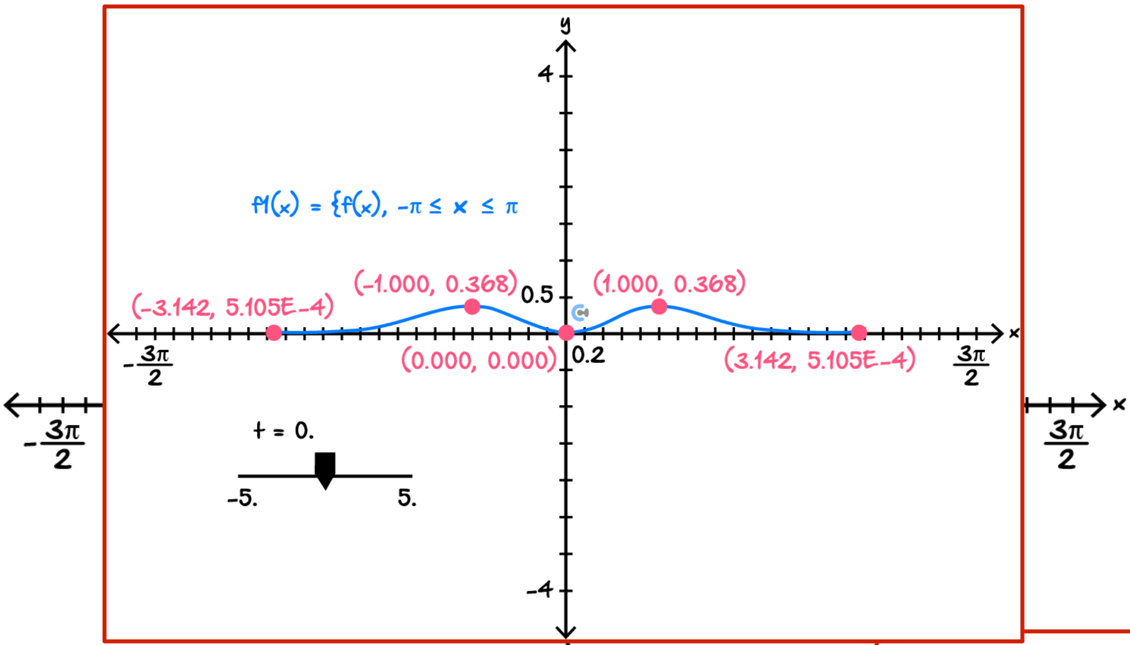
So, the centre of the sculpture dips lower as t increases. **1A**

Away from the centre, the shape flattens since $(x^2 - t^2)$ becomes smaller when $x^2 \approx t^2$. **1A**

As t continues to increase more of the function lies below the x-axis. **1A**

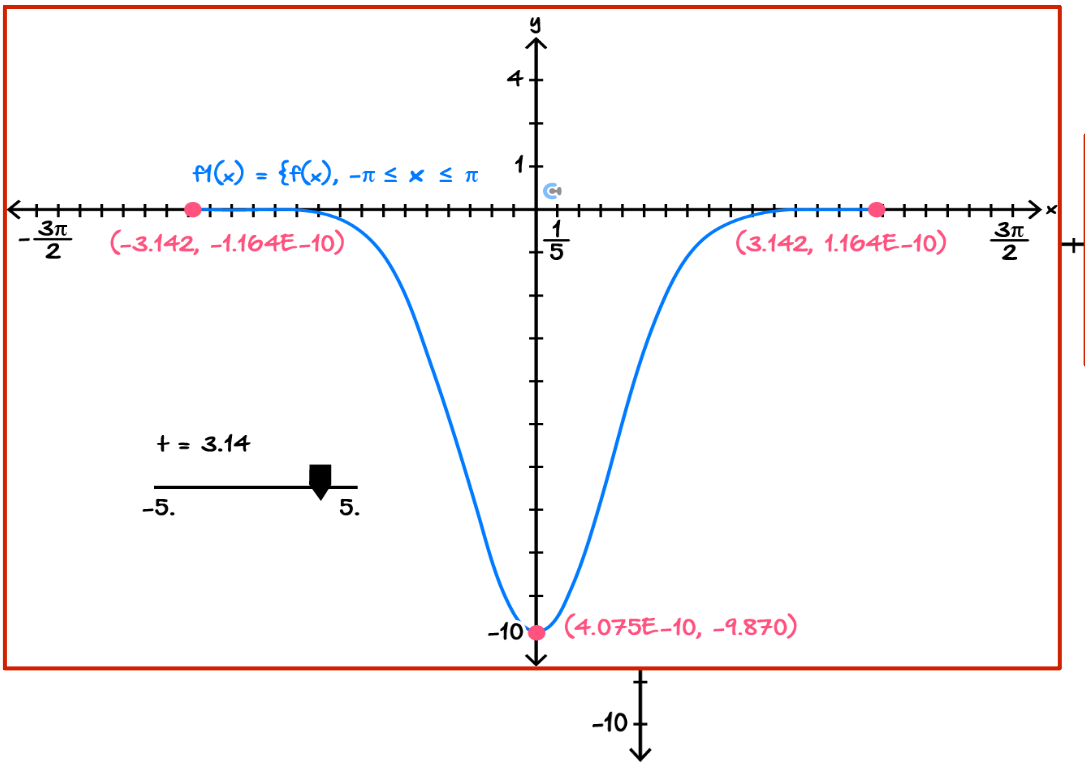
b.

i. Sketch the graph of the function when $t = 0$. Label the coordinates of the minimum. (3 marks)



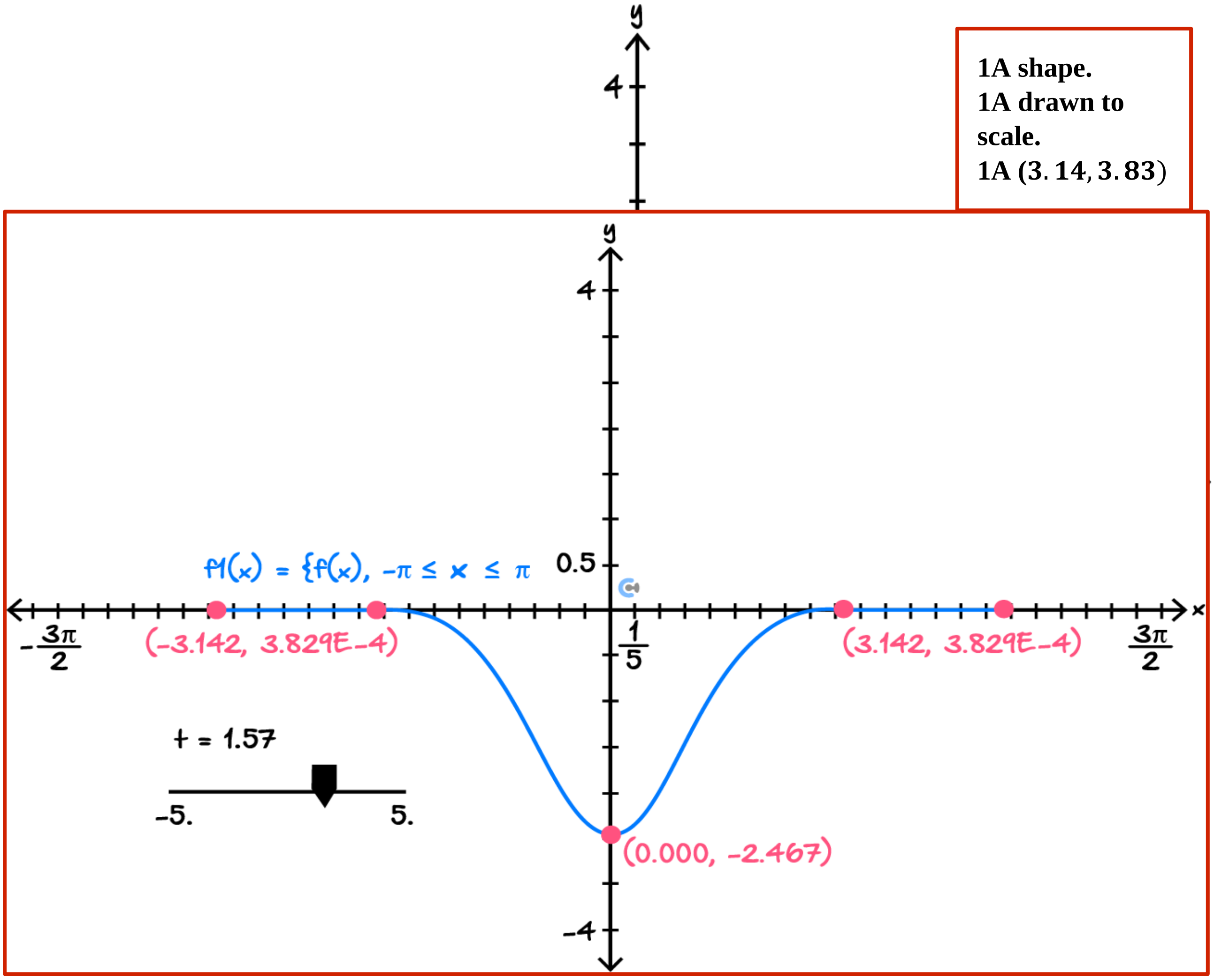
1A shape.
1A drawn to scale.
1A $(0, 0)$

ii. Sketch the graph of the function when $t = \pi$. Label the coordinates of the minimum correct to two decimal places. (3 marks)



1A shape.
1A drawn to scale.
1A $(0.00, -9.87)$

iii. Sketch the graph of the function when $t = \frac{\pi}{2}$. Label the coordinates of the minimum correct to two decimal places. (3 marks)



1A shape.
1A drawn to scale.
1A $(3.14, 3.83)$

c. The magnitude of the rate of change at a point is proportional to the amount of stress placed on that part of the sculpture. **Hint:** Magnitude is found by both squaring and square rooting a value, e.g., $\sqrt{a^2}$.

i. State the expression in terms of a constant of proportionality and t that represents the amount of stress at any x, S. (2 marks)

$S = k \cdot |f'(x)| = k \cdot \sqrt{(f'(x))^2}$, **1M** $k > 0$

$k \cdot \left|\frac{d}{dx}(f(x))\right| \quad 2 \cdot k \cdot e^{-x^2} \cdot \left|x \cdot \left(x^2 - t^2 - 1\right)\right|$ **1A**

ii. What is the amount of stress going through the point $x = 0$? (1 mark)

$2 \cdot k \cdot e^{-x^2} \cdot \left|x \cdot \left(x^2 - t^2 - 1\right)\right| \Big|_{x=0} \quad 0$ **1A**

iii. Two beams are used to support the stress. They are both tangential to the stress when the stress is at a minimum. State their equations. (3 marks)

From part ii. We know minimum stress is when x=0, Stress = 0

$\frac{d}{dx}\left(2 \cdot k \cdot e^{-x^2} \cdot \left|x \cdot \left(x^2 - t^2 - 1\right)\right|\right)\Big|_{x=0}$

1M $\pm 2 \cdot k \cdot \left(t^2 + 1\right)$

$\text{solve}\left(y - 0 = \pm 2 \cdot k \cdot \left(t^2 + 1\right) \cdot (x - 0), y\right)$

$y = 2 \cdot k \cdot \left(t^2 + 1\right) \cdot x$ or $y = -2 \cdot k \cdot \left(t^2 + 1\right) \cdot x$ **1A each**

Note tangent line function may not work

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Question 3 (17 marks)

A skate park has a ramp with a cross section that can be modelled with the following function:

$$r(x) = (x-6)^2 e^{-x} - 1, x \in (0, k]$$

The ground is taken to be the x-axis. Note that part of the skate park goes below ground level. All measurements are in metres.

a. Using Newton's method, approximate a, where a is the first x-intercept of $r(x)$, correct to 3 decimal places. Using $x = 0$ as your starting value. (4 marks)

Define $r(x)=(x-6)^2 \cdot e^{-x}-1$	Done	
Define $n(x)=x-\dfrac{r(x)}{\frac{d}{dx}(r(x))}$	Done	**1M**
$n(0)$	0.7291667	
$n(1.3999934985532)$	1.963394	**1M**
$n(1.9633941963913)$	2.339746	
$n(2.3397463161347)$	2.485458	**1A 2.504**
$n(2.4854582030618)$	2.503279	**1M**
$n(2.5032788529665)$	2.503516	
$n(2.503515821558)$	2.503516	

Stop when tolerance is less than 0.001

b. Skaters start slowing down when the rate of change of the ramp is at its maximum. Where does this occur? Provide your answer correct to 4 decimal places. (3 marks)

$\text{solve}\left(\frac{d^2}{dx^2}(r(x))=0,x\right)$ **1M**

1M $x=6.585786$ or $x=9.414214$

$\text{fMax}\left(\frac{d}{dx}(r(x)),x,0,7.503516\right)$ $x=6.585786$

$r(x)|x=6.5857864376269$ -0.9995265

$(6.5858, -0.9995)$ **1A**

c. After a skater starts slowing down, they start to travel in a parabolic path. The parabolic path and the ramp are smooth and continuous at the point found in **part c.** The skater lands 10 m horizontally from the origin after following this parabolic path.

i. Write three simultaneous equations that can be used to solve for the equation of the parabolic path. You chose to write values correct to three decimal places where required. (3 marks)

Let $q(x) = ax^2 + bx + c$

$$\begin{cases} \frac{d}{dx}(r(x)) = \frac{d}{dx}(q(x)) \mid x = 6.5857864376269 \\ r(6.5857864376269) = q(6.5857864376269) \\ q(10) = 0 \end{cases}$$

1A each – Note that q(x) must be defined

ii. Hence, state the equation of this parabolic path correct to two decimal places. (1 mark)

$a=0.085411$ and $b=-1.123854$ and $c=2.6974$

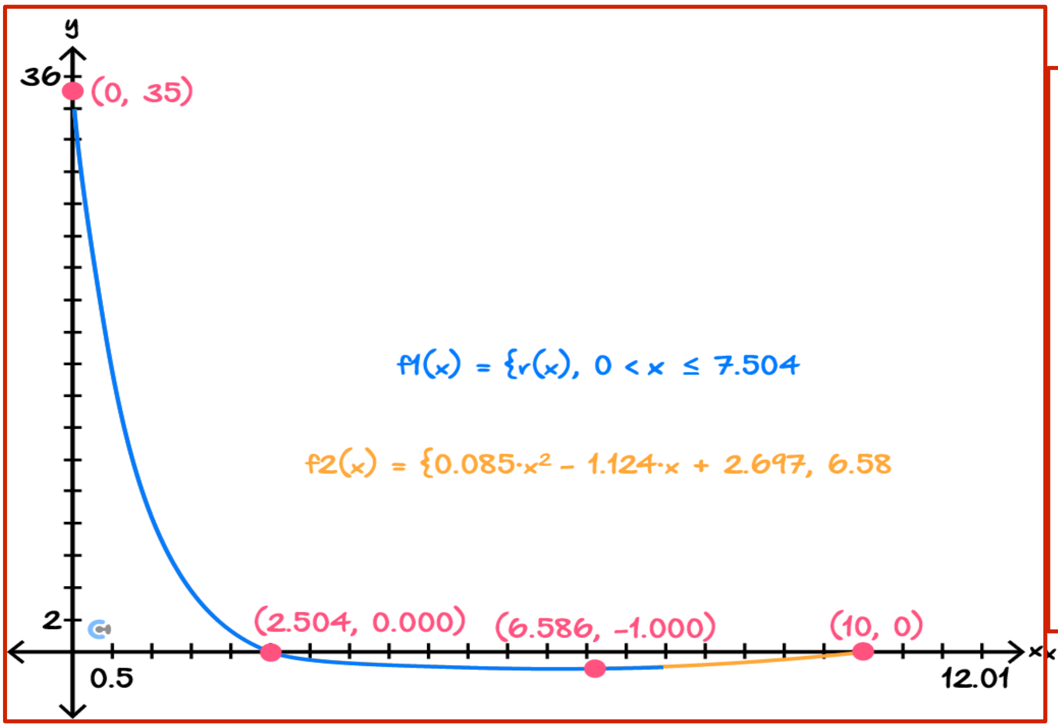
$q(x) \mid a=0.0854110131194$ and $b=-1.1238542$

$0.085411 \cdot x^2 - 1.123854 \cdot x + 2.697442$

1A $q(x) = 0.09x^2 - 1.12x + 2.70$

d.

i. Sketch the graph of $r(x)$. Label the turning points and endpoints correct to 2 decimal places. (4 marks)



i.
1A Shape.
1A y-intercept.
1A x-intercept.
1A endpoint.
ii.
1A shape with smooth continuity.
1A x-intercept.

ii. Sketch the parabola on the same set of axes above. (2 marks)

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