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VCE Mathematical Methods ¾

AOS 3 Revision [3.0]

SAC 1

52 Marks. 15 Minutes Reading. 75 Minutes Writing.

Section A: SAC Questions (Tech Active) (52 Marks)

Question 1 (10 marks)

Supersonic cars race against each other in a 100 km straight stretch of land in the desert. The position x km from the starting point after t minutes is given by:

$$x(t) = \frac{3}{4}t^{\frac{4}{3}} - 360t - 3(t-60)^{\frac{4}{3}} + c$$

where $t \geq 60$.

a. Find the exact value of c, assuming the car starts at $x = 0$ when $t = 60$. (2 marks)

b. Find the domain of $x'(t)$. (1 mark)

c. The velocity is given by $v(t) = x'(t)$. Find $v(t)$. (2 marks)

d. The acceleration is given by $a(t) = v'(t)$. Find $a(t)$. (2 marks)

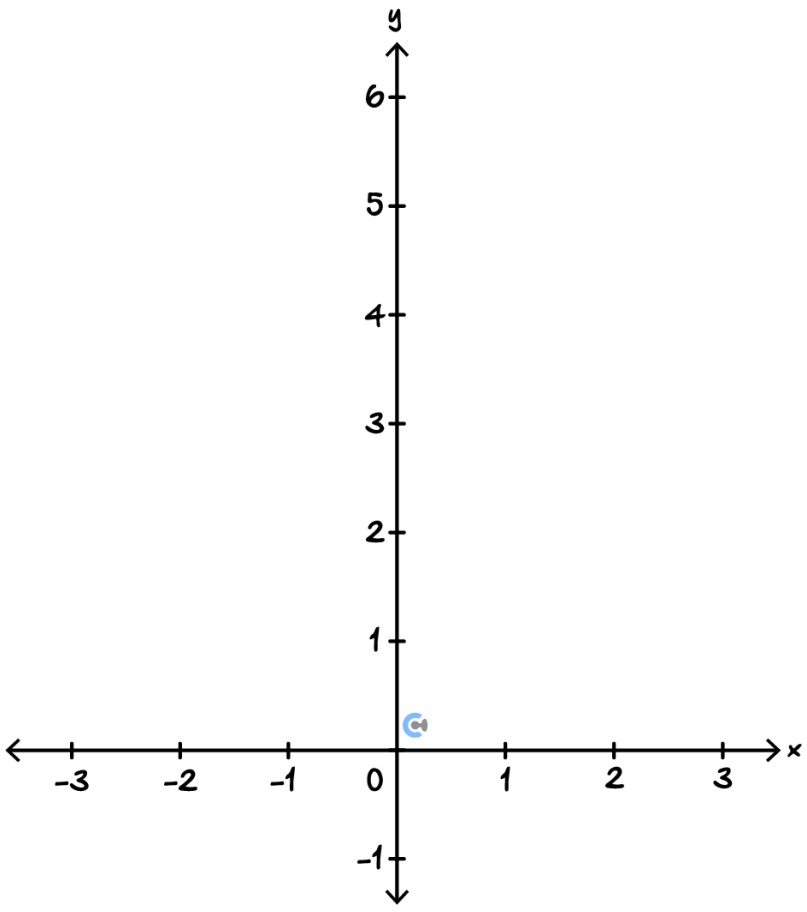
e. Hence, investigate the speed of the car for $t > 60$. Is there a maximum speed? Justify your answer.

Note: Speed is the magnitude (size) of velocity. That is, speed $= |v(t)|$. (3 marks)

Question 2 (16 marks)

Consider the two functions: $f(x) = x^2$ and $g(x) = a$, where $a \in \mathbb{R}^+$. A rectangle is inscribed between these two functions, such that one edge lies on the line $y = g(x)$ whilst the two opposite vertices lie on the graph of $y = f(x)$.

a. On a set of axes, sketch $f(x), g(x)$ for $a = 4$. An example of the inscribed rectangle is described. Label the point(s) of intersection between $f(x)$ and $g(x)$ in terms of a. (5 marks)



b. What can be said about the relationship between the x-coordinates of the vertices of the rectangles? (2 marks)

c. Hence, given that one of the vertices lies on $f(x)$ has an x-coordinate of b (where $0 < b \leq \sqrt{a}$), state the coordinates of the 4 vertices, in terms of a and b. (2 marks)

d. Hence, express the area A of the rectangle in terms of b and a. (2 marks)

e. Hence, find the exact value of b (in terms of a) for which the area of the rectangle will be a maximum, and state this exact maximum area. (3 marks)

f. For what value(s) of b in the domain $0 < b \leq \sqrt{a}$ is the area minimised? State the minimum area. (2 marks)

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Question 3 (21 marks)

Consider the function $f(x) = x^3 - 5x + 1$, for $x \in \mathbb{R}$.

a. State the equation of the tangent to the graph of $y = f(x)$ at $x = 2$. (1 mark)

b. Let $x_0 = 2$. Use Newton's method once to find x_1, and hence find the values of x_2 and x_3 using CAS. Give correct values to 4 decimal places. (3 marks)

c. Using $x_0 = 2$, state the smallest value of n such that x_n correctly approximates the value of the positive x-intercept of $f(x)$, correct to 4 decimal places. (2 marks)

A tangent line is drawn to the function $y = f(x)$ at the point where $x = a$.

d.

i. State the equation of this tangent line in terms of a. (2 marks)

ii. Given that this tangent line passes through the coordinate $(0, 1)$, state the possible value(s) of a. (2 marks)

iii. State the possible value(s) of a such that using $x_0 = a$ in Newton's method for $f(x) = 0$ causes an oscillating sequence (e.g., $x_0, x_1, x_0, \ldots$). Give the value(s) correct to 2 decimal places. (2 marks)

iv. State the possible value(s) of a such that using $x_0 = a$ in Newton's method for $f(x) = 0$ terminates immediately at a root. Give value(s) correct to 2 decimal places. (2 marks)

Let another function be $g(x) = \sqrt{x + 4}$, for $x \geq -4$.

e.

i. Determine which composite function, $f(g(x))$ or $g(f(x))$, has a domain that is a strict subset of the domain of the inner function, and state why.

Note: A strict is a subset that isn't equal to the original set. (1 mark)

ii. For the composite function identified in **part e.i.,** state its domain correctly to 3 decimal places. (2 marks)

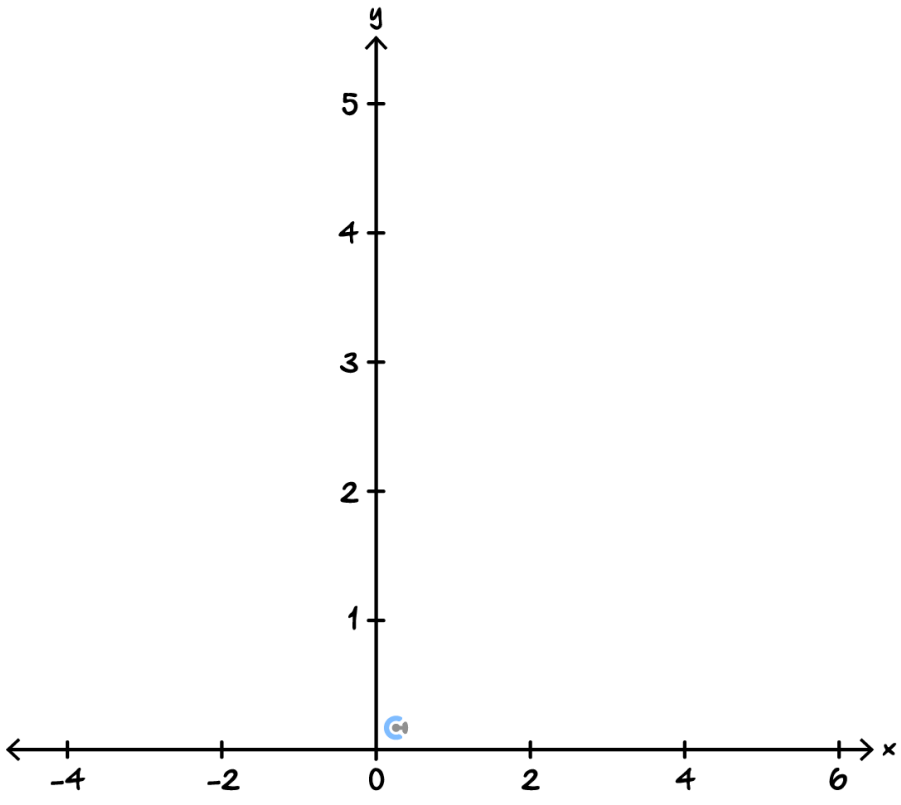
iii. State the range of the composite function identified in **part e.i.** (1 mark)

f. Using Newton's method, approximate the solution to the equation $f(x) = g(x)$. Use $x_0 = 2$ and stop when successive iterations differ by less than 10^{-4}. Give your answer correct to 4 decimal places. (3 marks)

Question 4 (5 marks)

Let the function be $h(x) = \sqrt{x + k} + 1$.

a. Sketch the graph of $y = h(x)$ on the axes below, given that $k = 3$. Label the coordinates of the endpoint clearly. (1 mark)



b. Define the inverse function $h^{-1}(x)$, stating its domain. (2 marks)

c. Find the possible value(s) of k such that $h(x)$ and its inverse function $h^{-1}(x)$ have exactly one point of intersection. (2 marks)

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