Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Pseudocode & its Exam Skills [2.7]
Workbook

Outline:

Learning Objectives:

- ☐ **MM34 [2.7.1]** - Evaluate Pseudocode with Conditional Statements and Loops
- ☐ **MM34 [2.7.2]** - Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Section A: Algorithm

Sub-Section: Introduction to Algorithm

What is an Algorithm?

Algorithm

- An algorithm is a clearly specified _____.

Question 1

Write down the steps necessary (create an algorithm) to find the gradient of a function at $x = 2$.

Space for Personal Notes

Sub-Section: Assigning Variables

Assigning Variables

- To construct algorithms for more mathematical/complex problems, _____ variables will be useful.

$A \leftarrow 3$ assigns the **value 3** to the **variable A.**

- We can also _____ our variables using the arrow.

$A \leftarrow A + 3$ assigns the value $\mathbf{A + 3}$ to the **variable A.**

  - Since the value of A was already 3, Its new value will be 6.

Question 2

What final value will be output by the algorithm below?

Step 1: $A \leftarrow 5$.

Step 2: $A \leftarrow A - 2$.

Step 3: $A \leftarrow 2A + 4$.

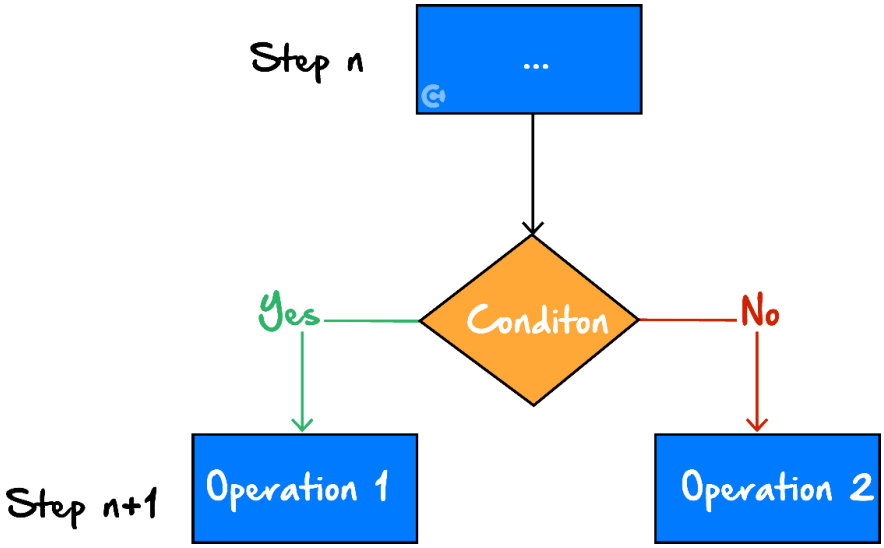
Step 4: Print A.

Sub-Section: Selections

Discussion: What could we do if we want to selectively perform an operation?

Selections

- Selections allow us to perform different operations at a given step, depending on a certain condition.



- We are _____ performing an operation.

Key Words for Conditions

If, then

Otherwise/Else

Else If

Question 3

Track the values of n and T in each step for the algorithm below.

Step 1: $n \leftarrow 3$.

Step 2: **If** n is odd, **then** $T \leftarrow 2n + 5$.

Otherwise $T \leftarrow n - 7$.

Step 3: $n \leftarrow n - 2$.

Step 4: Print n, T.

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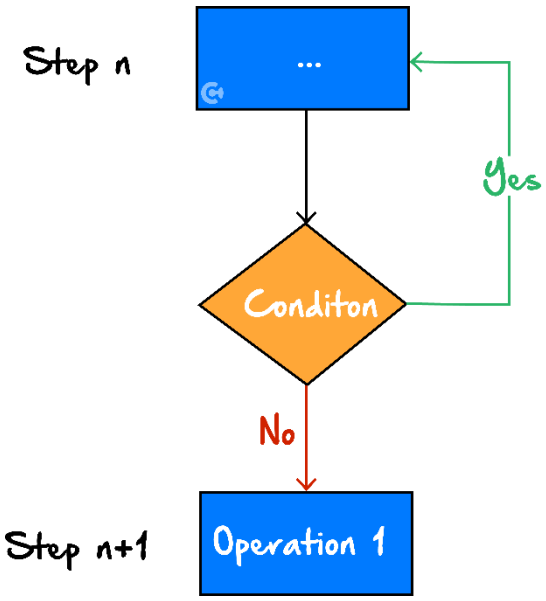
Sub-Section: Loops

Discussion: What do we do if we want to do something repeatedly?

Iteration (Loops)

- Iteration (a.k.a. looping) allows us to repeat steps in a _____.
- It is controlled by the _____.
  - E.g., we only loop when a condition is met.



Key Words for Iteration

Repeat...For...

Repeat...While...

Let's look at a while loop example!

Question 4 Walkthrough.

Step 1: $A \leftarrow 5$.

Step 2: $A \leftarrow 3A - 1$.

Step 3: **Repeat** from Step 2 **while** $A < 20$.

Step 4: Print A.

NOTE: We only stop the loop when $A \geq 20$. Hence, our final answer for A should be ≥ 20.

Question 5

Step 1: $B \leftarrow 10$.

Step 2: $B \leftarrow 1.5B - 2$.

Step 3: **Repeat** from Step 2 **while** $B < 15$.

Step 4: Print B.

Let's look at a for loop example!

Question 6 Walkthrough.

Step 1: A ← 7.

Step 2: A ← 2A + 1.

Step 3: **Repeat** from Step 2 **for** 2 iterations.

Step 4: Print A.

Discussion: Could you have told me how many loops we will be doing without running through the algorithm?

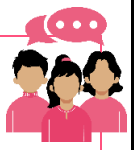
Question 7

Step 1: A ← 2.

Step 2: A ← A − 3.

Step 3: **Repeat** from Step 2 **for** 3 iterations.

Step 4: Print A.



Discussion: What are the differences between for and while loops?

Analogy: Your Parents Taking Care of You

- Consider the two parents below.
  - **Parent 1:**

    I'll take care of you for the next 10 years!

  - **Parent 2:**

    I'll take care of you while you are not married!

  - For which parent do we exactly know how many years they will take care of us?
  - Hence, which parent is a for loop and which parent is a while loop?

For vs While Loop

- For Loop:
  - We know how many iterations will happen.
- While Loop:
  - We don't know how many loops will happen.

Sub-Section: Pseudocode

What is a pseudocode?

Pseudocode

```
if  condition  then
    operation 1
else
    operation 2
end if
```

- A way to write the algorithm in a code-like style.

Pseudocode for Selections

- "If-then"

```
if  condition  then
    operation
end if
```

  - Allows us to perform an operation only when a certain condition is met.

- "Else"

```
if condition then
    operation 1
else
    operation 2
end if
```

  - Provides an opportunity to perform an operation only when a certain condition is met.

- "Else-If"

```
if condition 1 then
    operation 1
else if condition 2 then
    operation 2
else
    operation 3
end if
```

  - Provides an opportunity to add multiple pathways, each with different conditions.

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Question 8 Walkthrough.

Evaluate the final output from each of the following:

$x \leftarrow 7$
if $x < 11$
    $x \leftarrow x + 10$
else
    $x \leftarrow x - 15$
end if
print x

Question 9

Evaluate the final output from each of the following:

$a \leftarrow 7$
$b \leftarrow 3$
if $a + b < 15$
    $b \leftarrow b - 2$
    $a \leftarrow a + 5$
end if
print a, b

Pseudocode for Iteration

- **For loops:**

```
for  variable  from  lower bound  to  upper bound
                    condition

    operation

end for
```

  - Loops for which a variable increases by _____ each time it loops.
  - The variable gets moved from the _____ to the _____ by 1.

- **While loops:** Loops which do **not** change the value of any variable by default.

```
while  condition

    operation

end while
```

Question 10 Walkthrough.

Evaluate the final output from each of the following:

$sum \leftarrow 3$
for i from 3 *to* 7
    $sum \leftarrow sum + i$
end for
print sum

Question 11

Evaluate the final output from each of the following:

$total \leftarrow 3$
for i from 1 to 5
    $total \leftarrow total + 2i - 1$
end for
print $total$

Question 12 Walkthrough.

Find the output of the following algorithm written in pseudocode:

$a \leftarrow 2$
while $a < 15$
    $a \leftarrow a + 5$
end while
print a

Question 13

Evaluate the final output from each of the following:

$a \leftarrow 3$
$b \leftarrow 7$
while $a + b < 13$
    $b \leftarrow b - 1$
    $a \leftarrow a + 3$
end while
print a, b

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Section B: Test (15 Marks)

INSTRUCTION: 15 Marks. 15 Minutes Writing.

Question 14 (2 marks)

Find the output of the following algorithm written in pseudocode.

$a \leftarrow 2$
while $a < 15$
    if a is even
        $a \leftarrow a + 5$
    else
        $a \leftarrow a + 3$
    end if
end while
print a

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Question 15 (2 marks)

Evaluate the final output from each of the following:

$a \leftarrow 4$
$b \leftarrow 3$
for i from 1 to 4
    $b \leftarrow b - 2$
    $a \leftarrow a + 5$
end for
print a, b

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Question 16 (2 marks)

A transformation maps the graph of $y = f(x)$ to $y = af(bx + c) + d$, where a, b, c and d are positive real numbers. The following algorithm will be used to map any point (x, y) on the graph of $y = f(x)$ to the graph of $y = af(bx + c) + d$.

input x, y	# *Line* 1
x_new = _____	# *Line* 2
y_new = _____	# *Line* 3
print x_{new}	# *Line* 4
print y_{new}	# *Line* 5

Complete lines 2 and 3 of the algorithm.

_____

_____

_____

_____

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Question 17 (3 marks)

Using pseudocode, write an algorithm to calculate the sum of the first n terms of the sequence:

$$\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2} \ldots$$

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Question 18 (3 marks)

Evaluate the final output of the pseudocode below:

```
a ← 1
b ← 4
for i from 1 to 6
    for j from 1 to 8
        b ← b + j
        a ← a + i + j
    end for
end for
print a, b
```

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Question 19 (3 marks)

Find the final output of the pseudocode shown below:

$x \leftarrow 5$
$y \leftarrow 2$
$z \leftarrow 0$

for i from 1 to 4
  while $x > 0$
    if x is even then
      $x \leftarrow x - 1$
      $y \leftarrow y + i$
    else
      $x \leftarrow x - 2$
      $z \leftarrow z + y$
    end if
  end while
end for
print x, y, z

Space for Personal Notes

Section C: Pseudocode Exam Skills

Sub-Section: Pseudocode for Newton's Method

REMINDER: VCAA Formula For Newton's Method

Newton's Method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Question 20 Walkthrough. Tech-Active.

Conisder the implementation of Newton's method shown below:

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$

Define newton $(f(x), x0)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$\text{prev}_x \leftarrow$ x0
for i from 1: 3 **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$
$\text{prev}_x \leftarrow \text{next}_x$
End For
Return next_x

Find the return value of newton$(x^2 - 5, 4)$, correct to three decimal places.

Question 21 Tech-Active.

Conisder the implementation of Newton's method shown below:

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$
n, the number of iterations

Define newton $(f(x), x0, n)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
prev_x $\leftarrow$ x0
while $i < n$ **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$
$i \leftarrow i + 1$
$\text{prev}_x \leftarrow \text{next}_x$
End While
Return next_x

Find the return value of newton$(x^2 - 7,4,3)$, correct to three decimal places.

Space for Personal Notes

Little harder now!

REMINDER: Tolerance

- The maximum difference between x_n and x_{n+1} we can have when we stop the iteration.

$$\text{We stop when } |x_{n+1} - x_n| < Tolerance.$$

- The question will give us the tolerance level.

Question 22 Walkthrough. Tech-Active.

Consider the pseudocode shown below:

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$
ε, a tolerance level

Define newton $(f(x), x0, \varepsilon)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$\text{prev}_x \leftarrow$ x0
While $i \leq 1000$ **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$

If $-\varepsilon < \text{next}_x - \text{prev}_x < \varepsilon$ **Then**
Return next_x

Else
$\text{prev}_x \leftarrow \text{next}_x$
$i \leftarrow i + 1$
End While

The number of iterations required for Newton $(x^2 - 7{,}5{,}0.01)$ is given by:

A. 2

B. 4

C. 25

D. 1000

Question 23

A pseudocode to compute $\sqrt{3}$ using Newton's method is shown below.

define $h(x)$
    return $x^2 - 3$
define $h'(x)$
    return $2x$
$xnext \leftarrow 2$
$xprev \leftarrow 50$
while $|xnext - xprev| \geq 0.001$
    $xprev \leftarrow xnext$
    $xnext \leftarrow xprev - \dfrac{h(xprev)}{h'(xprev)}$
    print $xnext$
end while

The program will stop after how many iterations of the while loop?

A. 2

B. 3

C. 4

D. 5

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Now limitation of Newton's method.

REMINDER: Limitation of Newton's Method

- **Terminating Sequence**: Occurs when we hit a stationary point.
- **Approximating a Wrong Root**: Occurs when we start on the wrong side.
- **Oscillating Sequence**: Occurs when we oscillate between two values without getting closer to the real root.

Question 24 Walkthrough. Tech-Active.

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$

Define newton $(f(x), x0)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$\text{prev}_x \leftarrow$ x0
While $i \leq 1000$ **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$

If $-0.1 < \text{next}_x - \text{prev}_x < 0.1$ **Then**
Return next_x

Else
$\text{prev}_x \leftarrow \text{next}_x$
$i \leftarrow i + 1$
End While

The number of iterations required for Newton $\left(x(x+5)(x-5), \sqrt{5}\right)$ is given by:

A. 500

B. 999

C. 1000

D. 1001

Question 25 Tech-Active.

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$

Define newton $(f(x), x0)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$\text{prev}_x \leftarrow$ x0
While $i < 5000$ **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$

If $-0.1 < \text{next}_x - \text{prev}_x < 0.1$ **Then**
Return next_x

Else
$\text{prev}_x \leftarrow \text{next}_x$
$i \leftarrow i + 2$
End While

The number of iterations required for Newton $\left(x(x+1)(x-1), \frac{1}{\sqrt{5}}\right)$ is given by:

A. 5

B. 5000

C. 2499

D. 2500

Space for Personal Notes

Section D: Exam 2

Question 26

The algorithm shown below will print the value:

$a \leftarrow 3$
while $a < 18$
    $a \leftarrow 2a$
End While
Print (a)

A. 24

B. 28

C. 30

D. 32

Question 27

The algorithm shown below will print the value:

sum $\leftarrow 4$
for x **from** 1 **to** 3
    for y **from** 1 **to** 2
        sum $=$ sum $+ x + y$
    end for
end for
print (sum)

A. 19

B. 21

C. 23

D. 25

Space for Personal Notes

Question 28 Tech-Active.

The algorithm below, described in pseudocode, estimates the root of a function $f(x)$ up to a given tolerance.

Inputs: $f(x)$, a function of x
$df(x)$, the derivative of f
$x0$, an initial guess of the root of f
tol, the tolerance

Define newton $(f(x), df(x), x0, \text{tol})$
$E \leftarrow \text{tol} + 1$
While $E > \text{tol}$ **Do**
$x \leftarrow x0 - f(x0) \div df(x0)$
If $x > x0$ **Then** $E \leftarrow x - x0$
Else $E \leftarrow x0 - x$
End if
$x0 \leftarrow x$
End While
Return $x0$

Consider the algorithm implemented with the following inputs:

$$\text{Newton } (x^2 - 12, 2x, 3, 0.05)$$

The value of the variable E after the function is implemented, is closest to:

A. 0.0002

B. 0.0357

C. -0.0357

D. -0.0002

Space for Personal Notes

Question 29 Tech-Active.

Consider the algorithm below, which uses the Newton's method to estimate the x-intercept of a function with a tolerance of 0.001.

Inputs: $f(x)$, a function of x
$df(x) \leftarrow$ the derivative of $f(x)$
x0, an initial estimate $f$or the x-intercept of f

Define newton $(f(x), df(x), x_0)$
$\text{prev}_x \leftarrow x_0$
For i **From** 1 **to** 1000
$\text{next}_x \leftarrow \text{prev}_x - f(\text{prev}_x) \div df(\text{prev}_x)$
If $-0.0001 < \text{next}_x - \text{prev}_x < 0.0001$ **Then**
Return next_x
Else
$\text{prev}_x \leftarrow \text{next}_x$
End if
EndFor
Return "Error: Did not converge"

Consider the algorithm implemented with the following inputs:

$$\text{Newton } (x^3 - 5x, 3x^2 - 5, 1)$$

The value of the variable next_x after the function is implemented, is closest to:

A. -1

B. 1

C. $\sqrt{5}$

D. $-\sqrt{5}$

Space for Personal Notes

Question 30

One way of implementing Newton's method using pseudocode, with a tolerance level of 0.005, is shown below. The pseudocode is incomplete, with two missing lines indicated by an empty box.

Inputs: $f(x)$, a function of x
x0, an initial estimate $f$or the x- intercept of $f(x)$

Define newton $(f(x), x0)$
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$\text{prev}_x \leftarrow$ x0
while $i < 1000$ **Do**
$\text{next}_x \leftarrow \text{prev}_x - \text{f}(\text{prev}_x) \div \text{df}(\text{prev}_x)$
[ ]
Else
$\text{prev}_x \leftarrow \text{next}_x$
$i \leftarrow i + 1$
EndWhile

Which one of the following options would be most appropriate to fill the empty box?

A. If $\text{next}_x - \text{prev}_x < 0.005$ **Then**
Return prev_x.

B. If $\text{prev}_x - \text{next}_x < 0.005$ **Then**
Return next_x.

C. If $-0.005 < \text{next}_x - \text{prev}_x < 0.005$ **Then**
Return prev_x.

D. If $-0.005 < \text{next}_x - \text{prev}_x < 0.005$ **Then**
Return next_x.

Space for Personal Notes

Contour Check

Learning Objective: [2.7.1] – Evaluate pseudocode with conditional statements and loops

Key Takeaways

- **Assigning Variables:**
  - To construct algorithms for more mathematical/complex problems, _____ variables will be useful.

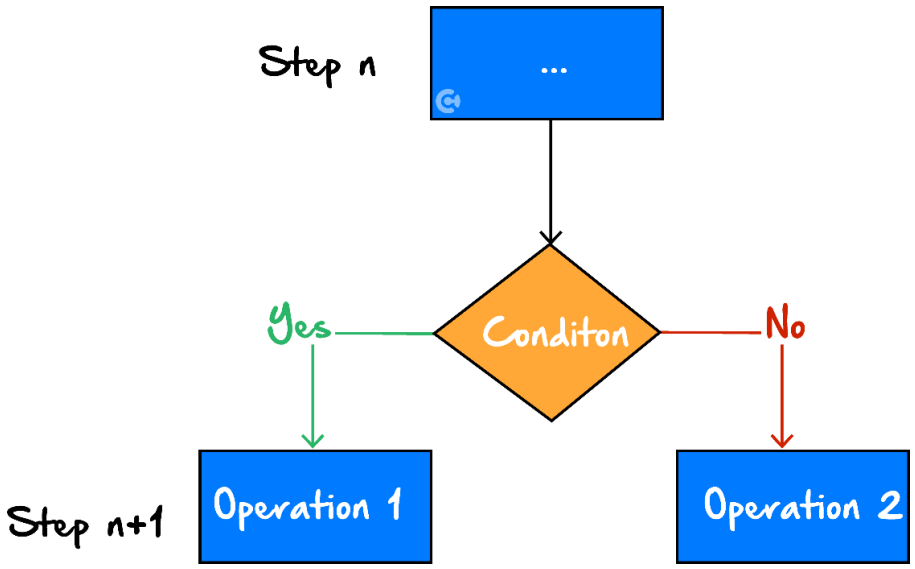
$A \leftarrow 3$ assigns the **value 3** to the **variable A.**

  - We can also _____ our variables using the arrow.

$A \leftarrow A + 3$ assigns the value $\mathbf{A + 3}$ to the **variable A.**

    - Since the value of A was already 3, Its new value will be 6.

- **Selections:**
  - Selections allow us to perform different operations at a given step, depending on a certain condition.



  - We are _____ performing an operation.

- "If-then"

```
if  condition  then
    operation
end if
```

  - Allows us to perform an operation only when a certain condition is met.

- "Else"

```
if  condition  then
    operation 1
else
    operation 2
end if
```

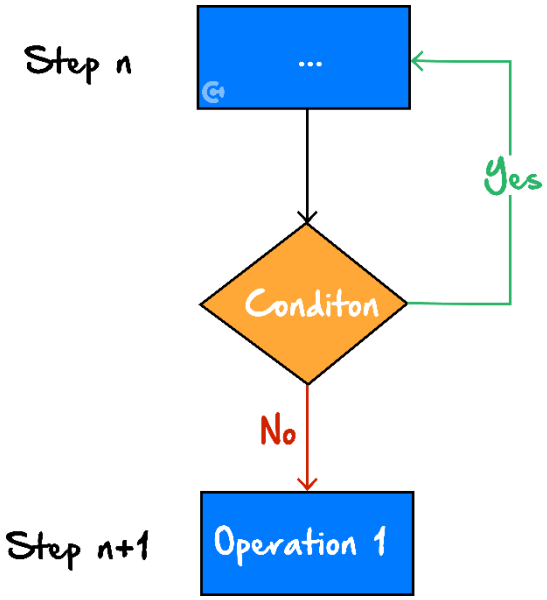
  - Provides an opportunity to perform an operation only when a certain condition is met.

- "Else-If"

```
if  condition 1  then
    operation 1
else if  condition 2  then
    operation 2
else
    operation 3
end if
```

  - Provides an opportunity to add multiple pathways, each with different conditions.

- **Iteration (Loops):**
  - Iteration (a.k.a. looping) allows us to repeat steps in a _____.
  - It is controlled by the _____.
    - E.g., we only loop when a condition is met.



  - **For loops:**

```
for variable from lower bound to upper bound
                 condition

    operation

end for
```

    - Loops for which a variable increases by _____ each time it loops.
    - The variable gets moved from the _____ to the _____ by 1.
  - **While loops:** Loops which do **not** change the value of any variable by default.

```
while condition

    operation

end while
```

☐ Learning Objective: [2.7.2] – Evaluate and understand the pseudocode for different implementations of Newton's method

Key Takeaways

- ☐ A key component of Newton's method is the recursive relationship.

$$x_{n+1} = x_n - \_\_\_\_\_$$

- ☐ Newton's method requires an input function $f(x)$, the derivative $f'(x)$ and an initial value x_0.
- ☐ The number of iterations that Newton's method performs can be limited in our pseudocode.
- ☐ The pseudocode can also specify a tolerance for Newton's method where the algorithm terminates if

$$|\_\_\_\_\_| < Tolerance$$

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