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VCE Mathematical Methods ¾
Pseudocode & its Exam Skills [2.7]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 13
Supplementary Questions	Pg 14 – Pg 22

Section A: Compulsory Questions

Sub-Section [2.7.1]: Evaluate Pseudocode with Conditional Statements and Loops

Question 1

The following pseudocode segments each print a value. Write down the value that is printed.

a.
$x \leftarrow 5$
if $x > 3$
  $x \leftarrow x + 2$
end if
print(x)

7

b.
$y \leftarrow 10$
if $y < 5$
  $y \leftarrow y - 3$
else
  $y \leftarrow y + 4$
end if
print (y)

14

c.
$z \leftarrow 1$
while $z < 4$
  $z \leftarrow z + 1$
end while
print (z)

4

Question 2

The following pseudocode segments each print a value. Write down the value that is printed.

a.
$a \leftarrow 3$
for i **from** 1 **to** 3
  if $a \leq 5$
    $a \leftarrow a + 2$
  end if
end for
print (a)

7

Iteration i	a before `if`	Condition $a \leq 5$?	a after `if`
1	3	Yes	5
2	5	Yes	7
3	7	No	7

b.
$b \leftarrow 6$
while $b > 2$
  if b is even
    $b \leftarrow b - 3$
  else
    $b \leftarrow b - 1$
  end if
end while
print (b)

2

Iteration	b before loop	Condition $b > 2$?	b even?	b after update
1	6	Yes	Yes	3
2	3	Yes	No	2
3	2	No	—	—

c.
```
c ← 1
for j ← 1 to 4
    if j is even
        c ← c + 3
    else
        c ← c + 1
    end if
end for
print (c)
```

9

Iteration j	c before update	j even?	c after update
1	1	No	2
2	2	Yes	5
3	5	No	6
4	6	Yes	9

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Question 3

The following pseudocode segments each print values. Write down the values that are printed.

a.

```
a ← 1
b ← 2
for i from 1 to 3
    for j from 1 to 2
        if a is even
            b ← b + a
        else
            a ← a + j
        end if
    end for
end for
print (a, b)
```

$a = 2, b = 12$

Outer i	Inner j	a before	a even?	Action taken	a after	b after
1	1	1	No	$a \leftarrow a + j = 2$	2	2
1	2	2	Yes	$b \leftarrow b + a = 4$	2	4
2	1	2	Yes	$b \leftarrow b + a = 6$	2	6
2	2	2	Yes	$b \leftarrow b + a = 8$	2	8
3	1	2	Yes	$b \leftarrow b + a = 10$	2	10
3	2	2	Yes	$b \leftarrow b + a = 12$	2	12

b.

```
x ← 3
y ← 10
while y > 5
    for j from 1 to 2
        if x is even
            y ← y − j
        else
            x ← x + 1
        end if
    end for
end while
print (x, y)
```

$x = 4, y = 5$

While Iteration	x before	y before	Inner j	x even?	Action Taken	x after	y after
1	3	10	1	No	$x \leftarrow x + 1 = 4$	4	10
	4	10	2	Yes	$y \leftarrow y - 2 = 8$	4	8
2	4	8	1	Yes	$y \leftarrow y - 1 = 7$	4	7
	4	7	2	Yes	$y \leftarrow y - 2 = 5$	4	5

c.
$p \leftarrow 2$
$q \leftarrow 8$
for j **from** 1 **to** 3
  for k **from** 1 **to** 2
    if q is even
      $p \leftarrow p + k$
      $q \leftarrow q - 1$
    else
      $q \leftarrow q - p$
    end if
  end for
end for
print (p, q)

$p = 4, q = -9$

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Sub-Section [2.7.2] : Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Question 4

An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n)$:
$df(x) \leftarrow$ the derivative of $f(x)$
  for i **from** 1 **to** n **do**
    if $df(x_0) = 0$ **then**
      return "Error: Division by zero"
    else
      $x_0 \leftarrow x_0 - \frac{f(x_0)}{df(x_0)}$
    end if
  end while
  return x_0

Consider calling the function newton$(x^2 - 3, 2, 3)$.

a. How many iterations are performed?

3

b. What is the final value of x_0. Give your answer correct to three decimal places.

1.732

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.	1.75	0.25
1	1.75	1.73214	0.0178571
2	1.73214	1.73205	0.0000920471
3	1.73205	1.73205	2.44585×10^{-9}

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Question 5

An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n, tol)$:
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_n + 1$
  $i \leftarrow i + 1$
end while
return x_n

Consider calling the function newton $(x^3 - x^2 + 1, 2, 30, 0.001)$.

a. Find the final return value. Give your answer correct to four decimal places.

-0.7549

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.	1.375	0.625
1	1.375	0.790107	0.584893
2	0.790107	-2.17979	2.96989
3	-2.17979	-1.42183	0.75796
4	-1.42183	-0.984494	0.437332
5	-0.984494	-0.795138	0.189356
6	-0.795138	-0.756433	0.0387053
7	-0.756433	-0.75488	0.0015525
8	-0.75488	-0.754878	2.44653×10^{-6}

b. State the value of i when the algorithm terminates.

$i = 8$

Question 6

An implementation of Newton's method is shown below.

```
define newton(f(x), x_0, n, tol):
df(x) ← the derivative of f(x)
i ← 0
x_n ← x_0
while i < n do
    if df(x_n) = 0 then
      return "Error: Division by zero"
    x_{n+1} ← x_n − f(x_n)/df(x_n)
    if −tol < x_{n+1} − x_n < tol then
      return x_{n+1}
    x_n ← x_{n+1}
    i ← i + 1
end while
return x_n
```

Consider calling the function.

a. Find the final return value of newton $(x^3 - 5x^2 + 1, 1, 2, 0.01)$.

0.4772

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.	0.571429	0.428571
1	0.571429	0.477217	0.0942118
2	0.477217	0.46988	0.00733677

b. The function newton $\left(x(x+2)(x-2), \frac{2}{\sqrt{5}}, 2000, 0.0001\right)$ is called. Find the final return value.

Oscillating sequence. We return $x_{2000} = \dfrac{2}{\sqrt{5}}$

Sub-Section: The 'Final Boss'

Question 7

a. Consider the pseudocode shown below. Determine the final values of m and n.

```
m ← 2
n ← 3
for i from 1 to 3
    for j from 1 to 2
        if m + n > 5
            m ← m + j
            n ← n − 1
        else
            m ← m + 2
            n ← n + 1
        end if
    end for
end for
print(m, n)
```

$m = 12, n = -1$

i	j	$m+n$ before	Condition > 5?	Action Taken	m after	n after
1	1	5	No	$m \leftarrow m+2=4, n \leftarrow n+1=4$	4	4
1	2	8	Yes	$m \leftarrow m+2=6, n \leftarrow n-1=3$	6	3
2	1	9	Yes	$m \leftarrow m+1=7, n \leftarrow n-1=2$	7	2
2	2	9	Yes	$m \leftarrow m+2=9, n \leftarrow n-1=1$	9	1
3	1	10	Yes	$m \leftarrow m+1=10, n \leftarrow n-1=0$	10	0
3	2	10	Yes	$m \leftarrow m+2=12, n \leftarrow n-1=-1$	12	-1

b. An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n)$:
$df(x) \leftarrow$ the derivative of $f(x)$
  for i **from** 1 **to** n **do**
    if $df(x_0) = 0$ **then**
      return "Error: Division by zero"
    else
      $x_0 \leftarrow x_0 - \frac{f(x_0)}{df(x_0)}$
    end if
  end while
  return x_0

The call newton$(\sin(x), x_0, 10)$ is made.

State all values of x_0 for which the function returns an error.

$f'(x) = \cos(x)$. Solve $\cos(x) = 0 \implies x = n\pi \pm \frac{\pi}{2}$, where $n \in \mathbb{Z}$.

So an error is returned if $x_0 = n\pi \pm \frac{\pi}{2}$, where $n \in \mathbb{Z}$.

c. The bisection algorithm is a method for approximating solutions to the equation $f(x) = 0$. As inputs, the algorithm takes a function f, an interval $[a, b]$ which should contain a root of f, and a tolerance ϵ, which the approximate solution is within of the exact solution. Pseudocode is shown below:

```
define Bisection (f, a, b, ϵ)
    if f(a) × f(b) ≥ 0
        print("Bisection method fails.")
        return None
    end if
    while |b − a| > 2ϵ
        m ← (a + b)/2
        if f(m) = 0
            return m
        end if

        if f(a) × f(m) < 0
            b ← m
        else
            a ← m
        end if
end while

return (a + b)/2
```

i. What happens when we call Bisection $(x^3 + 3x + 5, -4, -2, 0.01)$.

We get the output "Bisection method fails".
This is because $f(-4) = -71$ and $f(-2) = -9$

ii. Find the return value of Bisection $(x^3 + 3x + 5, -2, -1, 0.01)$.

-1.1484375

In[68]:=
```
ResourceFunction["BisectionMethodFindRoot"][x^3 + 3 x + 5,
 {x, -2, -1}, 8, 8, "Steps"]
```

Out[68]=

steps	a	f[a]	b	f[b]
1	-2.0000000	-9.	-1.0000000	1.
2	-1.5000000	-2.875	-1.0000000	1.
3	-1.2500000	-0.703125	-1.0000000	1.
4	-1.2500000	-0.703125	-1.1250000	0.201172
5	-1.1875000	-0.237061	-1.1250000	0.201172
6	-1.1562500	-0.0145569	-1.1250000	0.201172
7	-1.1562500	-0.0145569	-1.1406250	0.0941429
8	-1.1562500	-0.0145569	-1.1484375	0.0400033

In[62]:=
```
Abs[-1.15625 - (-1.140625)]
```

Out[62]= 0.015625

In[67]:=
```
NumberForm[(-1.15625 + (-1.140625))/2, 8]
```

Out[67]//NumberForm=
-1.1484375

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Section B: Supplementary Questions

Question 8

The following pseudocode segments each print a value. Write down the value that is printed.

a.
$x \leftarrow 8$
if $x > 5$
  $x \leftarrow x - 3$
end if
print(x)

5

b.
$y \leftarrow 4$
if $y < 3$
  $y \leftarrow y + 6$
else
  $y \leftarrow y - 1$
end if
print (y)

3

c.
$z \leftarrow 2$
while $z < 6$
  $z \leftarrow z + 2$
end while
print (z)

6

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Question 9

The following pseudocode segments each print a value. Write down the value that is printed.

a.
$a \leftarrow 2$
for $i \leftarrow 1$ **to** 4
  if a is even
    $a \leftarrow a + 3$
  else
    $a \leftarrow a + 2$
  end if
end for
print(a)

11

Iteration i	a before	a even?	Action Taken	a after
1	2	Yes	$a \leftarrow a+3=5$	5
2	5	No	$a \leftarrow a+2=7$	7
3	7	No	$a \leftarrow a+2=9$	9
4	9	No	$a \leftarrow a+2=11$	11

b.
$b \leftarrow 10$
while $b > 4$
  if b is even
    $b \leftarrow b - 2$
  else
    $b \leftarrow b - 3$
  end if
end while
print(b)

4

Iteration	b before	b even?	Action Taken	b after
1	10	Yes	$b \leftarrow b-2=8$	8
2	8	Yes	$b \leftarrow b-2=6$	6
3	6	Yes	$b \leftarrow b-2=4$	4

c.
$c \leftarrow 1$
for $j \leftarrow 1$ **to** 5
  if j is even
    $c \leftarrow c + 2$
  else
    $c \leftarrow c + 4$
  end if
end for
print(c)

17

Iteration j	c before	j even?	Action Taken	c after
1	1	No	$c \leftarrow c+4 = 5$	5
2	5	Yes	$c \leftarrow c+2 = 7$	7
3	7	No	$c \leftarrow c+4 = 11$	11
4	11	Yes	$c \leftarrow c+2 = 13$	13
5	13	No	$c \leftarrow c+4 = 17$	17

Space for Personal Notes

Question 10

The following pseudocode segments each print values. Write down the values that are printed.

a.

```
a ← 2
b ← 3
for i from 1 to 2
    for j from 1 to 3
        if a is even
            b ← b + j
            a ← a + 1
        else
            a ← a + 2
        end if
    end for
end for
print(a, b)
```

$a = 13, b = 4$

Outer i	Inner j	a before	a even?	Action Taken	a after	b after
1	1	2	Yes	$b \leftarrow b+1=4, a \leftarrow a+1=3$	3	4
1	2	3	No	$a \leftarrow a+2=5$	5	4
1	3	5	No	$a \leftarrow a+2=7$	7	4
2	1	7	No	$a \leftarrow a+2=9$	9	4
2	2	9	No	$a \leftarrow a+2=11$	11	4
2	3	11	No	$a \leftarrow a+2=13$	13	4

b.
$x \leftarrow 5$
$y \leftarrow 12$
while $y > 6$
  for j **from** 1 **to** 3
    if x is even
      $y \leftarrow y - 2$
    else
      $x \leftarrow x + 1$
    end if
  end for
end while
print(x, y)

$x = 6, y = 2$

✅ **First `while` iteration (y = 12):**

j	x	Even?	Action	New x	New y
1	5	No	$x \leftarrow x + 1 = 6$	6	12
2	6	Yes	$y \leftarrow y - 2 = 10$	6	10
3	6	Yes	$y \leftarrow y - 2 = 8$	6	8

✅ **Second `while` iteration (y = 8):**

j	x	Even?	Action	New x	New y
1	6	Yes	$y \leftarrow y - 2 = 6$	6	6
2	6	Yes	$y \leftarrow y - 2 = 4$	6	4
3	6	Yes	$y \leftarrow y - 2 = 2$	6	2

c.
$p \leftarrow 1$
$q \leftarrow 10$
for j **from** 1 **to** 4
  for k **from** 1 **to** 2
    if q is even
      $p \leftarrow p + k$
      $q \leftarrow q - 2$
    else
      $q \leftarrow q - 1$
    end if
  end for
end for
print(p, q)

$p = 13, q = -6$

j	k	q before	Even?	Action	p after	q after
1	1	10	Yes	$p \leftarrow p+1=2, q \leftarrow q-2=8$	2	8
1	2	8	Yes	$p \leftarrow p+2=4, q \leftarrow q-2=6$	4	6
2	1	6	Yes	$p \leftarrow p+1=5, q \leftarrow q-2=4$	5	4
2	2	4	Yes	$p \leftarrow p+2=7, q \leftarrow q-2=2$	7	2
3	1	2	Yes	$p \leftarrow p+1=8, q \leftarrow q-2=0$	8	0
3	2	0	Yes	$p \leftarrow p+2=10, q \leftarrow q-2=-2$	10	-2
4	1	-2	Yes	$p \leftarrow p+1=11, q \leftarrow q-2=-4$	11	-4
4	2	-4	Yes	$p \leftarrow p+2=13, q \leftarrow q-2=-6$	13	-6

Space for Personal Notes

Sub-Section [2.7.1] : Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Question 11

An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n)$:
$df(x)$ ← the derivative of $f(x)$
for i **from** 1 **to** n **do**
  if $df(x_0) = 0$ **then**
    return "Error: Division by zero"
  else
    $x_0 \leftarrow x_0 - \frac{f(x_0)}{df(x_0)}$
  end if
end while
return x_0

Consider calling the function newton $(x^2 - 5, 2, 5)$.

a. How many iterations are performed?

5

b. What is the final value of x_0. Give your answer correct to four decimal places.

2.2361

Iteration	x_n	x_{n+1}	$\lvert x_{n-1} - x_n \rvert$
0	2.	2.25	0.25
1	2.25	2.23611	0.0138889
2	2.23611	2.23607	0.0000431332
3	2.23607	2.23607	4.16014×10^{-10}
4	2.23607	2.23607	0.

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Question 12

An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n, tol)$:
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider caling the function newton $(x^3 - x^2 + 5, 2, 30, 0.0001)$.

a. Find the final return value. Give your answer correct to four decimal places.

-1.4334

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.	0.875	1.125
1	0.875	-8.09286	8.96786
2	-8.09286	-5.31609	2.77676
3	-5.31609	-3.49774	1.81836
4	-3.49774	-2.35292	1.14481
5	-2.35292	-1.71662	0.636306
6	-1.71662	-1.47176	0.244855
7	-1.47176	-1.43426	0.0374961
8	-1.43426	-1.43343	0.000836399
9	-1.43343	-1.43343	4.10702×10^{-7}

b. State the value of i when the algorithm terminates.

$i = 9$

Question 13

An implementation of Newton's method is shown below.

define newton$(f(x), x_0, n, tol)$:
$df(x) \leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider calling the function.

a. Find the final return value of newton $(x^3 - 2x^2 + 4, 1, 2, 0.001)$.

2.875

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.	4.	3.
1	4.	2.875	1.125
2	2.875	2.03026	0.844741

b. The function newton $\left(x(x+7)(x-7), \frac{7}{\sqrt{5}}, 5001, 0.0001\right)$ is called. Find the final return value.

Oscillating sequence. We return $x_{5001} = -\frac{7}{\sqrt{5}}$

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