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VCE Mathematical Methods ¾
Pseudocode & its Exam Skills [2.7]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
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Section A: Compulsory Questions

Sub-Section [2.7.1]: Evaluate Pseudocode with Conditional Statements and Loops

Question 1

The following pseudocode segments each print a value. Write down the value that is printed.

a. $x \leftarrow 5$
if $x > 3$
  $x \leftarrow x + 2$
end if
print(x)

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b. $y \leftarrow 10$
if $y < 5$
  $y \leftarrow y - 3$
else
  $y \leftarrow y + 4$
end if
print (y)

---

c. $z \leftarrow 1$
while $z < 4$
    $z \leftarrow z + 1$
end while
print (z)

Question 2

The following pseudocode segments each print a value. Write down the value that is printed.

a.
$a \leftarrow 3$
for i **from** 1 **to** 3
  if $a \leq 5$
    $a \leftarrow a + 2$
  end if
end for
print (a)

---

b.
$b \leftarrow 6$
while $b > 2$
  if b is even
    $b \leftarrow b - 3$
  else
    $b \leftarrow b - 1$
  end if
end while
print (b)

---

c. $c \leftarrow 1$
for $j \leftarrow 1$ **to** 4
    if j is even
        $c \leftarrow c + 3$
    else
        $c \leftarrow c + 1$
    end if
end for
print (c)

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Question 3

The following pseudocode segments each print values. Write down the values that are printed.

a.

```
a ← 1
b ← 2
for i from 1 to 3
    for j from 1 to 2
        if a is even
            b ← b + a
        else
            a ← a + j
        end if
    end for
end for
print (a, b)
```

b.

```
x ← 3
y ← 10
while y > 5
    for j from 1 to 2
        if x is even
            y ← y − j
        else
            x ← x + 1
        end if
    end for
end while
print (x, y)
```

c.

$p \leftarrow 2$
$q \leftarrow 8$
for j **from** 1 **to** 3
  for k **from** 1 **to** 2
    if q is even
      $p \leftarrow p + k$
      $q \leftarrow q - 1$
    else
      $q \leftarrow q - p$
    end if
  end for
end for
print (p, q)

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Sub-Section [2.7.2]: Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Question 4

An implementation of Newton's method is shown below.

```
define newton(f(x), x_0, n):
df(x) ← the derivative of f(x)
    for i from 1 to n do
        if df(x_0) = 0 then
            return "Error: Division by zero"
        else
            x_0 ← x_0 − f(x_0)/df(x_0)
        end if
    end while
    return x_0
```

Consider calling the function newton$(x^2 - 3, 2, 3)$.

a. How many iterations are performed?

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b. What is the final value of x_0. Give your answer correct to three decimal places.

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Question 5

An implementation of Newton's method is shown below.

```
define newton(f(x), x_0, n, tol):
df(x) ← the derivative of f(x)
i ← 0
x_n ← x_0
while i < n do
    if df(x_n) = 0 then
        return "Error: Division by zero"
    x_{n+1} ← x_n − f(x_n)/df(x_n)
    if −tol < x_{n+1} − x_n < tol then
        return x_{n+1}
    x_n ← x_n + 1
    i ← i + 1
end while
return x_n
```

Consider calling the function newton $(x^3 - x^2 + 1, 2, 30, 0.001)$.

a. Find the final return value. Give your answer correct to four decimal places.

b. State the value of i when the algorithm terminates.

Question 6

An implementation of Newton's method is shown below.

```
define newton(f(x), x_0, n, tol):
df(x) ← the derivative of f(x)
i ← 0
x_n ← x_0
while i < n do
    if df(x_n) = 0 then
      return "Error: Division by zero"
    x_{n+1} ← x_n − f(x_n)/df(x_n)
    if −tol < x_{n+1} − x_n < tol then
      return x_{n+1}
    x_n ← x_{n+1}
    i ← i + 1
end while
return x_n
```

Consider calling the function.

a. Find the final return value of newton $(x^3 - 5x^2 + 1, 1, 2, 0.01)$.

b. The function newton $\left(x(x+2)(x-2), \frac{2}{\sqrt{5}}, 2000, 0.0001\right)$ is called. Find the final return value.

Sub-Section: The 'Final Boss'

Question 7

a. Consider the pseudocode shown below. Determine the final values of m and n.

```
m ← 2
n ← 3
for i from 1 to 3
    for j from 1 to 2
        if m + n > 5
            m ← m + j
            n ← n − 1
        else
            m ← m + 2
            n ← n + 1
        end if
    end for
end for
print(m, n)
```

b. An implementation of Newton's method is shown below.

```
define newton(f(x), x0, n):
df(x) ← the derivative of f(x)
    for i from 1 to n do
      if df(x0) = 0 then
        return "Error: Division by zero"
      else
        x0 ← x0 − f(x0)/df(x0)
      end if
    end while
    return x0
```

The call newton$(\sin(x), x_0, 10)$ is made.

State all values of x_0 for which the function returns an error.

c. The bisection algorithm is a method for approximating solutions to the equation $f(x) = 0$. As inputs, the algorithm takes a function f, an interval $[a, b]$ which should contain a root of f, and a tolerance ϵ, which the approximate solution is within of the exact solution. Pseudocode is shown below:

```
define Bisection (f, a, b, ϵ)
    if f(a) × f(b) ≥ 0
        print("Bisection method fails.")
        return None
    end if
    while |b − a| > 2ϵ
        m ← (a + b)/2
        if f(m) = 0
            return m
        end if

        if f(a) × f(m) < 0
            b ← m
        else
            a ← m
        end if
end while

return (a + b)/2
```

i. What happens when we call Bisection $(x^3 + 3x + 5, -4, -2, 0.01)$.

ii. Find the return value of Bisection $(x^3 + 3x + 5, -2, -1, 0.01)$.

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Section B: Supplementary Questions

Question 8

The following pseudocode segments each print a value. Write down the value that is printed.

a.

$x \leftarrow 8$
if $x > 5$
  $x \leftarrow x - 3$
end if
print(x)

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b.

$y \leftarrow 4$
if $y < 3$
  $y \leftarrow y + 6$
else
  $y \leftarrow y - 1$
end if
print (y)

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c.

$z \leftarrow 2$
while $z < 6$
  $z \leftarrow z + 2$
end while
print (z)

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Question 9

The following pseudocode segments each print a value. Write down the value that is printed.

a.

```
a ← 2
for i ← 1 to 4
    if a is even
        a ← a + 3
    else
        a ← a + 2
    end if
end for
print(a)
```

b.

```
b ← 10
while b > 4
    if b is even
        b ← b − 2
    else
        b ← b − 3
    end if
end while
print(b)
```

c. $c \leftarrow 1$
for $j \leftarrow 1$ **to** 5
  if j is even
    $c \leftarrow c + 2$
  else
    $c \leftarrow c + 4$
  end if
end for
print(c)

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Question 10

The following pseudocode segments each print values. Write down the values that are printed.

a.

```
a ← 2
b ← 3
for i from 1 to 2
    for j from 1 to 3
        if a is even
            b ← b + j
            a ← a + 1
        else
            a ← a + 2
        end if
    end for
end for
print(a, b)
```

b.

```
x ← 5
y ← 12
while y > 6
    for j from 1 to 3
        if x is even
          y ← y − 2
        else
          x ← x + 1
        end if
    end for
end while
print(x, y)
```

c. $p \leftarrow 1$
$q \leftarrow 10$
for j **from** 1 **to** 4
  for k **from** 1 **to** 2
    if q is even
      $p \leftarrow p + k$
      $q \leftarrow q - 2$
    else
      $q \leftarrow q - 1$
    end if
  end for
end for
print(p, q)

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Sub-Section [2.7.1]: Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Question 11

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n$):
$df(x)$ ← the derivative of $f(x)$
for i **from** 1 **to** n **do**
  if $df(x_0) = 0$ **then**
    return "Error: Division by zero"
  else
    $x_0 \leftarrow x_0 - \frac{f(x_0)}{df(x_0)}$
  end if
end while
return x_0

Consider calling the function newton $(x^2 - 5, 2, 5)$.

a. How many iterations are performed?

b. What is the final value of x_0. Give your answer correct to four decimal places.

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Question 12

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n, tol$):
$df(x)$ ← the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider caling the function newton $(x^3 - x^2 + 5, 2, 30, 0.0001)$.

a. Find the final return value. Give your answer correct to four decimal places.

b. State the value of i when the algorithm terminates.

Question 13

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n, tol$):
$df(x)$ $\leftarrow$ the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider calling the function.

a. Find the final return value of newton $(x^3 - 2x^2 + 4, 1, 2, 0.001)$.

b. The function newton $\left(x(x+7)(x-7), \frac{7}{\sqrt{5}}, 5001, 0.0001\right)$ is called. Find the final return value.