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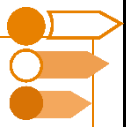
VCE Mathematical Methods ¾
Family of Functions and its Exam Skills [2.6]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 19

Section A: Compulsory Questions



Sub-Section [2.6.1] : Applying Family of Functions

Question 1

Consider the following family of functions $f(x) = e^{ax} - 1, a > 0$.

a. Identify the "surname" (common aspect(s) of the family) and the "first name" (unique aspect(s) of the family).

b. Hence, state what happens to the graph of f in terms of a transformation when the value of a increases.

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Question 2

Consider the following family of functions $f(x) = (x-2)^2 + k, k \in \mathbb{R}$.

a. Show that the graph of f always has a stationary point at $x = 2$ and find the nature of this stationary point.

b. Hence, identify the "surname(s)" and "first name(s)" of the family.

Question 3

Consider the family of functions $f(x) = \sin\left(kx + \frac{\pi}{2}\right), k \in \mathbb{R}$.

a. Identify the effects of k on the graph.

b. Hence, identify the "surname" and "first name(s)" of the family.

c. Express $f(x)$ without using sin when $k = \pm 1$.

Hint: List out the transformations and sketch the resulting graph if you get stuck!

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Question 4 Tech-Active.

Consider the family of functions $f(x) = e^{ax} - ax + 1, a \in \mathbb{R}^+$.

a. State one transformation that maps the graph of $g(x) = e^x - x + 1$ onto the graph of $f(x)$.

b. Identify a "surname" of the family.

c. Describe what happens to the shape of $f(x)$ as a increases.

d. By plotting $h(x) = f(x) - e^{ax}$ on the same axes as $f(x)$, state the equation of the asymptote of $f(x)$.

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Sub-Section [2.6.2] : Finding Unknowns for a Certain Number of Intersections

Question 5

Find the value of a where $a \in \mathbb{R}$ such that the graph of $f(x) = e^x + a$ intersects the line $y = x$ exactly once.

Question 6

Consider the functions $f: \mathbb{R} \to \mathbb{R}, f(x) = e^x + a$ where $a \in \mathbb{R}$ and $g(x) = x - 3$.

a. Find the inverse function of $f(x)$.

b. Find the value of a such that the graph of $f^{-1}(x)$ intersects with $g(x)$ exactly once.

Question 7

Consider the functions $f: [-4,4) \to \mathbb{R}, f(x) = x^2 + 1$ and $g(x) = mx, m \in \mathbb{R}$. Find the value(s) of m where f and g intersect exactly once.

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Question 8 Tech-Active.

Consider the function $f: [k, \infty) \to \mathbb{R}, f(x) = (x - k)^2 + 4$. Find the value(s) of k such that $f(x)$ and its inverse intersect exactly once.

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Sub-Section [2.6.3] : Finding Unknowns for Maximums and Minimums

NOTE: This entire section can be done tech-active.

Question 9

For what value of $k \in \mathbb{R}$, will the function, $f(x) = 2x + e^{kx}$ have a minimum on the x-axis?

Question 10

For what value(s) of $k \in \mathbb{R}$ will the function $f(x) = (x - k)^2 \log_e(x)$ have a minimum at $x = 4$?

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Question 11

Find the value(s) of $k \in \mathbb{R}$ such that the minimum of the function $f: [-4,10] \to \mathbb{R}, f(x) = 3\left(\frac{x}{2} - 3k\right)^2 - 6$ occurs at $x = -4$.

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Sub-Section: Exam 1 Questions

Question 12

Consider the function $f: [a, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + k, k \in \mathbb{R}$, where a is a real constant that ensures f is a one-to-one function.

a. Find the smallest possible value of a.

b. Find the value of k such that $y = 4x$ is a tangent to the graph of f.

c. Find the value(s) of k such that the graphs of f and f^{-1} do not intersect.

Question 13

Consider the function $f(x) = ae^x + k$, where $a \in \mathbb{R}^+$ and $k \in \mathbb{R}$.

Let the point P be the y-intercept of the graph of $f(x)$.

a. State the coordinates of the point P in terms of a and k.

b. Find the gradient of f at P in terms of a.

c. Given that the graph of $y = f(x)$ passes through the origin, express k in terms of a.

d. Given that the graph of f also goes through the point $(1,3(e-1))$, find the values of a and k.

Question 14

Consider the function $f(x) = x^2 + ax + b$.

Find the value(s) of $a, b \in \mathbb{R}$ such that $f(x)$ has a turning point at $(1,4)$.

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Sub-Section: Exam 2 Questions

Question 15

The graph of $y = kx + 2$ intersects the graph of $y = 2x - 3x^2$ at 2 distinct points for:

A. $k = 2$

B. $k \in [2 - 2\sqrt{6}, 2 + 2\sqrt{6}]$

C. $k \in (-\infty, 2 - 2\sqrt{6}) \cup (2 + 2\sqrt{6}, \infty)$

D. $k \in (2 - 2\sqrt{6}, 2 + 2\sqrt{6})$

Question 16

The graph with rule $f(x) = x^3 - 3x^2 + c, c \in \mathbb{R}$, has 3 distinct x-intercepts.

The set of all possible values of c is:

A. $\mathbb{R}^+$

B. $[0,4]$

C. $(-4,0)$

D. $(0,4)$

Question 17

For the parabola with the equation $y = ax^2 - 2bx + c$, the equation of the axis of symmetry is:

A. $x = \frac{b}{a}$

B. $x = -\frac{b}{a}$

C. $y = c$

D. $x = \frac{2b}{a}$

Question 18

The largest value of a such that the function $f: (-\infty, a] \to \mathbb{R}, f(x) = x^2 - 5x + 6$, where f is one-to-one, is:

A. 2.5

B. 6

C. -2.5

D. -5

Question 19

The function $f(x) = \frac{1}{3}x^3 + mx^2 + nx + p$, for $m, n, p \in \mathbb{R}$, has turning points at $x = -3$ and $x = 1$ and passes through the point (3,4).

The values of m, n and p are:

A. $m = 0, n = -\frac{7}{3}, p = 2$

B. $m = 1, n = -3, p = -5$

C. $m = -1, n = -3, p = 13$

D. $m = \frac{5}{2}, n = 6, p = -\frac{83}{4}$

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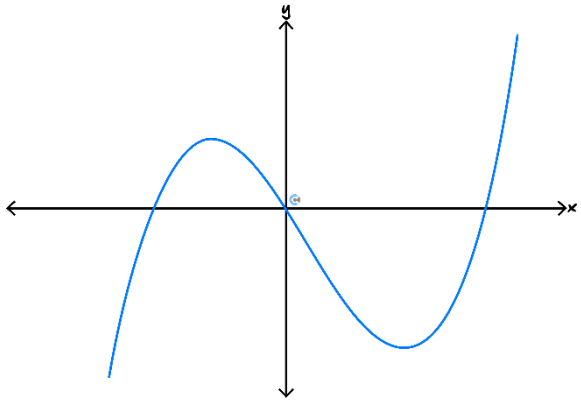
Question 20

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = x(x-3)(x+2)$. Part of the graph of f is shown below:



Consider the point $A\ (a, f(a))$.

a. State the gradient of the graph of f at the point A in terms of a.

b. Hence, find the equation of the tangent to the graph of f at point A in the form $y = mx + c$.

c. Find the value(s) of a where the tangent to f at A intersects the graph of f once.

d. Let $h: \mathbb{R} \to \mathbb{R}, h(x) = (x-a)(x-b)^2$, where $h(x) = f(x) + k$ and $a, b, k \in \mathbb{R}$.

Find the possible values of a and b.

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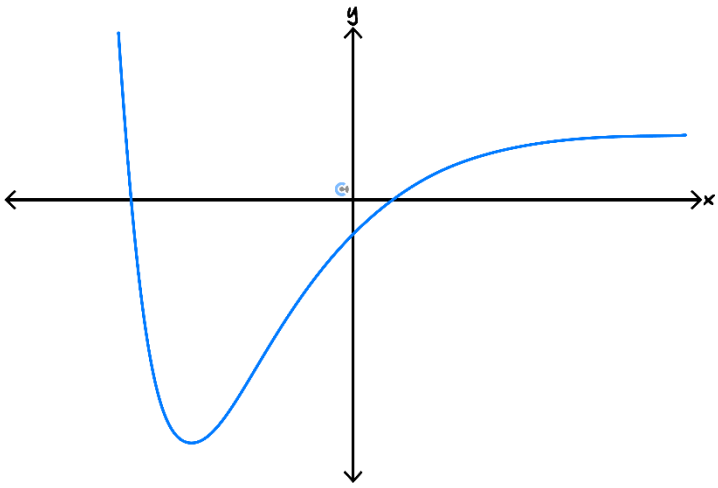
Question 21

Consider the function $p(x) = e^{-3x} - 4e^{-2x} + 2$.

a. Explain why $p(x)$ is not a one-to-one function.

The diagram below shows part of the graph of p.



b. Find the gradient of the tangent to the graph of p at $x = a$.

c. Find the value(s) of a for which the tangent to f at $x = a$ intersects p only at $x = a$.

d. Find the smallest value of $q \in \mathbb{R}$, such that $f: [q, \infty) \to \mathbb{R}, f(x) = p(x)$ is a one-to-one function.

e. Consider the function $g: [-2,2] \to \mathbb{R}, g(x) = p(x + k)$.

i. Find the value(s) of k such that the minimum of p occurs at $x = -1$.

ii. Find the value(s) of k such that the minimum of p occurs at $x = -2$.

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