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VCE Mathematical Methods ¾

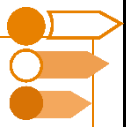
Applications of Differentiation Exam Skills [2.5]

Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 - Pg 24
Supplementary Questions	Pg 25 - Pg 49

Section A: Compulsory Questions



Sub-Section [2.5.1]: Advanced Tangents and Normal Questions

Question 1

a. Find the equation of the tangent to the function $f(x) = x^2 - 3x + 1$ that is parallel to the line $y = 4 - x$.

b. Find the equation of the tangent to the function $f(x) = x^2 - 4x + 5$, that is perpendicular to the line $y = \frac{1}{2}x + 1$.

c. Find the equation of a tangent to the function $f(x) = \frac{1}{x}$, that makes an angle of $135°$ with the positive x-axis.

Question 2

a. Find the equation of the tangent to the function $f(x) = \frac{3}{x}$ that is parallel to the tangent of f when $x = 1$.

b. Find the equation of the tangent to the function $f(x) = \frac{1}{3}x^3 - x$, that is parallel to the tangent of f when $x = -2$.

c. Find the equation of a tangent to the function $f(x) = \frac{1}{x}$, that passes through the point $(0,4)$.

Question 3

a. A tangent to $f(x) = 2x^3 - 5x$ makes an angle of $45°$ with the positive x-axis and passes through the point $(-2, b)$, where $b > 0$. Find the value of b.

b. A tangent to the function $f(x) = \sin(2x) + 1$, makes an angle of $60°$, with the positive x-axis and passes through the point $\left(0, \frac{3}{2} - \frac{\pi}{4\sqrt{3}}\right)$. Determine the point on f that this tangent is drawn to.

c. Find the equation of a tangent to the function $f(x) = x^2 - 5x + 5$, that makes an angle of 15° with the line $y = \sqrt{3}x$ on the side of x-axis.

Question 4 Tech-Active.

a. Find the equation of the tangent to $f(x) = 3x^2 + 2x - 1$ when $x = 2$.

b. Find the equation of the normal to $y = x^3 - 2x + 4$ when $x = 1$.

c. Find the equation of the normal to the function $f(x) = x^2 + 2x + 3$ that passes through the point $(9, 4)$.

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Sub-Section [2.5.2]: Advanced Maximum / Minimum Questions

Question 5

a. Find two positive numbers that sum to 8 and such that the sum of the cube of the first and the square of the second is as small as possible.

b. Find the maximum area of a pig pen that can be enclosed by 32 metres of fencing.

c. Find the minimum distance from the origin to a point on the line $y = 6 - x$.

Question 6

a. Water is being poured into a container. The volume of the containers at time t seconds, in mL, is given by:

$$V(t) = 8t^2 - \frac{1}{3}t^3, 0 \leq t \leq 16$$

At what time is the rate of increase in volume the greatest, and what is this rate of increase?

b. Part of a roller coaster track can be described by the rule $y = 12\cos\left(\frac{\pi x}{40}\right) + 10, x \in [0, 40]$.

State the coordinates of the point on the track for which the **magnitude** of the gradient is maximum.

c. The number $N(t)$ of bugs on an island at time t is given by $N(t) = 30te^{-t/10}$. Find the maximum rate of increase in the population and the time at which this occurs.

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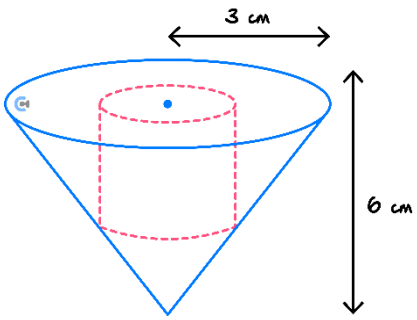
Question 7

A cylinder fits exactly in a right circular cone so that the base of the cone and one end of the cylinder are in the same plane, as shown in the diagram below.



a. Show that $h = 6 - 2r$.

b. The surface area of a cylinder of height h and radius r is given by $S = 2\pi rh + 2\pi r^2$. Find the value of r for which S is the maximum.

Sub-Section: Exam 1 Questions

Question 8

Let $h(x) = f(x)\sin(x)$. It is known that $f\left(\frac{\pi}{3}\right) = a$ and $f'\left(\frac{\pi}{3}\right) = b$.

a. Express $h'\left(\frac{\pi}{3}\right)$ in terms of a and b.

b. Find the exact value of $h'\left(\frac{\pi}{3}\right)$ if $f(x) = \log_e(x)$.

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Question 9

The line $y = 4x - 1$ is tangent to the curve $y = x^4 + c$. Find the exact value of c.

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Question 10

Consider the family of parabolas that pass through the point $(1,1)$ with gradient 2 at this point.

a. Find the general rule for the family of parabolas in the form $y = ax^2 + bx + c$, expressing a and b in terms of c.

b. Find the rule for the particular member of this family of parabolas that also passes through the point $(0,1)$.

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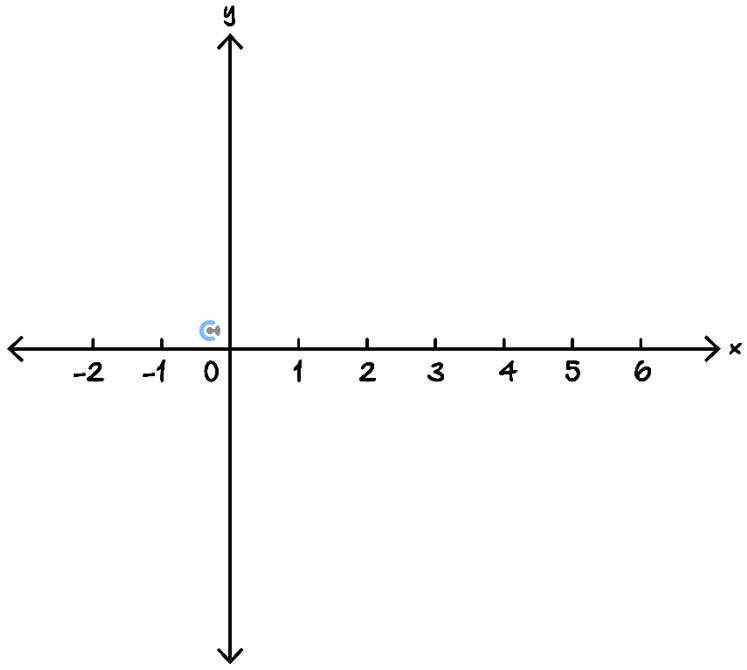
Question 11

On the set of axes provided below, sketch a continuous curve with equation $y = f(x)$ having the following properties:

$$f(0) = 0 \qquad f'(0) = 0$$
$$f(4) = 0 \qquad f'(3) = 0$$
$$f'(x) < 0 \text{ for } \{x : x > 3\}$$
$$f'(x) > 0 \text{ for } \{x : x < 3\} \setminus \{0\}$$



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Question 12

A **normal** to the graph of $y = \sqrt{x}$ is parallel to the line $y = -4x$. Find the equation of the normal.

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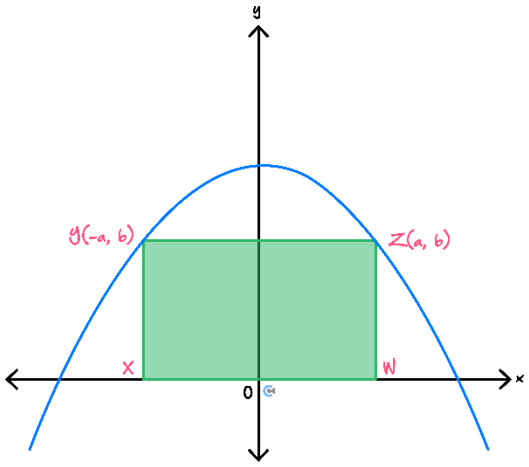
Question 13

A rectangle $XYZW$ has two vertices, X and W, on the x-axis and the other two vertices, Y and Z, on the graph of $y = 9 - 3x^2$. The coordinates of Z are (a, b), where a and b are positive real numbers.



a. Find the area, A, of rectangle $XYZW$ in terms of a.

b. Find the maximum value of A and the value of a for which this occurs.

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Sub-Section: Exam 2 Questions

Question 14

The gradient of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = \frac{3x}{x^2+5}$ is negative for:

A. $-\sqrt{5} < x < \sqrt{5}$

B. $x > 5$

C. $x < -\sqrt{5}$ and $x > \sqrt{5}$.

D. $x < 0$

Question 15

The function f is differentiable for all $x \in \mathbb{R}$ is and satisfies the following conditions:

- $f'(x) < 0$ where $x < -1$.
- $f'(x) = 0$ where $x = -1$.
- $f'(x) = 0$ where $x = 2$.
- $f'(x) > 0$ where $-1 < x < 2$.
- $f'(x) > 0$ where $x > 2$.

Which one of the following is true?

A. The graph of f has a local maximum point where $x = 2$.

B. The graph of f has a stationary point of inflection where $x = 2$.

C. The graph of f has a local minimum point where $x = 2$.

D. The graph of f has a stationary point of inflection where $x = -1$.

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Question 16

The normal to the curve with equation $y = x^{\frac{5}{2}} + x$ at the point $(4, 36)$ is parallel to the straight line with the equation:

A. $y + 21x = 7$

B. $y = \frac{x}{21} + 1$

C. $21y + x = 5$

D. $y + x = 21$

Question 17

Given that g is a differentiable function and k is a real number, the derivative of the composite function $g(e^{2kx})$ is:

A. $2kg'\left(e^{2kx}\right)e^{2kx}$

B. $2kg\left(e^{2kx}\right)$

C. $2ke^{2kx}g(e^{2kx})$

D. $2ke^{2kx}g'(e^{2x})$

Question 18

Newton's method is used to estimate the x-intercept of the function:

$$f : [0, \infty) \to \mathbb{R}, \quad f(x) = \log_e(2x + 3) - (6 - x^2)^{\frac{5}{2}}$$

With an initial estimate of $x_0 = 1$, the estimate for x_3 is closest to:

A. 2.1180

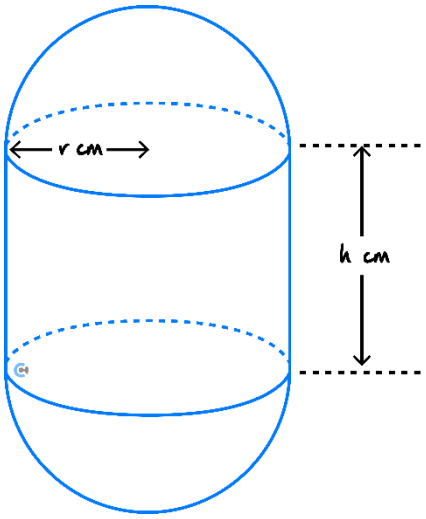
B. 1.9643

C. 2.1601

D. 2.1639

Question 19

Victoria Jones wants to construct a time capsule to bury some of her treasures. The time capsule will be a right circular cylinder of height $h\ cm$, and radius $r\ cm$, with hemispherical caps of radius $r\ cm$ on each end, as shown in the diagram.



Let the total volume of the capsule be $V\ cm^3$.

a. Express V in terms of r and h.

b. If the total volume of the capsule is $8000\ cm^3$, then it can be shown that:

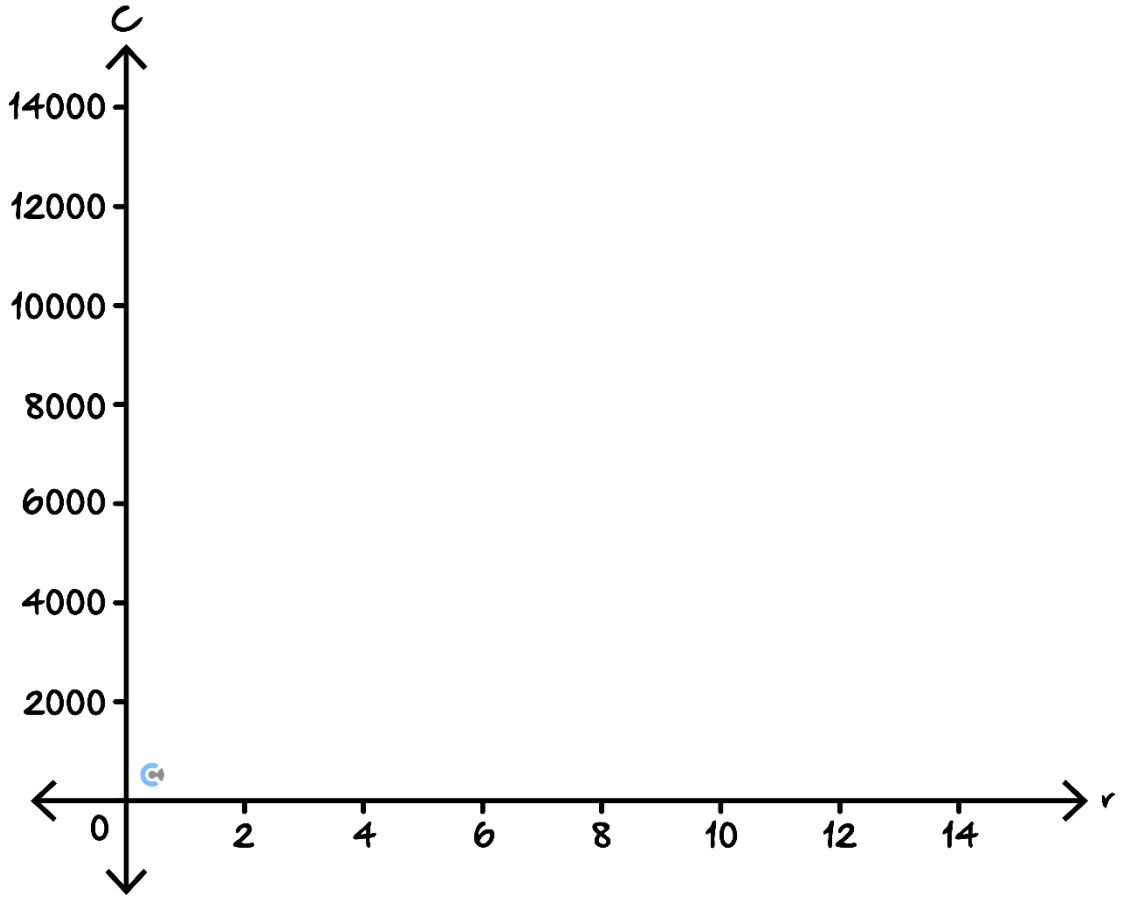
$$h = \frac{8000}{\pi r^2} - \frac{4r}{3}$$

The values that r may take lie in an interval. Find the exact end-points of this interval.

c. The material for the cylindrical part of the capsule costs 2 cents per cm^2 of surface. The material for the hemispherical caps costs 3 cents per cm^2 of surface. (The surface area of a sphere of radius r is $4\pi r^2$.)

Find an expression for C cents, the total cost of the materials for the capsule, in terms of r.

d. Sketch the graph of C over the appropriate domain on the axes below. Label any horizontal or vertical asymptotes with their equations and label the coordinates of the right endpoint correct to two decimal places.



e.

i. Find $\frac{dC}{dr}$ in terms of r.

_____

_____

ii. Hence, find the exact value of r for which C is a minimum. (You do not need to justify that the value you find is a minimum.)

_____

_____

iii. Find the minimum cost to the nearest dollar.

_____

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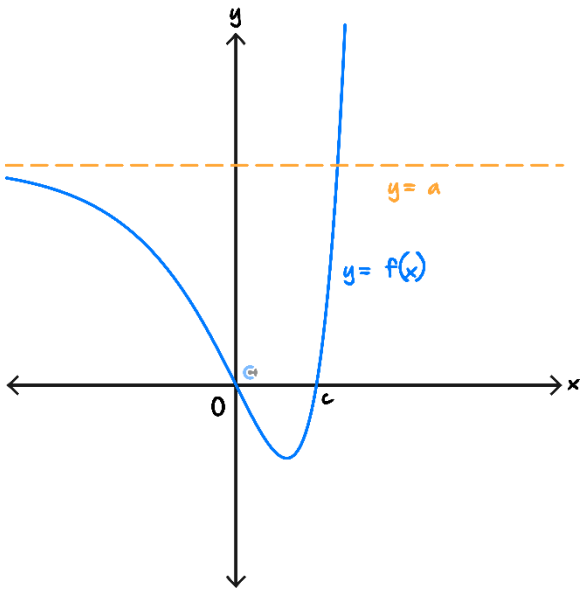
Question 20

The graph of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = e^{2x} - 4e^{x} + 3$.



The graph of f has a horizontal asymptote $y = a$.

a. Write down the exact value of a.

The graph of f passes through the origin and the point $(c, 0)$.

b. Find the exact value of c.

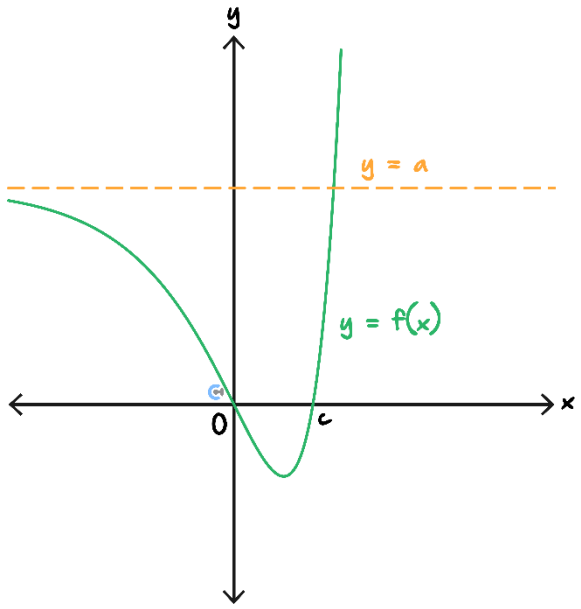
c. Find the exact values of the coordinates of the turning point of the graph of f.

d. Find the equation of the tangent to $y = f(x)$ that makes an angle of $135°$ with the positive x-axis and has a positive x-axis intercept.

Let g be the function whose graph is the reflection of the graph of f in the y-axis.

e.

i. Sketch the graph of g on the axes below.



ii. Write down the rule for g.

_____

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Consider the family of functions $f_k : \mathbb{R} \to \mathbb{R}, f_k(x) = e^{2x} - 2ke^x + 3$, where k is a real number.

i. For what values of k, will the graph of f_k have a turning point?

ii. Find the coordinates of this turning point when it exists.

iii. Find the value of k for which the graph of f_k touches the x-axis.

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Section B: Supplementary Questions

Sub-Section [2.5.1]: Advanced Tangents and Normal Questions

Question 21

a. Find the equation of the tangent to the function $f(x) = 2x^2 + 2x - 3$ that is parallel to the line $y = 3 - 2x$.

b. Find the equation of the tangent to the function $f(x) = 3x^2 - 13x + 8$, that is perpendicular to the line $y + x = 3$.

c. Find the equation of a tangent to the function $f(x) = e^x$, which makes an angle of 45° with the positive x-axis.

Question 22

a. Find the equation of the tangent to the function $f(x) = \frac{2}{x}$ that is parallel to the tangent of f when $x = -1$.

b. Find the equation of the tangent to the function $f(x) = \frac{1}{3}x^3 - 3x$, that is parallel to the tangent of f when $x = 3$.

c. Find the equation of the tangents to the curve $y = x^2 + 2x + 2$ that passes through the point $(-1, 0)$.

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Question 23

a. A tangent to $f(x) = \frac{x^3}{3} - x^2 - 2x + 4$ makes an angle of $45°$ with the positive x-axis and passes through the point $(3, b)$, where $b < 0$. Find the value of b.

b. A tangent to the function $f(x) = \cos(2x) + 2$, makes an angle of $120°$, with the positive x-axis and passes through the point $\left(0, \frac{\pi}{6} + \frac{5}{2\sqrt{3}}\right)$.

Determine the point on f that this tangent is drawn to.

c. Find the equation of a tangent to the function $f(x) = x^2 - 3x + 3$ that makes an angle of 15° with the line $\sqrt{3}y = x$. The slope of the tangent is greater than the slope of the line.

Question 24 Tech-Active.

a. Find the equation of the tangent to $f(x) = x^2 + 2x - 5$ when $x = 1$.

b. Find the equation of the normal to $y = 2x^3 - 3x + 1$ when $x = -2$.

c. Find the equation of the normal to the function $f(x) = x^2 - 2x - 1$ that passes through the point $(4, -2)$.

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Sub-Section [2.5.2]: Advanced Maximum/Minimum Questions

Question 25

a. The sum of three positive numbers is 26. The second number is 3 times as large as the first. If the sum of the squares of these numbers is minimum, find the numbers.

b. Find the maximum area of a field that can be enclosed by 40 metres of fencing.

c. Find the minimum distance from the origin to a point on the line $y = x - 3$.

Question 26

a. Water is being poured into a container. The volume of the container at time t seconds, in mL, is given by:

$$V(t) = 20t^2 - \frac{1}{3}t^3, 0 \leq t \leq 430$$

At what time is the rate of increase in volume the greatest, and what is this rate of increase?

b. Part of a roller coaster track can be described by the rule $y = 8\sin\left(\frac{\pi x}{20}\right) + 5, x \in [0, 40]$.

State the coordinates of the point on the track for which the magnitude of the gradient is maximum.

c. The population of foxes $P(t)$ on an island at time t in years is given by $P(t) = 20te^{-\frac{t}{2}}$. Find the maximum rate of increase in the population and the time at which this occurs.

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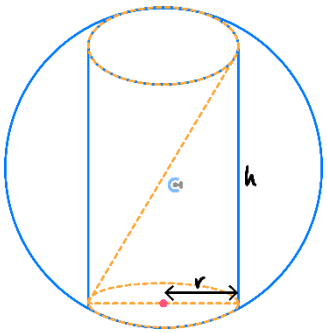
Question 27

A cylinder fits inside a sphere of radius $3\sqrt{3}\ cm$.



a. If the radius of the cylinder is r and the height of the cylinder is h, show that $r = \frac{1}{2}\sqrt{108 - h^2}$.

b. The volume of a cylinder is given by $V = \pi r^2 h$. Find the maximum volume of the cylinder.

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Sub-Section: Exam 1 Questions

Question 28

Let $h(x) = f(x)\tan(x)$. It is known that $f\left(\frac{\pi}{4}\right) = a$ and $f'\left(\frac{\pi}{4}\right) = b$.

a. Express $h'\left(\frac{\pi}{4}\right)$ in terms of a and b.

b. Find the exact value of $h'\left(\frac{\pi}{4}\right)$ if $f(x) = e^x$.

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Question 29

The line $y = 3x - 1$ is tangent to the curve $y = x^3 + c (c > 0)$. Find the exact value of c.

Question 30

Consider the family of parabolas that pass through the point $(2, 1)$ with gradient 4 at this point.

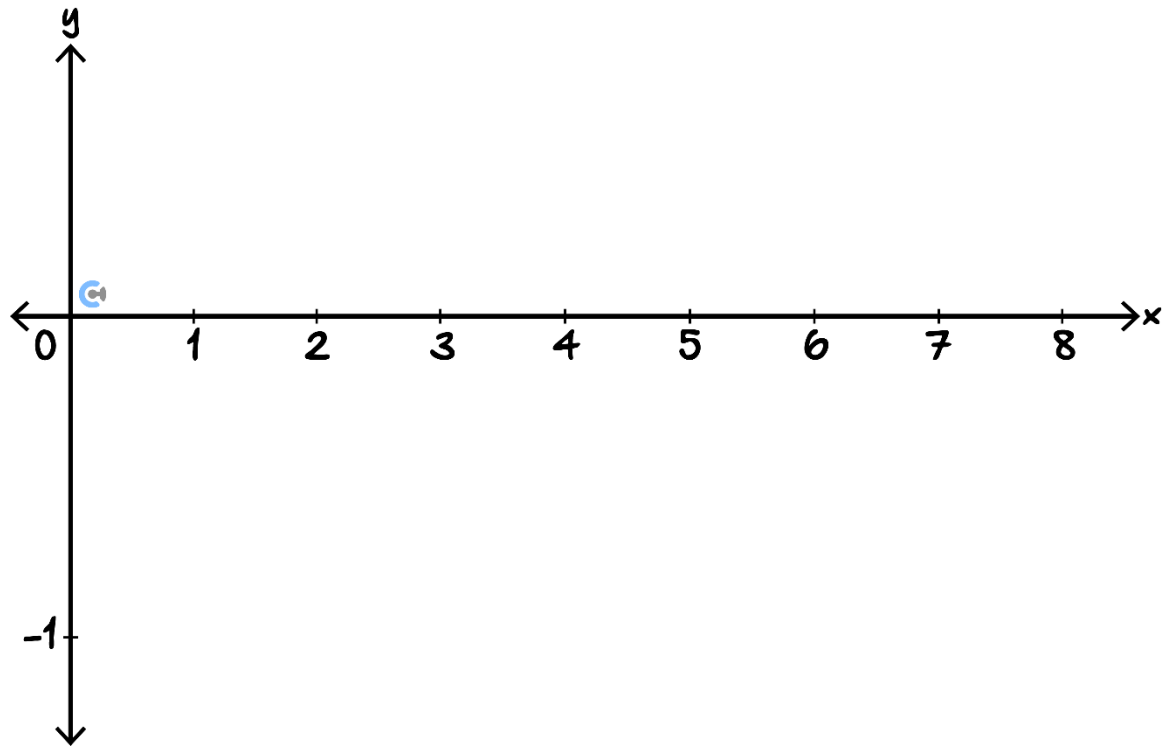
a. Find the general rule for the family of parabolas in the form $y = ax^2 + bx + c$, expressing a and b in terms of c.

b. Find the rule for the particular member of this family of parabolas that also passes through the point $(0, -3)$.

Question 31

Sketch a continuous curve with equation $y = f(x)$, on the set of axes provided below, having the following properties:

$$f(5) = 0 \text{ and } f(7) = 0$$
$$f'(5) = f'(6) = f'(7) = 0$$
$$f'(x) < 0 \text{ for } x \in (-\infty, 5) \cup (5, 6)$$
$$f'(x) > 0 \text{ for } x \in (6, 7) \cup (7, \infty)$$

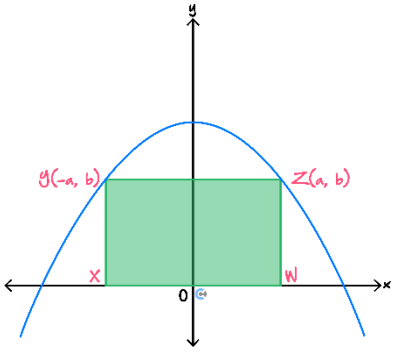


Question 32

A normal to the graph of $y = \sqrt{x-1}$ is parallel to the line $y = -2x + 1$. Find the equation of the normal.

Question 33

A rectangle $XYZW$ has two vertices, X and W, on the x-axis and the other two vertices, Y and Z, on the graph of $y = 25 - 5x^2$. The coordinates of Z are (a, b), where a and b are positive real numbers.



a. Find the area, A, of rectangle $XYZW$ in terms of a.

b. Find the maximum value of A and the value of a for which this occurs.

Space for Personal Notes

Sub-Section: Exam 2 Questions

Question 34

The gradient of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = \frac{x}{x^2+9}$ is positive for:

A. $-3 < x < 3$.

B. $x > 3$.

C. $x < -3$ and $x > 3$.

D. $-9 < x < 0$.

Question 35

The function g is differentiable for all $x \in R$ and satisfies the following conditions:

- $g'(x) > 0$ where $x < 0$.
- $g'(x) = 0$ where $x = 0$.
- $g'(x) < 0$ where $0 < x < 4$.
- $g'(x) = 0$ where $x = 4$.
- $g'(x) > 0$ where $x > 4$.

Which one of the following is true?

A. The graph of g has a local maximum at $x = 4$.

B. The graph of g has a local minimum at $x = 0$.

C. The graph of g has a stationary point of inflection at $x = 4$.

D. The graph of g has a local maximum at $x = 0$.

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Question 36

The normal to the curve $y = x^{\frac{3}{2}} + 2x$ at the point $(16, 40)$ is parallel to the straight line with the equation:

A. $8y + x = \frac{21}{4}$

B. $y = \frac{1}{8}x + 5$

C. $8y + x = 42$

D. $x + y = \frac{21}{4}$

Question 37

If f is a differentiable function and p is a real number, find the derivative of the function $f(\cos(a^2x))$:

A. $\mathrm{a}^2 \sin(a^2x)\, f'(\cos(a^2x))$

B. $\sin(a^2x)\, f'(\cos(x))$

C. $\mathrm{a}^2 \cos(a^2x)\, f'(\sin(a^2x))$

D. $-\mathrm{a}^2 \sin(a^2x)\, f'(\cos(a^2x))$

Question 38

Consider the function f given by:

$$f : [0, \infty) \to \mathbb{R}, f(x) = \log_e(5x - 4) - (9 - x^2)^{\frac{7}{2}}$$

If we use Newton's method to estimate the x-intercept of the function with an initial estimate of $x_0 = 1$, then the estimate for x_3 is closest to:

A. 2.4307

B. 2.1384

C. 2.6047

D. 2.7629

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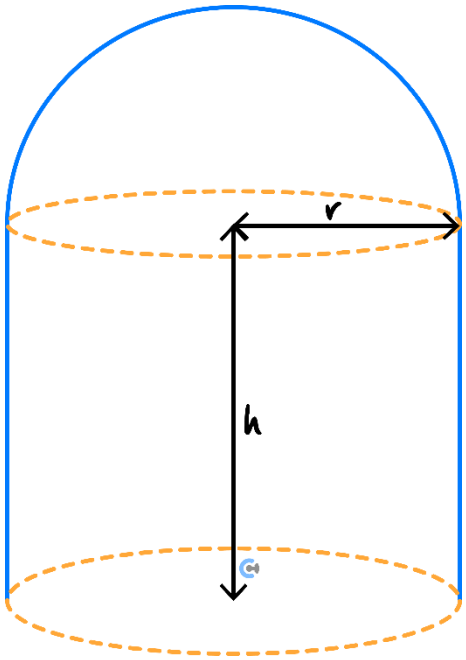
Question 39

A new toy is being manufactured for Christmas. The toy is in the form of a cylinder on top of a hemisphere as seen in the diagram below:



Let the total volume of the toy be $V\ cm^3$.

a. Express V in terms of r and h.

b. If the total volume of the toy is $4000\ cm^3$, then it can be shown that:

$$h = \frac{4000}{\pi r^2} - \frac{2}{3}r$$

The values that r may take lie in an interval. Find the exact end-points of this interval.

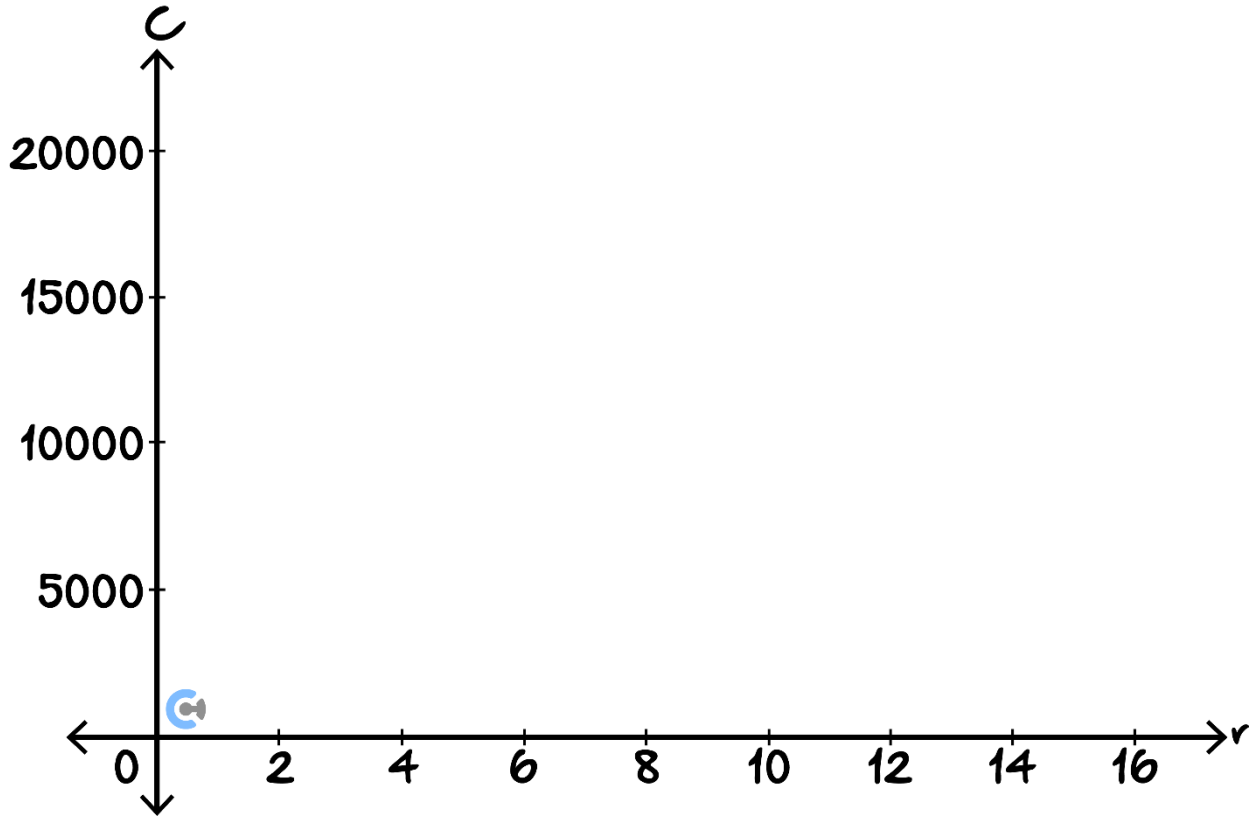
c. The material for the cylindrical part of the capsule costs 5 cents per cm^2 of surface.

The material for the hemispherical base costs 10 cents per cm^2 of surface.

(The surface area of a sphere of radius r is $4\pi r^2$.)

Find an expression for C cents, the total cost of the materials for the capsule, in terms of r.

d. Sketch the graph of C over the appropriate domain on the axes below. Label any horizontal or vertical asymptotes with their equations and label the coordinates of the right endpoint correct to two decimal places.



e.

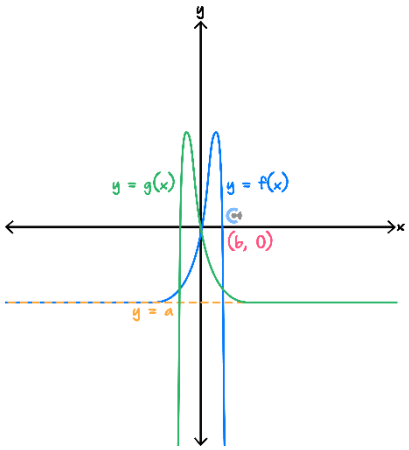
i. Find $\frac{dC}{dr}$ in terms of r.

ii. Hence, find the exact value of r for which C is a minimum. (You do not need to justify that the value you find is a minimum)

iii. Find the minimum cost to the nearest dollar.

Question 40

The graph of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = -e^{2x} + 8e^{x} - 7$.



The graph of f has a horizontal asymptote $y = a$.

a. State the exact value of a.

The function f passes through the origin and the other point $(b, 0)$.

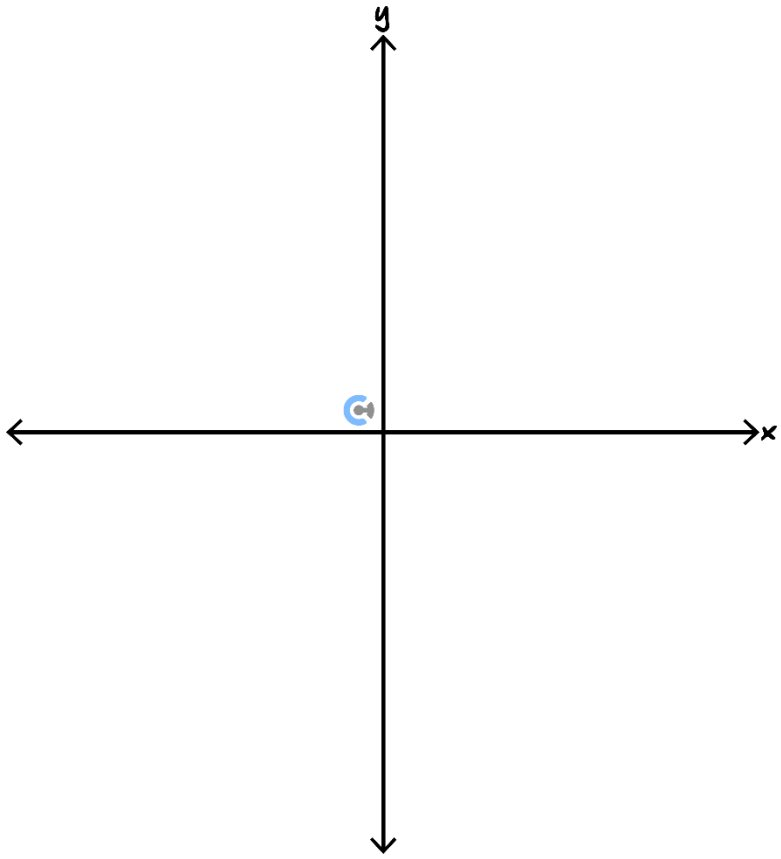
b. Determine the exact value of b.

c. Find the exact coordinates of the turning point of the function f.

d. Determine the equation of the tangent to the curve $y = f(x)$ that makes an angle of 60° with the positive x-axis and has a negative x-axis intercept.

e. Let g be the function obtained by reflecting the graph of f across the y-axis.

i. Sketch the graph of g on the axes below.



ii. Write down the rule for g.

f. Consider the family of functions defined as $f_k : R \to R$, where $f_k(x) = -e^{2x} + 2ke^x - 7$, where k is a real number.

iv. Identify which value of k will the graph of f_k as a turning point.

v. Determine the coordinates of this turning point when it exists.

vi. Find the value of k for which the graph of f_k touches the x-axis.

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- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➢ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➢ One active booking at a time (must attend before booking the next).	➢ Message +61 440 138 726 with questions. ➢ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726