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VCE Mathematical Methods ¾
Applications of Differentiation [2.4]
Test Solutions

25.5 Marks. 33 Minutes Writing.

Results:

Test Questions	____________ / 16.5
Extension Test Questions	____________ / 9

Section A: Test Questions (16.5 Marks)

INSTRUCTION: 16.5 Marks. 21 Minutes Writing.

Question 1 (3.5 marks)

Tick whether the following statements are **true** or **false**.

		True	**False**
a.	Tangents and normals are always perpendicular to each other if they are formed at the same point.	✓	
b.	To find the tangent line of $f(x)$ which passes through (a, b) we first need to solve $\frac{f(x)-b}{x-a} = f'(x)$ to identify where the tangent was made.	✓	
c.	To find the minimum or maximum of any function, we just need to find their stationary points.		✓
d.	For optimisation questions, it is important to construct the function for which we want to find the maximum or minimum.	✓	
e.	Newton's iterative formula is given by $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$.		✓
f.	The purpose of Newton's method is to approximate the solution to any equation.	✓	
g.	Terminating sequence for Newton's method occurs when the x-value of a stationary point is reached within the sequence.	✓	

Space for Personal Notes

Question 2 (2 marks)

Find the equation of the line that is normal to $y = x^3 - 2x^2 + 3x$ at $x = 1$.

```
In[135]:= f[x_] := x^3 - 2 x^2 + 3 x

In[136]:= -1/f'[1]

Out[136]= -1/2

In[139]:= y == -1/2 (x - 1) + f[1] // Expand

Out[139]= y == 5/2 - x/2
```

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Question 3 (3 marks)

Consider the function $f: (2, \infty) \to R, f(x) = \frac{3}{x-2}$. Find the equation(s) of the lines tangent to f, which are parallel to the line $y = -\frac{1}{3}x + 2$.

```
In[1]:= f[x_] := 3/(x-2)
        f'[x]
Out[2]= -3/(-2+x)^2

In[3]:= Solve[f'[x] == -1/3]
Out[3]= {{x -> -1}, {x -> 5}}

        y == -1/3 (x+1) + f[-1] // Expand (* Tangent at x = -1, parallel to the given line *)
Out[6]= y == -4/3 - x/3

        y == -1/3 (x-5) + f[5] // Expand (* Tangent at x = 5, parallel to the given line *)
Out[7]= y == 8/3 - x/3
```

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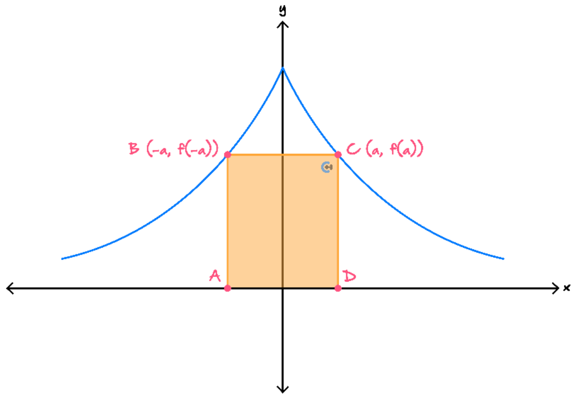
Question 4 (4 marks)

Consider the hybrid function $f(x) = \begin{cases} 2e^{x}, & -2 \le x < 0 \\ 2e^{-x}, & 0 \le x \le 2 \end{cases}$



A rectangle has vertices $ABCD$, as shown in the diagram below, with coordinates $A(-a, 0)$, $B(-a, f(-a))$, $C(a, f(a))$ and $D(a, 0)$, where $a > 0$.

a. Find the area A of rectangle $ABCD$ in terms of a. (1 mark)

By symmetry: $A = 2a \times f(a) = 4ae^{-a}$

b. Find the value of a of which A is the maximum. (2 marks)

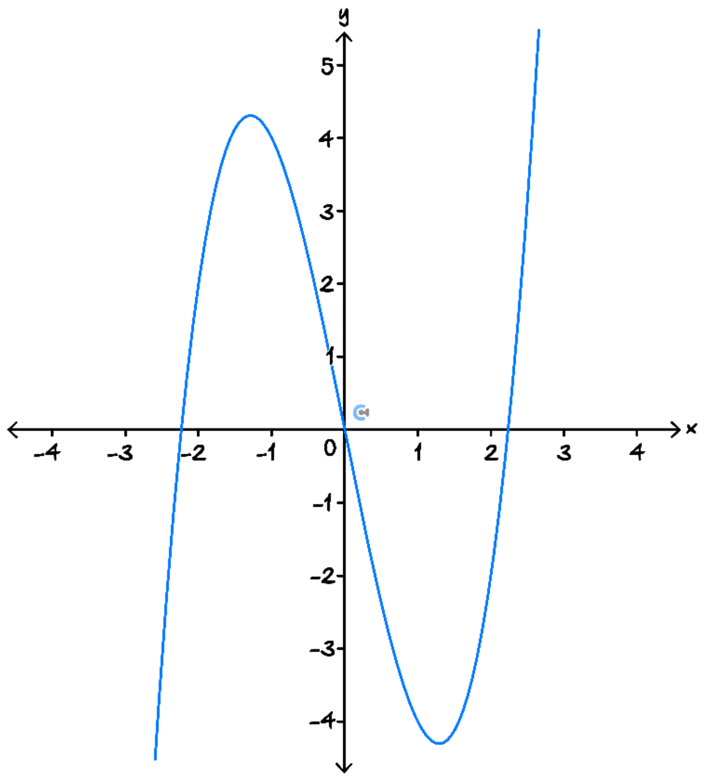
```
In[101]:= Solve[D[4 a e^-a, a] == 0, a]
Out[101]= {{a → 1}}
```

c. Hence, find the maximum area of $ABCD$. (1 mark)

```
In[102]:= 4 a e^-a /. a → 1
Out[102]= 4/e
```

Question 5 (4 marks)

Consider the function $f(x) = x^3 - 5x$ on the diagram below.



a. Find x_1 for $x_0 = -1$. (2 marks)

```
In[89]:= f[x_] := x^3 - 5 x
In[90]:= n[x_] := x - f[x]/f'[x]
In[91]:= n[x]
Out[91]= x - (-5 x + x^3)/(-5 + 3 x^2)
In[92]:= n[-1]
Out[92]= 1
```

b. Find x_2. (1 mark)

$$\boxed{-1}$$

c. What do you notice? (1 mark)

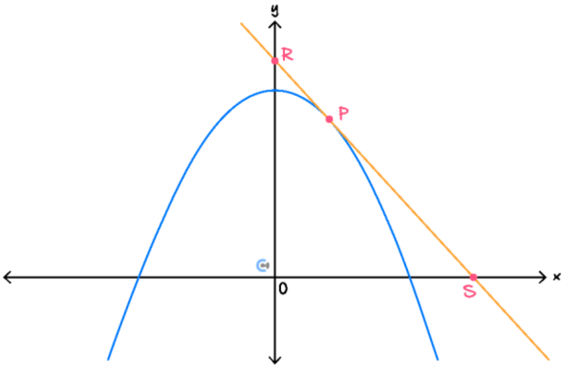
This is an oscillating sequence.

Section B: Extension Test Questions (9 Marks)

INSTRUCTION: 9 Marks. 2 Minutes Reading. 12 Minutes Writing.

Question 6 (9 marks) **Tech-Active.**

The diagram shows the graph of the function $f(x) = 9 - x^2$.



The graph of the tangent line to the curve at point $P\big(p, f(p)\big)$, where $1 \le p \le 3$ is also shown.

a. Determine the equation of this tangent line in terms of p. (1 mark)

$$-2\cdot p\cdot x+p^2+9$$

b. If the tangent line crosses the x-axis at the point S, and crosses the y-axis at the point R, find the coordinates of the points S and R in terms of p. (2 marks)

Define $t(x)=-2\cdot p\cdot x+p^2+9$	*Done*
$t(0)$	p^2+9
$\text{solve}(t(x)=0,x)$	$x=\dfrac{p^2+9}{2\cdot p}$

c. Hence, find the area A of the triangle OSR in terms of p. (1 mark)

$\frac{1}{2}\cdot(p^2+9)\cdot\frac{p^2+9}{2\cdot p}\to a(p)$	*Done*
$a(p)$	$\frac{(p^2+9)^2}{4\cdot p}$

d. Find the **minimum** area of the triangle OSR and the value of p for which the area is minimum. (3 marks)

$\text{fMin}(a(p),p,1,3)$	$p=\sqrt{3}$
$a(\sqrt{3})$	$12\cdot\sqrt{3}$

e. Find the **maximum** area of the triangle OSR and the value of p for which the area is maximum. (2 marks)

$\text{fMax}(a(p),p,1,3)$	$p=3$
$a(3)$	27

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